

Phenotypic Factor and Image Analysis of Type Traits of Holstein Cows in the Masstock Saudi Herds

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Abstract. Factor and image analysis were carried on 13,642 type traits records collected from Masstock dairy farms in Saudi Arabia. Principal component analysis showed that only 5 principal components (PC) were to be retained for factor analysis. These 5 principal components accounted for 59% of the total variance in all type traits. Factor 1 included udder traits, factor 2 included capacity traits, factor 3 included leg characters, factor 4 included angularity and rump angle, and factor 5 included teat characteristics. Factors computed from correlation matrix adjusted for discontinuity, or image correlation matrix, described similar type traits. Cows with well-developed fore and rear udders and with deep and strongly supported udders produced more milk than cows with opposite udder traits. Rear legs as viewed from the side and foot angle had no significant effect on milk production in first and second lactations. Pair-wise image correlation (r_i), correlation between the predictable part of the phenotypic value of type traits, was high between fore udder attachment and other udder characteristics ($r_i > .60$). Fore udder angle had a high image correlation (r_i) with rear udder width, udder support and udder depth ($r_i = .78, .73$ and $.73$ respectively).

Introduction

The recording system in some developing countries is still in the initial stage; thus, pedigree and progeny information is limited and has not yet formed the basis for estimating reliable genetic parameters. Therefore, phenotypic information, such as the correlation matrix, must be used to clarify phenotypically the relationship among type traits. Factor analysis is a statistical procedure that can be used to analyze type traits records to (1) find the relationships among traits, (2) partition each trait response into a covariant and a specific part, (3) summarize the information from the observed traits into a few unobserved derived factors.

Sieber *et al.* [1, 2] applied factor analysis on a phenotypic and genetic correlation matrix to analyze relationships among type traits. Ali [3] used factor analysis to

analyze simulated data on scored linear management traits. The primary defect of the model of comon factor analysis is that it fails to provide explicit definitions for the common and unique parts of variables. Therefore, Guttman [4] and Kaiser [5] provided an alternative determinate model, the image model, which retains many of the features of common factor analysis. Objectives of this study were to (1) investigate the potential of factor analysis and to explain linear scored type traits by a few unobserved derived factors and (2) use image analysis to investigate the relationships or covariances among the components of the observed linear traits.

Factor model

The factor model can be written as if the multivariate system consisted of $Z_{p \times 1}$ random vector whose coordinates are the observed values of the random traits Z_1, \dots, Z_p ; $X_{r \times 1}$ a random vector whose coordinates are (r) common factors X_1, \dots, X_r and $V_{p \times 1}$ a random vector whose coordinates are the(n) unique-factor variables V_1, \dots, V_p , Mulaik [6, pp. 191-197]. This model can be expressed in matrix form as:

$$Z = FX + UV$$

where:

$F_{p \times r}$ = matrix of factor pattern or loading.

$U_{p \times p}$ = diagonal matrix with the square root of the unique-factor variance on the main diagonal.

Assumptions

$$E(X) = E(Z) = E(V) = 0$$

$$E(ZZ') = R, E(XX') = R_{xx} \text{ and } E(VV') = 1$$

$$R_{xv} = R_{vx} = 0$$

The variance-covariance matrix of the observed type traits (Σ) can be partitioned such that:

$$\Sigma = FF' + U$$

where:

FF' = variance-covariance matrix of common variables.

whose diagonal-elements (C_i^2) are the communalities.

Correlation matrix (R) between traits can be partitioned such that:

$$R = FR_{xx}F' + U^2$$

R = correlation matrix of the observed type traits.

R_{xx} = correlation matrix of common factors.

Image model

Guttman [4] and Kaiser [7] developed the image analysis theory and explicitly defined 1) the components of observed value (P) which are the image (G) and the anti-image (V). Image is the predictable part of the phenotypic value, and anti-image is the unpredictable part; so $P = G + V$. 2) The matrix $W = (1 - R^{-1} D^2)$, where R is the correlation matrix among the observed traits, and $D^2 = (\text{diag } R^{-1})^{-1}$. The matrix W can be used to predict the image (G) from the phenotypic value of the trait (P). In other words, $G = W'P$; where W is a $p \times p$ multiple correlation weight matrix for predicting each linear score trait in p observed traits from p-1 random traits.

Materials and Methods

The data used in this study included two sets of records obtained from Masstock, one of the largest specialized dairy farms in Saudi Arabia, located in the central region of the Kingdom. The first set of records was the type traits file which has 13,642 records with the following information: farm number, herd book number, date of classification, date of calving (last calving date before classification), lactation number, stature, chest width, body depth, angularity, rump angle, rump width, rear legs side view, rear legs rear view, foot angle, fore udder attachment, fore udder angle, rear udder width, udder support, udder depth, teat placement rear view, teat placement side view, and teat length. The 17 biological traits were assessed using a nine-point scale, where 1 and 9 are applied to the extremes of the trait. The same scoring system has been used by the Holstein Friesian Cattle Society of Great Britain and Ireland (HFS) since 1983. The second set of records was the production file, which contained 14,161 records with the following information: farm number, herd book number, lactation number, yield1 (milk production of first lactation) and yield2 (milk production of second lactation). First and second lactation records of 305n days milk production from 10 dairy farms that belong to Masstock, were adjusted for age and month of calving using adjustment factors derived by [8].

Data were collected during the period 1985 to 1988. Masstock farm cows were evaluated by evaluators from the United Kingdom. Four evaluators visit Masstock farms annually in the spring when most of the cows have passed peak production and are in the 5th to 7th month of lactation. Each cow must be evaluated once in her lifetime. If a cow was not evaluated during any one lactation, she would be evaluated during the next visit to the farm. After type trait records and production records had been edited for missing scores and for lactation length < 159 days, type traits and production files were merged by cow number within farm number, to establish a combined file that was then used for analysis. Correlation coefficients between type traits for the overall data were computed between residuals after fitting a model with herd and lactation number effects. However, correlation coefficients between type traits for lactations 1 to 5 were computed after fitting a model with herd effect. Residual correlation coefficients for the different lactations and for the overall data were adjusted for discontinuity according to method of Cox [9].

Principal components (PC) were computed from the adjusted correlation matrices of the type traits, for the overall data and for different lactations. PC's with eigenvalues greater than one were kept to represent the minimum number of factors which explain the data. Factor analysis with varimax rotation as described by Kaiser [7] was carried out, to obtain factor pattern coefficients. The varimax rotation provides axis with few large loadings. In other words, varimax rotation seems to maximize the interpretability of factors by simplifying them so that each factor has only a minimum number of traits with large loadings.

Computing the factor loading and communalities is not sufficient in factor analysis. Production traits must also be linked with factors of type traits by estimation of factor scores, to predict milk production of first and second lactations from factor scores 1 to 5.

A factor score is calculated such that $F = p' * T$, where:

F = the matrix of factor scores

P = the matrix of factor pattern or loading

T = vector of a standardized traits $t_i = Z_i - Z/\sigma_i$

The factor score denoted by F_1 to F_5 were used in multiple regression model

$$Y = FB + e$$

where

Y is the milk production

F = matrix of factor scores.

B = vector of Unknown parameters B_1, \dots, B_5 .

e = a vector of random errors.

Image analysis

The adjusted correlation matrix R was used to 1) compute two matrices: $D^2 = (\text{diag } R^{-1})^{-1}$ and $W = 1 - R^{-1} D^2$; 2) compute covariance matrices between the components of the observed type traits $P = G + V$ according to Kaiser's equations [7], i.e $E(GG')$, $E(VV')$, $E(GV')$, $E(GP')$ and $E(VP')$ (Appendix); and 3) carry out factor analysis on the image correlation matrix of the overall data (correlation matrix among images).

Results and Discussion

Factor analysis of type traits

Adjusted phenotypic correlation matrix of the type traits (Table 1) shows that the highest correlation coefficients were between fore udder angle and fore udder attachment $r = .97$. The second high correlation coefficient was between stature and body depth $r = .61$. Correlation coefficients between udder characteristics (fore udder attachment, fore udder angle rear udder width, udder support and udder depth

Table 1. Cox adjustable correlation (above diagonal) and product moment correlation (lower diagonal)

Trait ¹	ST	CW	BD	ANG	RA	RW	RLS	RLR	FA	FUA	FANG	RUW	US	UD	TPR	TPS	TL
ST	.303		.607	.052	-.014	.410	.055	.072	.024	.138	.188	.208	.007	.045	.075	.038	.094
CW	.368		.392	-.439	-.072	.373	-.059	.147	.076	.070	.016	.136	-.055	-.039	-.023	-.065	.052
BD	.681	.539		-.057	-.124	.400	.027	.264	-.002	.164	.233	.266	.044	-.043	.042	.007	.056
ANG	.108	-.489	-.022		-.032	-.281	.081	-.048	-.095	-.036	-.058	.004	.130	-.012	.020	.220	-.008
RA	-.018	-.102	-.150	-.033		-.045	-.046	-.105	.032	-.040	-.060	-.038	-.053	-.009	.048	-.031	.073
RW	.442	.506	.487	-.272	-.055		-.022	.181	.068	.140	.188	.218	-.001	.001	.049	-.054	.074
RLS	.074	-.093	.036	.106	-.054	-.036		-.359	-.235	.012	.033	-.064	.003	.039	.019	-.027	-.050
RLR	.066	.159	.256	-.236	-.101	.169	-.373		.153	.159	.197	.330	.135	.016	.012	.041	-.063
FA	-.006	.104	-.022	-.126	.031	.075	-.316	.156		.083	.075	.047	.016	.040	.080	.015	.070
FUA	.099	.008	.110	-.019	-.034	.080	.034	.140	.087		.966	.488	.355	.436	.469	-.075	.000
FANG	.107	.021	.132	-.034	-.041	.091	.049	.137	.067	.808		.536	.359	.494	.532	-.104	-.020
RUW	.219	.118	.261	.093	-.035	.189	-.054	.288	.039	.471	.430		.392	.195	.313	.054	.042
US	-.016	-.126	-.010	.124	-.041	-.054	.024	.116	.017	.401	.349	.375		.282	.347	.029	-.005
UD	-.052	-.200	-.180	-.004	.025	-.135	.075	.008	.033	.549	.534	.231	.409		.250	-.124	-.101
TPR	.085	-.072	.028	.057	.052	.012	.045	.011	.091	.525	.496	.339	.397	.347		.117	-.010
TPS	.106	-.029	.080	.249	-.058	-.013	-.033	.049	.011	-.116	-.134	.063	-.018	-.242	.114		.034
TL	.129	.127	.122	-.001	.053	.125	-.066	.022	.073	-.076	-.095	.013	-.081	-.253	-.054	.095	

¹ ST = stature, CW = chest width, BD = body depth, ANG = angularity, RA = rump angle, RW = rump width, RLS = rear leg side view, RLR = rear legs rear view, FA = fore angle, FUA = fore udder attachment, FANG = fore udder angle, RUW = rear udder width, US = udder support, UD = udder depth, TPR = teat placement rear view, TPS = teat placement side view, and TL = teat length.

ranged from .20 to .97. Rear legs side view, rump angle and teat length had low correlations with other type traits $-.10 < r > .10$.

Principal components (PC) were computed from the adjusted correlation matrix to ensure that type traits were weighted equally in the PC analysis (Table 2). The first PC accounted for 21% of the total variation among the 17 type traits. The second, third, fourth and fifth PC accounted for 13.5%, 8.8%, 8.5% and 6.9% of the total variance respectively. So only those PC were retained which were above the cut-off point when any additional PC would account for less variance than any standard type traits, which is equivalent to retaining components with eigenvalues greater than or equal to 1. Thus five PC were retained for the analysis, and these accounted for 59% of the total variances in all type traits.

Table 2. Eigenvalues and percentage of total variance explained by principal components (PC) for phenotypic and image correlation matrix of type traits for overall data

PC	Phenotypic correlation matrix		Image correlation matrix	
	Eigenvalues	%	Eigenvalues	%
1	3.57	21.01	5.844	43.37
2	2.29	13.45	3.426	20.15
3	1.56	8.81	2.156	12.69
4	1.44	8.49	1.662	9.78
5	1.17	6.89	1.275	7.50
6	.92	5.42	.743	4.37
7	.91	5.34	.680	4.00
8	.88	5.20	.364	2.14
9	.73	4.28	.282	1.66
10	.68	3.99	.234	1.38
11	.62	3.66	.164	.96
12	.61	3.61	.109	.64
13	.49	2.89	.062	.34
14	.44	2.57	.049	.29
15	.41	2.40	.011	.07
16	.32	1.87	.001	.01
17	.02	.01	.001	.01

Table 3 shows the magnitude and proportion of the total variance for each eigenvalue across lactations; the trend is similar to that observed for the overall data. Sieber *et al* [1,2] found that the first eight components accounted for more than 69% of the total variance in type score.

Table 3. Eigenvalues and percentage of total variance explained across lactation (L) by principal component (PC) from phenotypic correlation matrix of type traits

PC	L1		L2		L3		L4		L5	
	Eigen-values	%	Eigen-values	%	Eigen-values	%	Eigen-values	%	Eigen-values	%
1	3.34	19.66	3.27	19.21	3.23	19.00	2.88	16.96	3.35	19.68
2	2.61	15.38	2.27	13.35	2.25	13.24	2.20	12.93	1.96	11.56
3	1.65	9.69	1.67	9.84	1.60	9.40	1.75	10.29	1.51	8.88
4	1.50	8.84	1.44	8.44	1.42	8.35	1.36	8.02	1.30	7.65
5	1.10	6.48	1.16	6.80	1.20	7.08	1.16	6.82	1.25	6.62
6	.92	5.41	.99	5.79	.97	5.71	.98	5.78	1.00	5.87
7	.91	5.34	.96	5.64	.88	5.19	.88	5.16	.93	5.46
8	.89	5.22	.89	5.22	.84	4.96	.84	4.97	.83	4.89
9	.78	4.56	.80	4.68	.76	4.45	.76	4.49	.82	4.80
10	.60	3.54	.68	4.01	.68	4.03	.70	4.11	.79	4.67
11	.56	3.30	.66	3.91	.60	3.55	.62	3.67	.71	4.18
12	.51	3.02	.55	3.25	.59	3.52	.61	3.58	.66	3.91
13	.47	2.75	.50	2.95	.46	2.72	.54	3.16	.60	3.54
14	.40	2.36	.42	2.49	.45	2.68	.52	3.03	.52	3.05
15	.34	1.99	.40	2.34	.42	2.45	.47	2.79	.49	2.90
16	.27	1.60	.28	1.70	.40	2.34	.43	2.51	.26	1.54
17	.15	.86	.06	.35	.23	.13	.29	1.73	.14	.01

Table 4 shows the factor pattern coefficients after varimax rotation. The absolute magnitude of the factor coefficients determine the importance of a certain trait to be included in a specific factor. A trait with a large absolute coefficient contributes more to the factor than a trait with a small one.

Table 5 shows the distribution of type traits for the overall data. For factor 1, traits with the largest absolute values were the udder traits, such as fore udder attachment, fore udder angle, rear udder width, udder support, udder depth and teat placement rear view. These six single traits contributed the most to the score for factor 1, and a cow with a large score for this factor would have high wide large rear udders, supported by strong suspensory ligaments with well attached fore udders, udder floor above the hock and good teat placement rear view. Factor 2 included stature, chest width, body depth and rump width; thus, a cow with a large score for this factor was a tall, big, strong cow with wide chest, deep body, and wide rump. Rear legs rear view, rear legs side view, and foot angle were separated by Factor 3 a cow with a large score for this factor has squarely placed legs, that are straight in the hocks, and has a steep foot angle. Factor 4 describes the sharpness, flatness of bone, openings of side and length of neck, and teat placement side view. Finally, rump angle and teat length

Table 4. Factor pattern ($\times 100$) and the communalities of type traits from phenotypic correlation matrix for overall data

Trait ¹	F1 ²	F2	Fe	F4	F5	Communality
ST	9.30	78.49	-13.52	15.39	9.41	67.56
CW	- 5.12	61.13	15.00	-43.46	- .47	58.78
BD	8.46	82.78	1.07	8.48	-12.67	71.57
FANG	5.89	-17.80	-17.80	77.80	- 8.39	67.95
RA	.14	-14.06	.27	- 7.32	66.18	46.31
RW	6.55	67.09	9.08	-24.09	4.89	53.35
RLS	5.54	7.09	-79.58	.75	- 4.18	64.32
RLR	14.14	23.63	69.11	8.58	-35.67	68.89
FA	8.05	- 1.63	54.95	- 9.61	29.98	40.78
FUA	88.14	12.39	2.57	11.94	- .40	80.71
FANG	91.09	18.57	1.67	-13.89	- 3.45	88.50
RUW	60.58	31.59	23.78	14.92	- 7.71	55.16
US	54.29	- 4.47	7.83	22.80	- 9.94	42.15
UD	62.40	-13.94	-10.50	-21.72	- 8.75	47.47
TPR	67.20	.24	- .78	12.06	20.96	51.01
TPS	- 5.68	6.97	12.55	65.05	9.42	45.59
TL	- 4.11	18.75	5.94	13.15	64.25	47.05

¹ ST = stature, CW = chest width, BD = body depth, ANG = angularity, RA = rump angle, RW = rump width, RLS = rear leg rear side view, RLR = rear leg rear view, FA = foot angle, FUA = fore udder attachment, FANG = fore udder angle, RUW = rear udder width, US = udder support, UD = udder depth, TPR = teat placement rear view, TPS = teat placement side view, and TL = teat length.

² F1 = strong support, well attached, high and wide rear udder and teat placement rear view; F2 = tall, big, strong cow and wide rump; F3 = cows with squarely placed legs, steep foot angles and straight in the hooks; F4 = sharp, lean and flatness of bone; F5 = sloped rump from hocks to pins and adequate teat length.

were described by Factor 5; a cow with a high score had good length of teat to hold the machine milking cup adequately. Although rump angle is mainly within Factor 5, rump angle is more appropriately included in Factor 4. Thus, interpretation of Factor 5 would then involve only measures of teat length. The same technique, omission of a variable from a Factor to include it in another factor to make a meaningful interpretation, was used by Sieber *et al.* [1].

Sieber *et al.* [1,2] found that 18 type traits and milkout were described by eight factors. This factor pattern was somewhat different from that found in the present study because the first factor included the basic form, strength, rump width, body depth, and disposition. However, udder width and height were included in Factor 2. The discrepancy between the Sieber *et al.* [1, 2] study and this study may be due to the

Table 5. The distribution of type traits¹ from phenotypic and image correlation matrix for the overall data

Phenotypic correlation matrix				
F1 ²	F2	F3	F4	F5
FUA	ST	RLS	ANG	RA
FANG	CW	RLR	TPS	
RUW	BD	FA		
US	RW			TL
UD				
TPR				
Image correlation matrix				
F1 ²	F2	F3	F4	F5
FUA	ST	RLS	ANG	
FANG	CW	RLR	TPS	RA
RUW	BD	FA		
US	RW			TL
UD				
TPR				

¹ ST = stature, CW = chest width, BD = body depth, ANG = angularity, RA = rump angle, RW = rump width, RLS = rear leg rear side view, RLR = rear leg rear view, FA = foot angle, FUA = fore udder attachment, FANG = fore udder angle, RUW = rear udder width, US = udder support, UD = udder depth, TPR = teat placement rear view, TPS = teat placement side view, and TL = teat length.

² F1 = strong support, well attached, high and wide rear udder and teat placement rear view; F2 = tall, big, strong cow and wide rump; F3 = cows with squarely placed legs, steep foot angles and straight in the hooks; F4 = sharp, lean and flatness of bone; F5 = sloped rump from hooks to pins and adequate teat length.

different scores from different cows. The Masstock data include a large proportion of the data in good to good plus category (another score used by Masstock), for breed character, udder and teat, body capacity and legs and feet [10]. However, Sieber *et al.* [1, 2] used data from the Twenty - First Century Genetics Mating Appraisal for Profit program.

Factor analysis across lactations 2, 3 and 4 (Table 6) showed similar factor patterns (F1 to F5) to those computed from the overall data. On the other hand, first lactation cows showed some disagreement, because F1 included body capacity traits (stature, chest width, body depth and rump width). However, F2 described udder traits (fore udder attachment, fore udder angle, rear udder width, udder support, udder depth and teat placement rear view). Therefore, body capacity precede udder conformation traits in importance for first lactation cow, factor 3 included angularity

and teat placement side view. However, leg traits were included in F4. Factor 5 described rump angle and teat length. Fifth lactation cows, showed discrepancy only for F3 and F4 because F3 included angularity and teat placement side view. However, rear leg rear view, rear leg side view and foot angle were described by Factor 4.

The communality estimates (C^2) are the proportion of variance of a type trait that are shared with other type traits via the common factor variate. Communality estimates (Table 4) of fore udder angle and fore udder attachment were 89 and 81% and were the highest, followed by body depth, rear legs rear view, angularity and stature ($C^2 > .67$). Foot angle had the lowest communality estimates (41%). In other words, the unique variance of foot angle was the highest. Therefore one can argue that foot angle does not share much variability with other type traits.

Table 7 gives the parameter estimates, standard errors and multivariate T values for the regression of factor scores of milk production in first and second lactations. Four of five factor scores were highly significant for first and second lactation milk production. The regression coefficient of the first factor (F1) was negative in the first lactation, indicating that the tallest, biggest and strongest cow did not produce the most milk. Positive regression coefficients and significant T values for the second factor (F2) in the first lactation and the first factor (F1) in second lactation, indicate that cows with well-developed and deep, strongly supported fore and rear udders were able to produce more milk than cows with the opposite udder traits. Sharp, lean, clearly defined angles and appropriate slope of rump from the hocks to the pins significantly increased milk production. Teat placement side view, teat placement rear view and teat length had highly significant effect on milk production in both lactations. Rear leg rear view and foot angle had no significant effect on milk production during first and second lactations.

Image analysis of type traits

Image analysis can complement factor analysis to clarify relationships among traits. Factor analysis was carried out on the image correlation matrix to investigate the distribution of type traits among factors based on the image correlation matrix (Table 2). The first PC of the image correlation matrix accounted for 43% of total variability. However, only 21% was accounted for by the first PC of the adjusted correlation matrix. The first five PC of the image correlation matrix accounted for 93% of total variability. However, those of the p-phenotypic correlation matrix accounted for only 59%. So dispersion of the eigenvalues of the image matrix is greater than that of the eigenvalues of the adjusted phenotypic correlation matrix. Table 5 shows the distribution of factor patterns of type traits from the image correlation matrix. Factors 1 to 5, obtained from the image matrix, included traits similar to factors 1 to 5 which were computed from the adjusted phenotypic correlation matrix. The image matrix was used to explain the relationships among the predicted parts of phenotypic measurements in the absence of pedigree information (Table 8). The magnitude of

Table 6. The distribution of type traits¹ by factor across lactations.

Lactation	F1 ²	F2	F3	F4	F5
First	ST	FUA	ANG	RLS	RA
	CW	FANG	TPS	RLR	TL
	BD	RUW		FA	
	RW	US			
		UD			
		TPR			
Second	FUA	ST	RLS	ANG	RA
	FANG	CW	RLR	TPS	TL
	RUW	BD	FA		
	UD	RW			
	US				
	TPR				
Third	FUA	ST	RLS	ANG	RA
	FANG	CW	RLR	TPS	TL
	RUW	BD	FA		
	US	RW			
	UD				
	TPR				
Fourth	FUA	ST	RLS	ANG	RA
	FANG	CW	RLR	TPS	TL
	RUW	BD	FA		
	US	RW			
	UD				
	TPR				
Fifth	FUA	ST	ANG	RLS	RA
	FANG	CW	TPS	RLR	TL
	RUW	BD	FA		
	US				
	UD				
	TPR				

¹ ST = stature, CW = chest width, BD = body depth, ANG = angularity, RA = rump angle, RW = rump width, RLS = rear leg rear side view, RLR = rear leg rear view, FA = foot angle, FUA = fore udder attachment, FANG = fore udder angle, RUW = rear udder width, US = udder support, UD = udder depth, TPR = teat placement rear view, TPS = teat placement side view, and TL = teat length.

² F1 = strong support, well attached, high and wide rear udder and teat placement rear view; F2 = tall, big, strong cow and wide rump; F3 = cows with squarely placed legs, steep foot angles and straight in the hooks; F4 = sharp, lean and flatness of bone; F5 = sloped rump from hocks to pins and adequate teat length.

Table 7. Linear regression analysis for milk production using factors developed from type traits data as independent variables.

Factor	First lactation			Second lactation		
	Coefficient	SE	t	Coefficient	SE	t
Intercept	7050.03	31.21	251.56***	8939.75	37.27	227.63***
F1	-81.00	12.50	-6.48***	113.96	15.55	7.33***
F2	37.88	12.49	3.03**	65.28	15.61	4.18***
F3	297.30	33.66	8.83***	40.69	25.40	1.60 ^{NS}
F4	-25.42	19.87	-1.28 ^{NS}	160.38	37.47	4.28***
F5	107.50	32.26	3.33***	140.37	41.84	3.36***

NS = Not significant

** = $P < .01$ *** = $P < .001$

F1 = strong support, well attached, high and wide rear udder and teat placement rear view; F2 = tall, big, strong cow and wide rump; F3 = cows with squarely placed legs, steep foot angles and straight in the hooks; F4 = sharp, leaqn and flatness of bone; F5 = sloped rump from hocks to pins and adequate teat length.

the image correlation reflects the pairwise relationship between each pair of traits. A high pairwise image correlation (r_i) was observed between fore udder attachment and udder characteristics ($r_i > .63$). Fore udder angle had a high image correlation and fore udder attachment and with rear udder width ($r_i = .97$ and $.78$). Udder conformation traits have a close mutual relationship. For example, fore udder attachment and udder depth had image correlation ($r_i = .82$), image correlation of fore udder attachment with teat placement rear view was ($r_i = .82$). The pairwise image correlation between chest width and stature; chest width and body depth; chest width and rump width were $.58$, $.66$ and $.91$, respectively. High positive r_i between two traits indicate the same behavior of the two traits. However, negative r_i values indicate different directions of the behavior of the two traits. Anti-image correlation coefficients among type traits ranged from $-.95$ to $.42$.

In simulated data of continuous type traits, Ali [11] found a high r_i between final score and udder characteristics ($r_i = .50$). Also, final score had high r_i with the stature of the cow ($r_i = .83$). Stature had high correlation with rump length ($r_i = .82$) and rump width ($r_i = .86$). Udder conformation traits have a close mutual relationship. For example, fore udder attachment and rear udder width had $r_i = .70$, rear udder width and udder support had $r_i = .80$. Also a substantial correlation was found between udder height and udder width ($r_i = .72$).

The image coefficients of type traits, the matrix W, (Table 9) has some characteristics 1) It is a $p \times p$ multiple correlation weight matrix for predicting each trait in p

Table 8. Image (above diagonal) and anti-image (below diagonal) correlation coefficient from the correlation matrix corrected for the discontinuity

Trait ¹	ST	CW	BD	ANG	RA	RW	RLS	RLR	FA	FUA	FANG	RUW	US	UD	TPR	TPS	TL
ST	.577	.741	.741	-.215	-.425	.670	.161	-.386	-.009	.213	.280	.415	.147	-.019	.145	.066	.379
CW	-.129	.659	.659	-.485	-.183	.907	-.140	.408	.304	.123	.177	.292	-.151	-.043	.002	-.358	.301
BD	-.482	-.195	.415	-.187	-.422	.877	-.027	.401	.143	.276	.291	.565	.053	.149	.209	-.030	.266
ANG	-.224	.415	-.033	.123	-.395	.176	-.208	-.311	-.075	-.092	-.041	.172	.002	.002	.092	.409	-.096
RA	-.077	.059	.076	.061	-.247	-.021	-.439	.248	-.238	-.184	-.467	-.039	-.207	-.189	.231	.285	
RW	-.255	-.101	-.083	.227	.015	-.132	.420	.297	.276	.321	.489	.020	.113	.168	-.229	.315	
RLS	-.007	.029	-.057	-.048	.050	-.016	-.344	-.647	.080	.020	-.147	-.100	.232	.202	-.221	.025	
RLR	.128	-.010	-.175	-.029	.056	-.068	.369	.469	.332	.306	.582	.286	.196	.299	-.064	.188	
FA	-.037	-.028	.048	.054	.016	-.023	.174	-.094	.249	.284	.353	.198	.078	.145	.035	.147	
FUA	.016	.013	.183	-.033	.073	.059	.119	.142	-.052	.967	.805	.631	.824	.815	-.296	-.054	
FANG	-.014	-.012	-.212	.041	.089	-.069	-.137	-.161	.040	-.953	.784	.733	.731	.724	-.179	-.048	
RUW	-.045	-.036	-.006	-.031	-.048	-.063	.030	-.182	.029	.137	-.238	.633	.664	.781	-.094	-.008	
US	-.073	.010	-.040	-.112	.058	.010	-.036	-.069	.019	-.151	.138	-.269	.467	.568	.313	-.151	
UD	-.090	.037	.193	.019	-.027	.057	.035	-.079	-.034	.282	-.390	.110	-.195	.848	-.496	-.285	
TPR	-.018	.039	.108	.016	-.107	.025	.063	.162	-.071	.329	-.439	.039	-.232	.147	-.200	-.017	
TPS	-.031	-.022	-.023	-.173	-.058	.007	-.014	-.072	-.012	-.166	.209	-.110	.043	.014	-.240	.243	
TL	-.050	-.014	-.019	-.005	-.060	-.042	.061	.109	-.064	-.055	.049	-.058	-.016	.080	.010	-.015	

¹ ST = stature, CW = chest width, BD = body depth, ANG = angularity, RA = rump angle, RW = rump width, RLS = rear leg side view, RLR = rear legs rear view, FA = foot angle, FUA = fore udder attachment, FANG = fore udder angle, RUW = rear udder width, US = udder support, UD = udder depth, TPR = teat placement rear view, TPS = teat placement side view, and TL = teat length.

from $p - 1$ traits, where p is the number of traits. 2) W is asymmetric with diagonal elements equal to 0, so a trait being predicted from $p-1$ other traits will receive a weight 0 in the prediction equation involving the full set of p traits; for example, to predict stature, multiply the first row (.000, .142, .494, .250, .084, .295, -.035, -.115, .039, .042, -.039, .086, -.109, .164, .090, -.002, .056) by the observed score of 17 traits and add these products together. 3) If R can be partitioned, such that

$$R = \begin{bmatrix} 1 & R_{12} \\ R_{12} & R_{22} \end{bmatrix}$$

Then,

$$W = (1 - R^{-1} D_2) = R_{22} (R^{-1}_{22} - D_2),$$

and the matrix $R_{22} - D_2$, represents the covariance between the original type traits and their predictable parts. The matrix D_2 represents the errors of estimate of each trait with respect to the $p-1$ other type traits as predictors of the traits and $D_2 = \text{Diag} (.540, .639, .490, .676, .942, .685, .791, .674, .912, .049, .038, .576, .718, .634, .565, .857, .949)$. Table 10 shows the covariance between image and anti-image of type traits. Covariance was low and ranged from -.269 to .173.

The relationship exists between common factor analysis and image analysis. Guttman [4] stated that "for the common factor model to be determinate, the unique variance of each trait (factor analysis) must equal the error of estimate in predicting the trait from $p-1$ traits (image analysis) i.e. $u_j^2 = d_j^2$. Moreover, if the number of common factors in a universe of traits is finite then common factor model would be determinate in that universe". Mulaik [6] found that if a common factor model is determinate then, as the number of traits increases without bound:

$$a) \lim_{p \rightarrow \infty} d_j^2 = u_j^2$$

Therefore, the error of estimate for predicting a trait from $p-1$ traits (image analysis) approaches as a limit the unique variance of that trait (factor analysis).

$$b) \lim_{p \rightarrow \infty} R_j^2 = C_j^2$$

The square of the multiple correlation coefficient for predicting a trait from $p-1$ traits (image analysis) approaches as a limit the communality of the trait (factor analysis).

$$c) \lim_{p \rightarrow \infty} S = R^2 - U^2 = FR_{xx} F'$$

Table 9. Image coefficients (W) matrix from the correlation matrix corrected for the discontinuity

Trait ¹	ST	CW	BD	ANG	RA	RW	RLS	RLR	FA	FUA	FANG	RUW	US	UD	TPR	TPS	TL
ST	.000	.142	.494	.250	.084	.295	.035	-.115	.039	.042	-.039	.086	-.109	.164	.090	.002	.056
CW	.115	.000	.116	-.428	-.040	.083	-.078	-.038	.049	-.083	.076	-.032	.032	-.150	-.158	.094	.034
BD	.508	.222	.000	.039	-.108	.099	.077	.210	-.067	.052	-.054	.013	.044	-.207	-.104	.024	.025
ANG	.191	-.411	-.099	.000	.002	-.275	-.0532	-.085	-.030	-.177	.161	-.128	.213	-.285	-.292	.352	.048
RA	.058	-.048	-.062	-.052	.000	-.015	-.052	-.053	.017	.007	-.009	.029	-.045	.009	.068	-.045	.063
RW	.225	.095	.042	-.226	.000	.000	-.007	.042	.034	-.058	.055	.023	.012	-.115	-.086	.029	.059
RLS	.006	-.025	.051	.044	-.058	.017	.000	-.336	-.188	-.022	.023	-.018	.030	-.019	-.040	.007	-.068
RLR	-.114	.010	.153	.029	-.069	.069	-.398	.000	.108	-.034	.034	.173	.068	-.068	-.140	.075	-.131
FA	.027	.024	-.041	-.047	.019	.018	-.167	.075	.000	.004	.000	-.030	-.012	.016	.043	.019	.068
FUA	-.051	-.044	-.564	.121	.311	-.212	-.467	-.513	.219	.000	.834	-.453	.557	-.972	-.1074	.652	.238
FANG	.052	.048	.748	-.175	-.434	.282	.616	.665	-.194	1.087	.000	.911	-.582	1.552	1.649	-.945	-.241
RUW	.044	.038	.010	.033	.058	.069	-.314	.202	-.038	-.034	.057	.000	.294	-.106	-.029	.125	.073
US	-.063	-.009	.047	.109	-.075	-.004	.050	.080	-.024	.060	-.051	.260	.000	.211	.236	-.063	.014
UD	.083	-.037	-.173	-.020	.034	-.059	-.042	-.083	.042	-.084	.101	-.109	.207	.000	-.142	-.012	-.097
TPR	.018	-.042	-.102	-.011	.139	-.027	-.076	-.179	.091	-.101	.118	-.041	.260	-.158	.000	.290	-.012
TPS	.026	.020	.038	.154	-.072	.001	.029	.082	.008	.069	-.071	.115	-.054	.029	.237	.000	.009
TL	.038	.012	.014	.005	.060	.035	-.055	-.092	.063	.013	-.011	-.046	.013	-.064	-.066	.013	.000

¹ ST = stature, CW = chest width, BD = body depth, ANG = angulinity, RA = rump angle, RW = rump width, RLS = rear leg side view, RLR = rear legs rear view, FA = foot angle, FUA = fore udder attachment, FANG = fore udder angle, RUW = rear udder width, US = udder support, UD = udder depth, TPR = teat placement rear view, TPS = teat placement side view, and TL = teat length.

Table 10. Covariance matrix between image and anti-image

Trait ¹	ST	CW	BD	ANG	RA	RW	RLS	RLR	FA	FUA	FANG	RUW	US	UD	TPR	TPS	TL
ST	.000	.076	.248	.135	.055	.154	.004	-.077	.026	-.002	.002	.025	-.046	.053	.010	.021	.036
CW	.000	.000	.109	-.273	-.046	.067	-.020	.007	.022	-.002	.002	.022	-.007	-.023	-.024	.016	.011
BD	.000	.000	.000	.019	-.051	.048	.038	.100	-.032	-.029	.029	.003	.024	-.107	-.057	.015	.013
ANG	.000	.000	.000	.000	-.049	-.154	.035	.019	-.042	.006	-.007	.019	.078	-.013	-.006	.132	.004
RA	.000	.000	.000	.000	.000	-.012	-.043	-.044	.014	.015	-.017	.035	-.048	.021	.078	-.052	.058
RW	.000	.000	.000	.000	.000	.000	-.011	.046	.019	-.010	.011	.039	-.007	-.037	-.015	-.005	.034
RLS	.000	.000	.000	.000	.000	.000	.000	-.269	-.148	-.024	.024	-.020	.027	-.025	-.042	.011	-.053
RLR	.000	.000	.000	.000	.000	.000	.000	.000	.074	-.026	.026	.114	.048	-.052	-.100	.054	-.087
FA	.000	.000	.000	.000	.000	.000	.000	.000	.000	.011	-.007	-.021	-.015	.026	.052	.011	.060
FUA	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.041	-.023	.028	-.050	-.054	.034	.012
FANG	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.035	-.022	.060	.064	-.037	-.009
RUW	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.173	-.067	-.022	.077	.043
US	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.132	.148	-.033	.013
UD	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	-.088	-.010	-.062
TPR	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.167	-.007
TPS	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.013
TL	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

¹ ST = stature, CW = chest width, BD = body depth, ANG = angularity, RA = rump angle, RW = rump width, RLS = rear leg side view, RLR = rear legs rear view, FA = foot angle, FUA = fore udder attachment, FANG = fore udder angle, RUW = rear udder width, US = udder support, UD = udder depth, TPR = teat placement rear view, TPS = teat placement side view, and TL = teat length.

The square of the multiple correlation coefficient for predicting a trait from $p-1$ traits (image analysis) approaches as a limit the communality of the trait (factor analysis).

$$c) \lim_{p \rightarrow \infty} S = R^2 - U^2 = FR_{xx} F'$$

The image covariance matrix approaches as a limit the reduced correlation matrix with communality coefficients on the main diagonal.

$$d) \lim_{p \rightarrow \infty} Q = D^2 = U^2$$

The anti-image covariance matrix approaches as a limit the diagonal matrix of errors of estimate and in turn the diagonal matrix of unique variances.

Appendix

The covariance between components of phenotypic value could be obtained using R and D^2 according to Kaiser's equation [7] as follows:

Because $E(pp') = E[(G + V)(G + V)']$

$$= E(GG') + E(VV') + E(VG') + E(GV')$$

and if the matrix $W = (I - R^{-1}D^2)$, where $R = E(PP')$ is a $p \times p$ correlation matrix for the p traits, and $D^2 = [\text{diag } R^{-1}]^{-1}$. Then the matrix of covariances among p different images (Table 8) is:

$$E(GG') = E(W'PP'W) = W'RW$$

$$S = R + D^2 R^{-1} D^2 - 2 D^2,$$

The covariance matrix among different anti-images.

$$E(W'W) = (I - W')PP'(I - W) = Q$$

$$= [I - (I - D^2 R^{-1})] R [I - (I - R^{-1} D^2)]$$

$$Q = D^2 R^{-1} D^2,$$

The covariance matrix between the image and the original type traits.

$$E(GP') = E(W'PP') = W'R$$

$$= R - D^2,$$

The covariance matrix between the anti-image and type traits.

$$E(VP') = E[(I - W')PP'] = (I - W')R = D^2$$

The covariance matrix between images and anti-images (Table 10).

$$E(GV') = E(I - D^2 R^{-1})PP'R^{-1}D^2$$

$$= D^2 - D^2 R^{-1} D^2$$

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التحليل العائلي المظهري والتخلي للصفات الشكلية لأبقار الهولستين في قطاعان ماستوك بالسعودية

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ملخص البحث: أجري التحليل العائلي والتخلي على ١٣٦٤٢ سجلاً للصفات الشكلية لأبقار اللبن والتي جمعت من مزارع ماستوك في المملكة العربية السعودية. وقد أظهرت الدراسة أن هناك خمسة عوامل يجب أن تدخل في التحليل وقد أسهمت هذه العوامل الخمسة بنسبة ٥٩٪ من كمية التباين الكلي للصفات الشكلية. وقد اشتمل العامل الأول على صفات الضرع، بينما اشتمل العامل الثاني على صفة الحجم، والعامل الثالث اشتمل على صفات الأرجل، بينما اشتمل العامل الرابع على صفات زوايا الميل وزوايا القطن، وكان العامل الخامس ممثلاً لصفات الحلمة. لقد وصف كل من التحليل العائلي والتحليل التخلي المحسوب من مصفوفة الارتباط بعد إزالة التأثير غير المتصل للبيانات، الصفات الشكلية نفسها.

وقد لوحظ أن الأبقار التي تتميز بعمق الضرع واتصاله وتميز الأرباع الأمامية والخلفية للضرع أنتجت كمية أكبر من اللبن بعكس الأبقار التي ليس لها مثل صفات الضرع. لم يكن هناك تأثير معنوي لمنظر الأرجل الخلفي والجانبية وكذلك زاوية القدم على إنتاج اللبن في الموسم الأول والثاني. كان الارتباط الازدواجي التخلي بين اتصال الأرباع الأمامية للضرع وباقي صفات الضرع (الارتباط بين الجزء المتبأ للصفات الشكلية) مرتفعاً أكثر من ٠,٦، وكان الارتباط التخلي بين زاوية أرباع الضرع الأمامية وكل من اتساع الضرع واتصاله وعمقه هي ٠,٧٨، ٠,٧٣، ٠,٧٣، على الترتيب.