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Graduating the Saudi Crude Mortality Rates and Constructing Their Monetary Tables

Ahmed D. Abid*, Ahmed A. Kamhawey and Othman I. Alsalloum*****

** Associate Professor, Department of Quantitve Methods, College of Administrative Sciences, King Saud University*

*** Visiting Professor, Department of Finance and Financial Institutioons, University of Kuwait*

**** Assistant Professor, Department of Quantitve Methods, College of Administrative Sciences, King Saud University*

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Abstract. The actuarial method to develop a law of mortality or a mathematical expression to graduate the Saudi crude mortality rates have been applied, also monetary tables have been constructed to assist calculating the actuarial present value for the Saudi life annuities (SPVA).

1. Introduction

Graduation may be regarded as the principles and methods by which a set of observed (or crude) probabilities are adjusted in order to provide a suitable basis for inferences to be drawn and further practical computations to be made.

The fundamental justification for the graduation of a set of observed probabilities like q_x is the premise that if the number of individuals in the group on whose experience the data are based, n_x had been considerably large, the set of observed probabilities would have displayed a much more regular progression with x. In the limit with n_x indefinitely large, the set of probabilities would thus exhibited a smooth progression with x. Therefore, the observed data may be regarded as a sample from a larger population so that the observed probabilities, derived there from, are subject to sampling errors. Providing these errors are random in nature, they may be reduced by increasing the size of the sample and thereby extending the scope of the investigation. A simpler, cheaper and more practicable alternative is often to use graduation to remove these random errors.

2. Grad[illegible]Methods

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The actuarial methods of graduation can be described broadly as [1]:

2.1. The graphic method consists of drawing a graph through or near the crude values of the function to be smoothed. Smoothness has been alleged to be achieved or improved by hand-polishing the values to make the third differences as small as possible.

2.2. The summation method consists of applying a finite difference forms involving successive summations to the crude values. The accepted 'smoothing coefficients' or 'smoothing index' is related to third differences and it follows that the more powerful smoothing index of the formula chosen, the closer it comes to the fitting of polynomial.

2.3. The Kernel's method is an elegant variation of the summation methods, which avoids the necessity of filling in the smoothed values at the two ends of the tables [2].

2.4 The method of osculatory interpolation, which aims at achieving identical third differences at points where different sections of the smoothed function meet one another.

2.5. The spline method is a variation on the method of osculatory interpolation, but third differential coefficients take the place of the third differences.

2.6. The curve fitting or parametric method, or the fitting of a formula which produces a suitable shape, it has sometimes been said to give ideally smooth results, but whether it does must surely depend upon which formula or curve has been fitted.

2.7. Graduation by reference to a standard table which depends on establishing a relationship between the crude rate and the rate obtained from the standard table by applying them on a simple formula. Clearly the degree of smoothness must depend very largely upon the smoothness of the standard table used.

2.8. The difference equation method determines the graduated series by a difference equation derived from an analytic measure of the relative emphasis placed on smoothness and fit.

2.9. The linear programming method determines the graduated series by minimizing the divergence index subject to constraints.

Graduation is characterized by two essential qualities, smoothness and fit. An increase in smoothing results a reduction in fit. Conversely, when fit is improved, smoothness usually suffers. The following preliminary test for the Saudi crude mortality rates suggest using the curve fitting method (mathematical model) for graduation.

3. The Preliminary Test for a Mathematical Model

The first important contribution towards finding a mathematical model was made by Benjamin Gompertz (1825), who argued that the force of mortality μ_x can be represented by the formula BC^x .

A development of Gompertz's law was subsequently made by Makeham (1860), who adapted the formula:

$$\mu_x \text{ or } q_x = A + BC^x \tag{1}$$

when Gompertz's law holds the graph of the following equation is a straight line [3]:

$$-\ln(-\ln(1-q_x)) = \ln\left(\frac{\ln C}{B(C-1)}\right) - x\ln C \tag{2}$$

Also, a Makeham gradution appears justified and estimates of C and B can be obtained by sketching the line and noting it's slope and y-intercept:

$$\hat{C} = \exp(-slope) \tag{3}$$

$$\hat{B} = \frac{\ln\hat{C}}{(\hat{C}-1)}\exp(-intercept) \tag{4}$$

Graduated rates can be then calculated by the use of an eight-parameter formula [4] for the age curve of mortality:

$$q_x = A^{(x+B)^C} + D\exp\left(-E\left(\ln\frac{x}{F}\right)^2\right) + \frac{GH^x}{1+GH^x} \tag{5}$$

where q_x is the probability of someone exact age x dying before exact age $x+1$, and A, B,…, H are parameters to be estimated. The parameters are estimated by least squares criteria, minimizing the sum of squares of the proportional differences of the fitted from the observed mortality probabilities, after regrouping into age groups. The least squares fitting criterion produce a smoothed set of ${}_5q_x$ values and single-year q_x values which aggregate to the smoothed ${}_5q_x$ values. The fit is usually very good and this procedure appears to provide excellent fits for the additional life table columns also [5].

The mathematical formula in Eq. (5) contains three terms each representing a distinct component of mortality [4]:

- The first term representing a rapidly declining exponentially, reflects the fall in mortality during the early childhood years as the child adapts to his or her new environment and gains immunity from the diseases of the outside world, this term has three parameters:

- A: which is nearly equal to q_i .
- B: is an age displacement, it measures the location of infant mortality.
- C: measures the rate of mortality decline in childhood, the higher the value of C, the faster mortality declines with increasing age.

- The second term is a function similar to the lognormal, reflects accident mortality for males and accident plus maternal mortality for the female population. It has three parameters:

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  - D: measuring the severity.
  - E: representing spread.
  - F: indicating location.

- The third term in the formula, the well-known Gompertz exponential, reflects the near geometric rise in mortality and it is generally considered the aging of the body, i.e. senescent mortality, and it has two parameters:

  - G: represents the base level of senescent mortality.
  - H: represent the rate of increase of that mortality.

The mortality rates of insured Saudi persons were assumed equal to those of the general population. Mortality rates of invalidity pensioners and survivors were assumed equal to the mortality of active insured persons. Crude rates appear in Table 1 [6]:

Table 1. Saudi crude mortality rates

Age interval	q_x^0 **Male**	q_x^0 **Female**
10 -	0.00209	0.00126
15 -	0.00292	0.00175
20 -	0.00408	0.00243
25 -	0.00569	0.00337
30 -	0.00794	0.00468
35 -	0.01106	0.00651
40 -	0.01541	0.00904
45 -	0.02151	0.01255
50 -	0.02998	0.01744
55 -	0.04183	0.02423
60 -	0.05834	0.03365
65 -	0.08136	0.04675
70 -	0.11463	0.06494
75 -	0.15825	0.09021
80 -	0.22068	0.12531
85 -	0.30778	0.17407
90 -	0.42926	0.24180

Applying the preliminary test on the above data, it suggests a straight line in each case for male (Fig. 1) and female (Fig. 2).

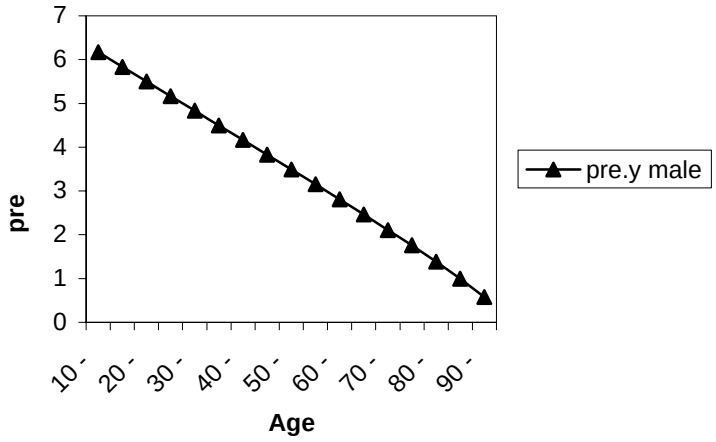

Fig. 1. Preliminary tests (male).

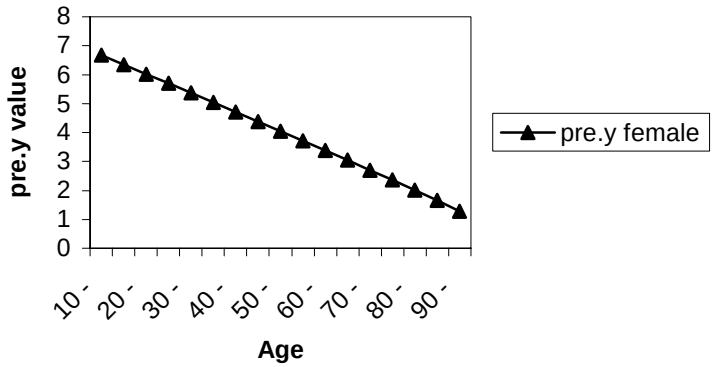

Fig. 2. Preliminary tests (female).

Both figures suggesting fitting a mathematical formula to graduate and represent the Saudi rates.

4. Fitting the Model

Various curve fitting have been tried [7, 8], which failed to represent the Saudi data. Heligman and Pollard mathematical formula Eq. (5) could represent the Saudi rates. The central mortality rates were transformed to q_x values by the classical formula:

$$q_x^0 = \frac{2m_x}{2+m_x} \tag{6}$$

The parameters of the curve were estimated by least squares Gauss-Newton iteration [9]. The function minimized was:

$$S^2 = \sum_{x=0}^{85} \left(\frac{q_x}{q_x^0} - 1.0\right)^2 \tag{7}$$

where q_x is fitted value at age x and q_x^0 is the observed (crude) mortality rate. That is, the sum of the squares of the proportional difference between the fitted and observed rates was minimized. The observed rates above age 85 will be excluded from the calculation because they appeared to be less reliable. Using the procedure UNABR from the United Nations Software Package for Mortality Measurement (MortPak-Lite), the graduated life tables for both sex are presented in Appendix I, also their parameters are presented in Table 2.

Table 2. Graduation parameters for Saudi mortality experience 1990-1993

Parameter	Males	Females
A	0.02779	0.01776
B	0.56113	0.82686
C	0.43809	0.37970
D	0.00119	0.00012
E	0.86895	1.83524
F	63.37608	23.35901
G	0.00012	0.00007
H	1.08145	1.08175

Comparing between the Saudi parameters of males and females in Table 2, males have experienced higher child mortality than females (156.48%), which appears from the values of parameter A. Also, the higher B value for females indicate that they have relatively lower infant mortality, in the same time males have higher value of C, i.e. their rate of mortality decline with age has been faster. Parameter D for male is higher than the female, may be that return to some accident mortality, and the following figure (Fig. 3) representing Saudi pattern of mortality for both genders.

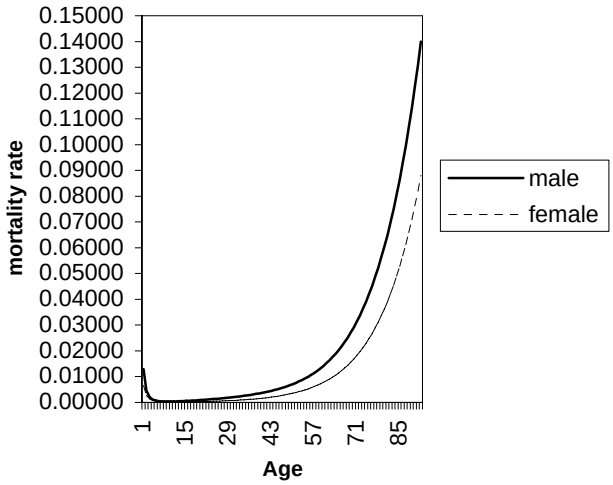

Fig. 3. Saudi pattern of mortality.

5. Monetary Saudi Tables

Commutation functions are derived by combining life table functions with compound interest functions, to construct the monetary tables based on the Saudi mortality rates obtained over the available range of ages. Life tables can be built up by starting with any suitable radix, say 100000, and applying successfully the relations [10]:

$$d_x = l_x q_x \tag{8}$$

$$l_{x+1} = l_x - d_x \tag{9}$$

where l_x indicating the survivors to the exact age x, and d_x indicating the number of deaths between x and $x+1$. Also, a computer program has been written in VISUAL BASIC.NET to compute the commutation functions in Eqs. (10) and (11) for ages

starting from 10 to 99 inclusively for both Saudi males and females, and interest rate $v = 5\%$ (as an example). The monetary tables have been represented in (Appendix II):

$$\left.\begin{aligned} D_x &= v^x \cdot l_x \\ N_x &= \sum_{t=0}^{\infty} D_{x+t} \\ S_x &= \sum_{t=0}^{\infty} N_{x+t} \end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned} C_x &= v^{x+1} \cdot d_x \\ M_x &= \sum_{t=0}^{\infty} C_{x+t} \\ R_x &= \sum_{t=0}^{\infty} M_{x+t} \end{aligned}\right\} \tag{11}$$

There are also other commutation functions used for the techniques of valuation of pension benefits that the researchers will deal with them in another article. To show some comparisons between Saudi life annuities for male and female, we use the calculated tables (Appendix II) and the following equation:

$$\ddot{a}_x = \frac{N_x}{D_x} \tag{12}$$

where, $\ddot{a}_x$: a life annuity-due which is a series of payments of one unit at the beginning of each year, payable as long as a life aged x is alive, and it can be expressed in terms of the elementary commutation functions as indicated. If the payments are made at the end of each year, the annuity is termed an immediate life annuity. It is denoted by the symbol a_x and has the following expression:

$$a_x = \frac{N_{x+1}}{D_x} \tag{13}$$

Some comparisons are made to distinguish between the annuities of both sexes at different ages when the interest rate equal 5% in Table 3 and Table 4:

Table 3. The probable present value of a Saudi life annuity-due

Age (x)	$\ddot{a}_x$ Male (1)	$\ddot{a}_x$ Female (2)	Ratio (2/1)%
30	17.95	18.86	103.07
40	16.61	17.77	106.98
50	14.81	16.24	109.66
60	12.56	14.20	113.06

Table 4. The probable present value of a Saudi immediate life annuity-due

Age (x)	a_x Male (1)	a_x Female (2)	Ratio (2/1)%
30	16.95	17.86	105.37
40	15.61	16.77	107.43
50	13.31	15.24	110.35
60	11.56	13.20	114.19

It is clear from Tables 3 and 4 that both life annuities are higher for females, which will represent higher premium for them. This is acceptable if we consider their lighter mortality rates, which will give the females few years to maturity.

6. Conclusion

This article is adopting a law of mortality suggested by Heligman and Pollard [4] to graduate the Saudi crude rates. Its mathematical formula contains three terms, each representing a distinct component of mortality. The curve is continuous and applicable over the entire age range. It has relatively few parameters all of which have demographic interpretation and together fully describe the Saudi age pattern of mortality for males and females. Also, monetary tables have been constructed by using a computer program written in VISUAL BASIC.NET to assist calculating the actuarial present values for the Saudi life annuities (SPVA). Some comparisons are made to distinguish between the annuities of both sexes at different ages.

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Appendix I

Saudi Life Table (Females)

AGE	M(X,N)	Q(X,N)	I(X)	E(X)	AGE	M(X,N)	Q(X,N)	I(X)	E(X)
0	.06486	.06205	100000	68.27	52	.00840	.00836	81712	26.65
1	.01306	.01297	93795	71.76	53	.00899	.00895	81028	25.87
2	.00463	.00462	92578	71.70	54	.00964	.00959	80303	25.10
3	.00209	.00209	92151	71.03	55	.01033	.01028	79532	24.33
4	.00111	.00111	91959	70.18	56	.01108	.01102	78715	23.58
5	.00069	.00069	91856	69.25	57	.01188	.01181	77848	22.84
6	.00049	.00049	91793	68.30	58	.01275	.01267	76929	22.11
7	.00040	.00040	91748	67.34	59	.01369	.01360	75954	21.38
8	.00036	.00036	91711	66.36	60	.01471	.01460	74921	20.67
9	.00036	.00036	91678	65.39	61	.01580	.01568	73827	19.97
10	.00038	.00038	91645	64.41	62	.01698	.01684	72670	19.28
11	.00040	.00040	91610	63.43	63	.01826	.01809	71446	18.60
12	.00044	.00044	91573	62.46	64	.01964	.01944	70153	17.93
13	.00049	.00049	91533	61.49	65	.02112	.02090	68789	17.28
14	.00054	.00054	91488	60.52	66	.02273	.02247	67351	16.64
15	.00060	.00060	91438	59.55	67	.02446	.02416	65838	16.01
16	.00067	.00067	91383	58.58	68	.02633	.02599	64247	15.39
17	.00074	.00074	91322	57.62	69	.02835	.02795	62577	14.79
18	.00081	.00081	91255	56.67	70	.03053	.03007	60828	14.20
19	.00089	.00089	91181	55.71	71	.03287	.03234	58999	13.63
20	.00097	.00097	91100	54.76	72	.03541	.03479	57091	13.07
21	.00105	.00105	91012	53.81	73	.03814	.03743	55104	12.52
22	.00114	.00114	90916	52.87	74	.04109	.04026	53042	11.99
23	.00124	.00124	90812	51.93	75	.04427	.04331	50906	11.47
24	.00134	.00134	90699	50.99	76	.04769	.04658	48702	10.96
25	.00144	.00144	90578	50.06	77	.05138	.05009	46433	10.48
26	.00155	.00155	90448	49.13	78	.05536	.05386	44107	10.00
27	.00166	.00166	90308	48.21	79	.05964	.05791	41731	9.54
28	.00178	.00177	90159	47.29	80	.06424	.06224	39315	9.10
29	.00190	.00190	89999	46.37	81	.06920	.06689	36868	8.67
30	.00203	.00203	89828	45.46	82	.07454	.07186	34402	8.25
31	.00217	.00216	89646	44.55	83	.08027	.07718	31929	7.85
32	.00231	.00231	89452	43.64	84	.08644	.08286	29465	7.47
33	.00246	.00246	89245	42.74	85	.09306	.08893	27024	7.10
34	.00262	.00262	89025	41.85	86	.10018	.09540	24620	6.74
35	.00280	.00279	88792	40.96	87	.10781	.10230	22272	6.40
36	.00298	.00297	88544	40.07	88	.11600	.10964	19993	6.07
37	.00317	.00317	88281	39.19	89	.12477	.11744	17801	5.76
38	.00338	.00337	88002	38.31	90	.13417	.12573	15711	5.46
39	.00359	.00359	87705	37.44	91	.14422	.13452	13735	5.17
40	.00383	.00382	87390	36.57	92	.15498	.14383	11887	4.90
41	.00408	.00407	87056	35.71					
42	.00435	.00434	86702	34.85					
43	.00463	.00462	86326	34.00					
44	.00494	.00493	85927	33.16	**PARAMETERS:**	A= .02779	B= .56113		
45	.00527	.00526	85503	32.32		C= .43809	D= .00119		
46	.00563	.00561	85054	31.49		E= .86895	F=63.37608		
47	.00601	.00599	84577	30.66		G= .00012	H= 1.08145		
48	.00642	.00640	84070	29.84					
49	.00686	.00684	83532	29.03					
50	.00733	.00731	82961	28.23					
51	.00785	.00782	82355	27.43					

Saudi Life Table (Males)

AGE	M(X,N)	Q(X,N)	I(X)	E(X)	AGE	M(X,N)	Q(X,N)	I(X)	E(X)
0	.06486	.06205	100000	68.27	52	.00840	.00836	81712	26.65
1	.01306	.01297	93795	71.76	53	.00899	.00895	81028	25.87
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5	.00069	.00069	91856	69.25	57	.01188	.01181	77848	22.84
6	.00049	.00049	91793	68.30	58	.01275	.01267	76929	22.11
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17	.00074	.00074	91322	57.62	69	.02835	.02795	62577	14.79
18	.00081	.00081	91255	56.67	70	.03053	.03007	60828	14.20
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22	.00114	.00114	90916	52.87	74	.04109	.04026	53042	11.99
23	.00124	.00124	90812	51.93	75	.04427	.04331	50906	11.47
24	.00134	.00134	90699	50.99	76	.04769	.04658	48702	10.96
25	.00144	.00144	90578	50.06	77	.05138	.05009	46433	10.48
26	.00155	.00155	90448	49.13	78	.05536	.05386	44107	10.00
27	.00166	.00166	90308	48.21	79	.05964	.05791	41731	9.54
28	.00178	.00177	90159	47.29	80	.06424	.06224	39315	9.10
29	.00190	.00190	89999	46.37	81	.06920	.06689	36868	8.67
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34	.00262	.00262	89025	41.85	86	.10018	.09540	24620	6.74
35	.00280	.00279	88792	40.96	87	.10781	.10230	22272	6.40
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44	.00494	.00493	85927	33.16					
45	.00527	.00526	85503	32.32					
46	.00563	.00561	85054	31.49					
47	.00601	.00599	84577	30.66					
48	.00642	.00640	84070	29.84					
49	.00686	.00684	83532	29.03					
50	.00733	.00731	82961	28.23					
51	.00785	.00782	82355	27.43					

PARAMETERS: A= .02779 B= .56113
C= .43809 D= .00119
E= .86895 F=63.37608
G= .00012 H= 1.08145

Appendix II

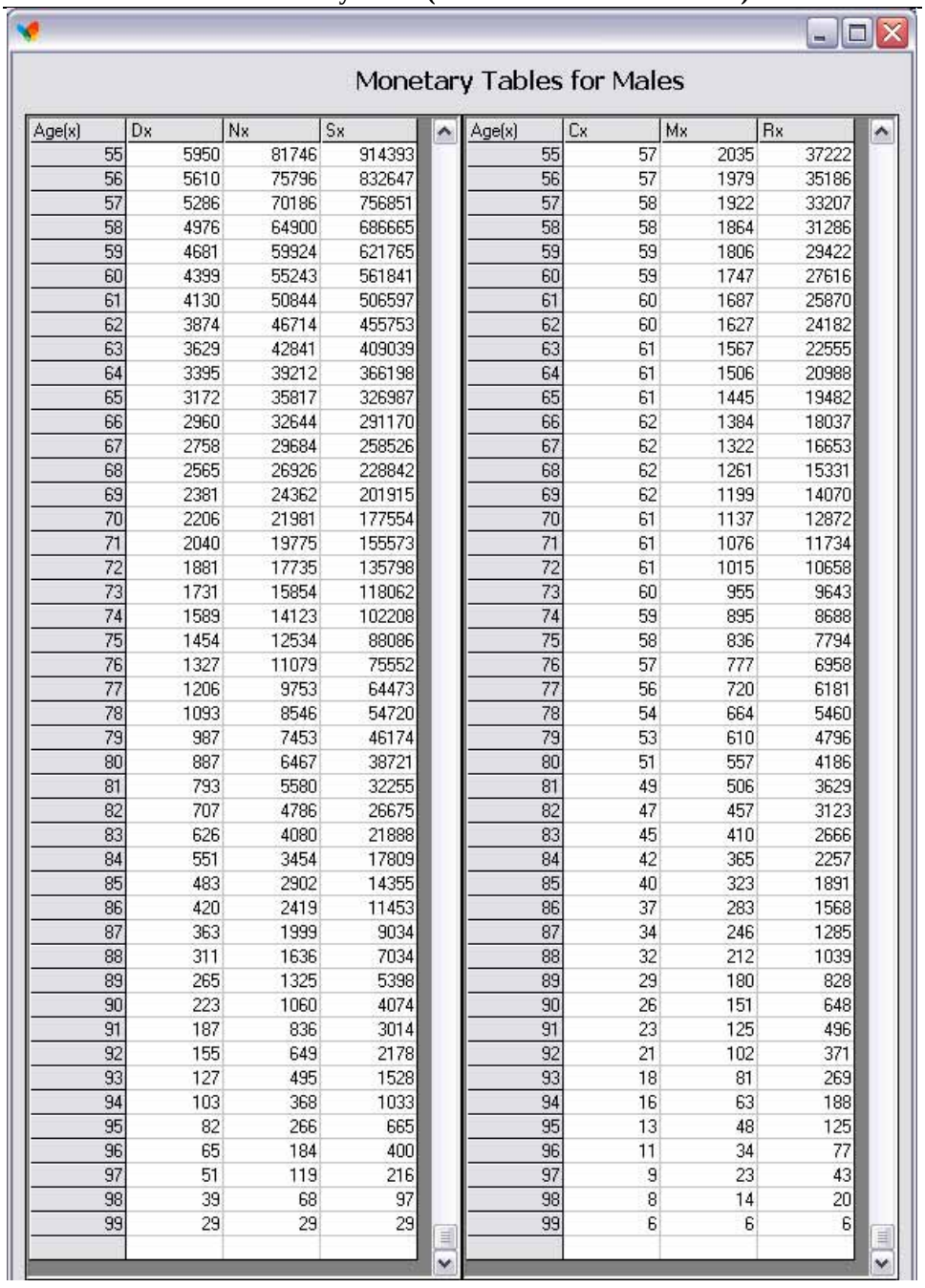
Saudi Monetary Table (Interest rate 5% and Males)

Monetary Tables for Males

Age(x)	Dx	Nx	Sx
10	61391	1206456	21559879
11	58446	1145065	20353423
12	55641	1086619	19208358
13	52969	1030977	18121739
14	50422	978008	17090762
15	47996	927586	16112753
16	45683	879590	15185167
17	43480	833907	14305578
18	41379	790427	13471671
19	39378	749048	12681244
20	37470	709670	11932196
21	35652	672201	11222526
22	33919	636549	10550325
23	32267	602630	9913776
24	30694	570363	9311146
25	29194	539669	8740783
26	27764	510476	8201113
27	26402	482711	7690637
28	25104	456309	7207926
29	23867	431205	6751617
30	22688	407338	6320411
31	21565	384650	5913073
32	20494	363085	5528423
33	19474	342591	5165338
34	18502	323117	4822747
35	17576	304615	4499630
36	16693	287039	4195015
37	15852	270346	3907976
38	15051	254494	3637631
39	14287	239443	3383137
40	13559	225156	3143694
41	12865	211598	2918538
42	12203	198733	2706940
43	11573	186530	2508207
44	10972	174957	2321677
45	10400	163984	2146721
46	9854	153585	1982737
47	9333	143731	1829152
48	8837	134398	1685421
49	8364	125561	1551023
50	7912	117197	1425463
51	7482	109285	1308265
52	7071	101803	1198980
53	6680	94732	1097177
54	6306	88052	1002445
55	5950	81746	914393
56	5610	75796	832647

Age(x)	Cx	Mx	Rx
10	22	3919	177832
11	22	3898	173913
12	23	3876	170015
13	24	3853	166139
14	25	3829	162286
15	27	3803	158457
16	28	3776	154654
17	30	3748	150878
18	31	3718	147130
19	33	3687	143412
20	34	3654	139725
21	35	3620	136071
22	36	3585	132451
23	37	3549	128866
24	38	3512	125317
25	39	3473	121805
26	40	3434	118332
27	41	3394	114898
28	42	3353	111504
29	42	3312	108151
30	43	3269	104839
31	44	3226	101570
32	44	3183	98343
33	45	3139	95161
34	45	3094	92022
35	46	3049	88929
36	46	3003	85880
37	47	2957	82877
38	47	2910	79920
39	48	2863	77010
40	48	2815	74148
41	49	2767	71333
42	49	2718	68566
43	50	2669	65848
44	50	2619	63179
45	51	2569	60559
46	51	2518	57990
47	52	2467	55472
48	52	2415	53005
49	53	2363	50590
50	54	2310	48227
51	54	2256	45918
52	55	2202	43662
53	55	2147	41460
54	56	2091	39313
55	57	2035	37222
56	57	1979	35186

Saudi Monetary Table (Interest Rate 5% and Males)



Monetary Tables for Males

Age(x)	Dx	Nx	Sx
55	5950	81746	914393
56	5610	75796	832647
57	5286	70186	756851
58	4976	64900	686665
59	4681	59924	621765
60	4399	55243	561841
61	4130	50844	506597
62	3874	46714	455753
63	3629	42841	409039
64	3395	39212	366198
65	3172	35817	326987
66	2960	32644	291170
67	2758	29684	258526
68	2565	26926	228842
69	2381	24362	201915
70	2206	21981	177554
71	2040	19775	155573
72	1881	17735	135798
73	1731	15854	118062
74	1589	14123	102208
75	1454	12534	88086
76	1327	11079	75552
77	1206	9753	64473
78	1093	8546	54720
79	987	7453	46174
80	887	6467	38721
81	793	5580	32255
82	707	4786	26675
83	626	4080	21888
84	551	3454	17809
85	483	2902	14355
86	420	2419	11453
87	363	1999	9034
88	311	1636	7034
89	265	1325	5398
90	223	1060	4074
91	187	836	3014
92	155	649	2178
93	127	495	1528
94	103	368	1033
95	82	266	665
96	65	184	400
97	51	119	216
98	39	68	97
99	29	29	29

Age(x)	Cx	Mx	Rx
55	57	2035	37222
56	57	1979	35186
57	58	1922	33207
58	58	1864	31286
59	59	1806	29422
60	59	1747	27616
61	60	1687	25870
62	60	1627	24182
63	61	1567	22555
64	61	1506	20988
65	61	1445	19482
66	62	1384	18037
67	62	1322	16653
68	62	1261	15331
69	62	1199	14070
70	61	1137	12872
71	61	1076	11734
72	61	1015	10658
73	60	955	9643
74	59	895	8688
75	58	836	7794
76	57	777	6958
77	56	720	6181
78	54	664	5460
79	53	610	4796
80	51	557	4186
81	49	506	3629
82	47	457	3123
83	45	410	2666
84	42	365	2257
85	40	323	1891
86	37	283	1568
87	34	246	1285
88	32	212	1039
89	29	180	828
90	26	151	648
91	23	125	496
92	21	102	371
93	18	81	269
94	16	63	188
95	13	48	125
96	11	34	77
97	9	23	43
98	8	14	20
99	6	6	6

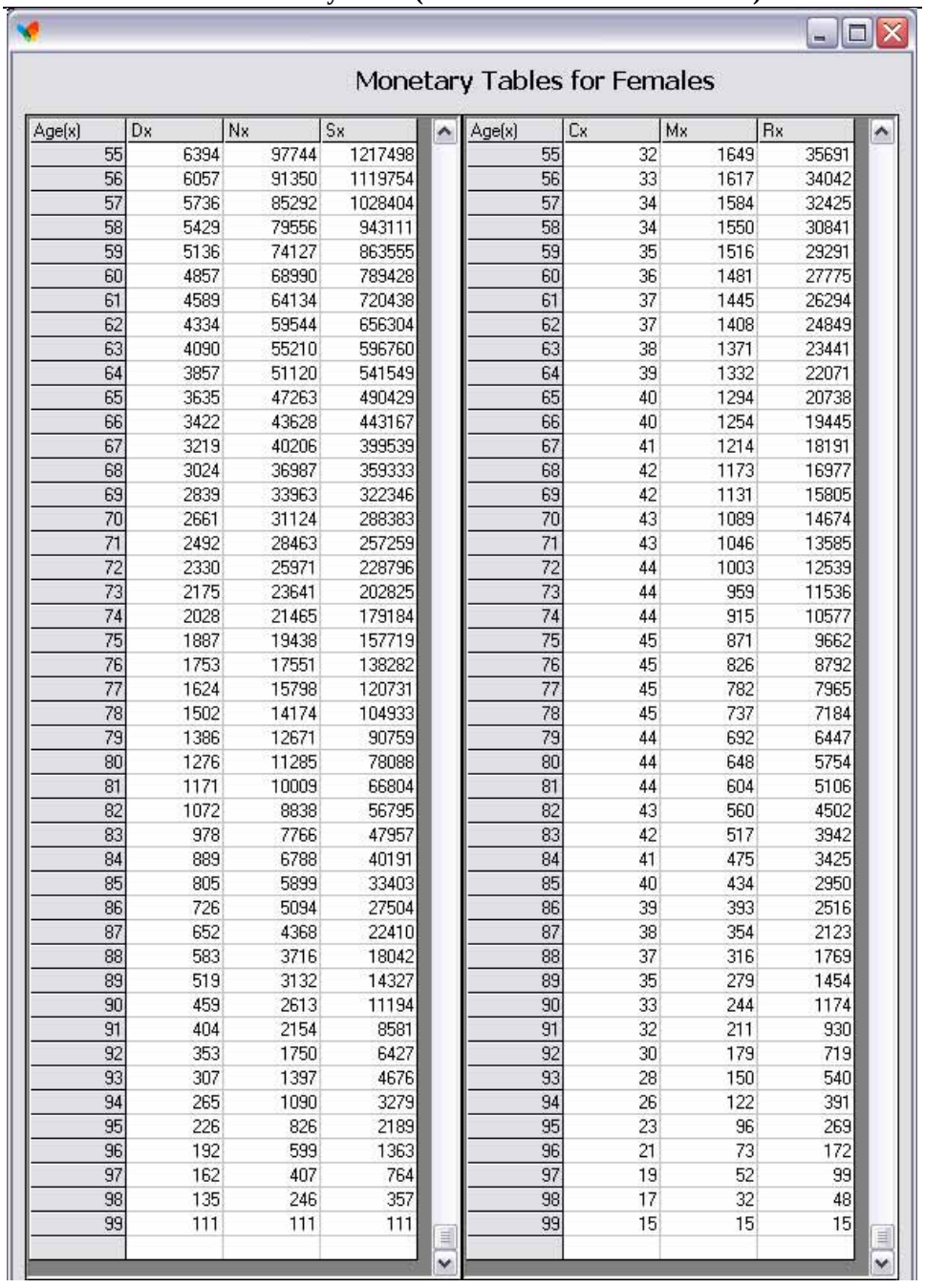
Saudi Monetary Table (Interest Rate 5% and Females)

Monetary Tables for Females

Age(x)	Dx	Nx	Sx
10	61391	1233399	22917821
11	58454	1172008	21684422
12	55658	1113553	20512414
13	52993	1057896	19398861
14	50456	1004902	18340966
15	48039	954446	17336063
16	45736	906408	16381617
17	43543	860671	15475209
18	41454	817128	14614538
19	39465	775674	13797410
20	37569	736209	13021736
21	35764	698640	12285527
22	34045	662875	11586887
23	32407	628830	10924012
24	30847	596423	10295182
25	29361	565577	9698758
26	27945	536216	9133182
27	26597	508271	8596966
28	25313	481674	8088695
29	24089	456361	7607022
30	22924	432272	7150661
31	21814	409348	6718389
32	20756	387534	6309042
33	19749	366777	5921508
34	18789	347028	5554731
35	17875	328239	5207702
36	17003	310365	4879463
37	16173	293361	4569099
38	15382	277188	4275738
39	14628	261806	3998549
40	13910	247178	3736743
41	13225	233269	3489565
42	12572	220044	3256296
43	11950	207472	3036252
44	11357	195522	2828780
45	10791	184165	2633259
46	10252	173374	2449093
47	9738	163121	2275720
48	9248	153383	2112598
49	8781	144135	1959215
50	8335	135354	1815080
51	7909	127020	1679726
52	7503	119110	1552706
53	7116	111607	1433595
54	6747	104491	1321988
55	6394	97744	1217498
56	6057	91350	1119754

Age(x)	Cx	Mx	Rx
10	13	2567	133921
11	13	2554	131353
12	14	2541	128799
13	14	2527	126259
14	14	2513	123732
15	15	2498	121219
16	15	2484	118721
17	15	2468	116237
18	16	2453	113769
19	16	2437	111316
20	16	2421	108879
21	16	2405	106458
22	17	2389	104053
23	17	2372	101664
24	17	2355	99292
25	17	2338	96937
26	17	2321	94599
27	18	2303	92278
28	18	2285	89975
29	18	2267	87690
30	19	2249	85422
31	19	2230	83173
32	19	2212	80943
33	19	2193	78731
34	20	2173	76538
35	20	2154	74365
36	21	2133	72211
37	21	2113	70078
38	21	2092	67965
39	22	2070	65873
40	22	2049	63803
41	23	2026	61754
42	23	2003	59728
43	24	1980	57725
44	25	1956	55745
45	25	1931	53789
46	26	1906	51858
47	26	1880	49952
48	27	1854	48072
49	28	1826	46219
50	28	1799	44392
51	29	1770	42594
52	30	1741	40824
53	31	1711	39083
54	31	1680	37372
55	32	1649	35691
56	33	1617	34042

Saudi Monetary Table (Interest Rate 5% and Females)



Monetary Tables for Females

Age(x)	Dx	Nx	Sx
55	6394	97744	1217498
56	6057	91350	1119754
57	5736	85292	1028404
58	5429	79556	943111
59	5136	74127	863555
60	4857	68990	789428
61	4589	64134	720438
62	4334	59544	656304
63	4090	55210	596760
64	3857	51120	541549
65	3635	47263	490429
66	3422	43628	443167
67	3219	40206	399539
68	3024	36987	359333
69	2839	33963	322346
70	2661	31124	288383
71	2492	28463	257259
72	2330	25971	228796
73	2175	23641	202825
74	2028	21465	179184
75	1887	19438	157719
76	1753	17551	138282
77	1624	15798	120731
78	1502	14174	104933
79	1386	12671	90759
80	1276	11285	78088
81	1171	10009	66804
82	1072	8838	56795
83	978	7766	47957
84	889	6788	40191
85	805	5899	33403
86	726	5094	27504
87	652	4368	22410
88	583	3716	18042
89	519	3132	14327
90	459	2613	11194
91	404	2154	8581
92	353	1750	6427
93	307	1397	4676
94	265	1090	3279
95	226	826	2189
96	192	599	1363
97	162	407	764
98	135	246	357
99	111	111	111

Age(x)	Cx	Mx	Rx
55	32	1649	35691
56	33	1617	34042
57	34	1584	32425
58	34	1550	30841
59	35	1516	29291
60	36	1481	27775
61	37	1445	26294
62	37	1408	24849
63	38	1371	23441
64	39	1332	22071
65	40	1294	20738
66	40	1254	19445
67	41	1214	18191
68	42	1173	16977
69	42	1131	15805
70	43	1089	14674
71	43	1046	13585
72	44	1003	12539
73	44	959	11536
74	44	915	10577
75	45	871	9662
76	45	826	8792
77	45	782	7965
78	45	737	7184
79	44	692	6447
80	44	648	5754
81	44	604	5106
82	43	560	4502
83	42	517	3942
84	41	475	3425
85	40	434	2950
86	39	393	2516
87	38	354	2123
88	37	316	1769
89	35	279	1454
90	33	244	1174
91	32	211	930
92	30	179	719
93	28	150	540
94	26	122	391
95	23	96	269
96	21	73	172
97	19	52	99
98	17	32	48
99	15	15	15

تدريج معدلات الوفاة السعودية الخام وبناء جداول رموزها الحسابية

أحمد درويش عابد*، وأحمد فمحاوي أباظة، وعثمان إبراهيم السلوم*****

** أستاذ مشارك، قسم الأساليب الكمية، كلية العلوم الإدارية، جامعة الملك سعود*

*** أستاذ زائر، قسم التحويل والمؤسسات المالية، جامعة الكويت*

**** أستاذ مساعد، قسم الأساليب الكمية، كلية العلوم الإدارية، جامعة الملك سعود*

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ملخص البحث. تم في هذا البحث تطبيق الطرق الإكتوارية لإيجاد قانون للوفاة أو تعبير رياضي يمثله وباستخدامه تم تهذيب وتدريج معدلات الوفاة الخام لكل من الذكور والإناث السعوديين، كذلك تم استخدام هذه الاحتمالات المهذبة ومعدل استثمار مركب (٥٪ - كمثال) لبناء جداول الرموز الحسابية اللازمة لتقدير القيم الحالية الإكتوارية لدفعات الحياة السعودية.