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A Production Planning Model for an Aluminum Company

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Abstract. In this paper, we present a real life case study of the design of a multi-product, multi-stage, multi-period production planning model for an aluminum company. The objective of the model is to obtain an optimal operational policy that minimizes the total costs of aluminum and final casting products. The model is formulated as a linear program that takes into acount the materials flow from the purchasing of raw materials to the reduction and casting stages. The model will assist the company in its decision making with respect to uncontrollable parameters changes and any modification in its production policy.

Introduction

This paper deals with a real life case study of the design of a multi-product, multi-stage, multi-period production planning model for an aluminum company. The purpose of the study is to develop a mathematical model that helps managers in their decision making, and to propose an optimal operational policy that minimizes the total costs of aluminum and finished products production.

Company background*

The company under study is an aluminum smelting company that has grown rapidly to become one of the Gulf's largest non-oil companies. It has five major operational areas:

1. The seaport terminal: the first transit location for the imported raw materials.
2. The power station: provides the electricity required in the reduction process (the production process of aluminum hot metal).

* All names, places, and figures have been disguised to protect for confidentiality.

3. The carbon department: produces the anodes consumed in the reduction process.
4. The potrooms: the location where the smelting reduction process takes place.
5. The casthouse: the place where the molten aluminum is cast into finished products (standard ingot, (SI), T-ingot, (TI), extrusion billet, (EX), rolling ingot, (RI), and bus bar, (BS).

The company purchases five major raw materials (petroleum coke, (C), alumina, (A), fluoride, (F), cryolite, (Y), pitch, (P)) from outside suppliers and stores them until needed for production. The flow of materials starts from the acquisition of these raw materials and continues till the smelting process and the casting of hot-metal. The finished products are then stored in regional warehouses before their distribution. Next, we describe in detail the flow of materials from the receipt of raw materials to the delivery of finished products. Figure 1 is a pictorial representation of this flow of materials.

The company imports the five major raw materials from different sources. The Petroleum coke and alumina are unloaded at the seaport terminal. They are transported to the plant's warehouse by ropeway road. The other main raw materials are imported through another seaport. There are no special storage areas for the company in this port since the raw materials are directly transported to the local warehouse by trucks.

All the above raw materials are unloaded in the company's local warehouse which is located near the potrooms in the plant. There is special and limited space for each kind of raw material.

The potrooms are where the smelting process, which is known as electrolytic reduction, takes place. Through this process alumina is transformed into aluminum hot metal. Pots of each pair of potrooms are electrically connected in series to form one potline.

A quantity of the liquid metal produced in the potrooms is transported in crucibles to subsidiary companies located near the plant.

In the casthouse the hot-metal is mixed with addition elements such as silicon, copper or iron to meet the required alloy specification. Titanium and boron can also be added for the purpose of grain refining. The liquid metal is then transferred to holding furnaces for casting. After that aluminum is cast in five different types of finished products.

The final products are then transported to the aluminum storage before they are shipped to internal and external customers.

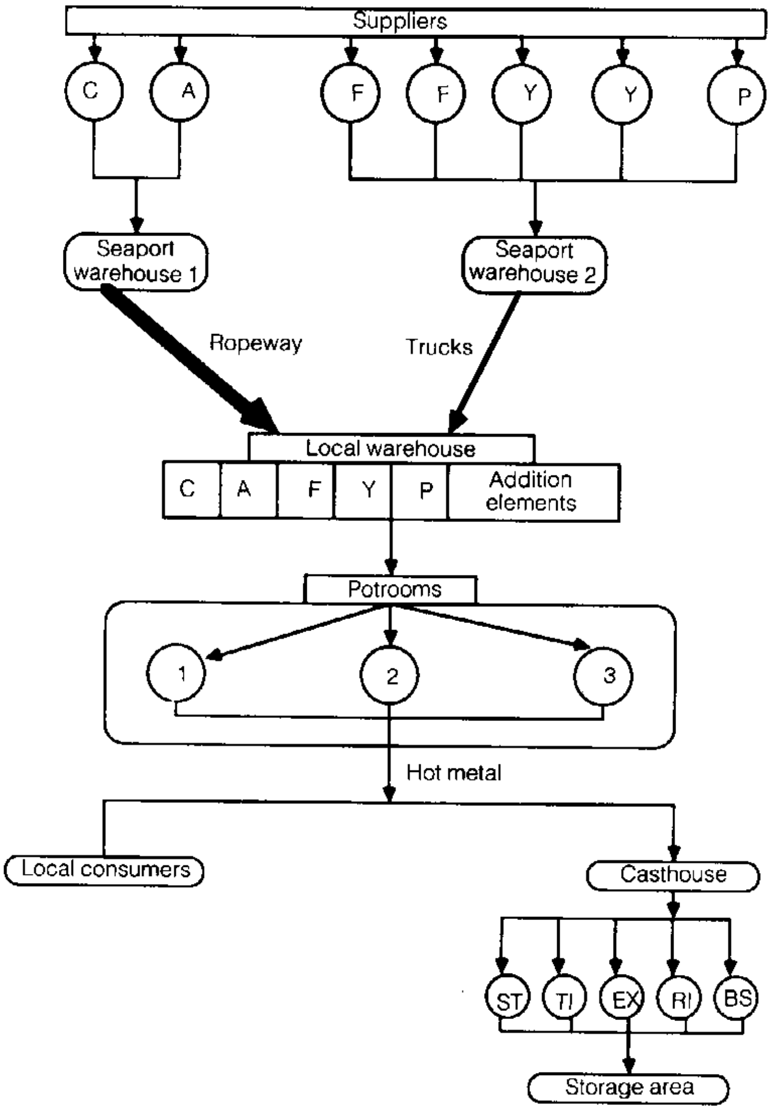

Fig. 1. Materials flow in the company

An outline of this paper is as follows. In the section on Model Formulation, we draw up a linear programming model for the company's operations. We present the necessary data for the model in the seciton on Data Gathering, while a sensitivity analysis is conducted for the output of the model in the next section. Finally, in the last section, we make some concluding remarks.

Model Formulation

After having a clear idea about the flow of materials in the company, the next step is to develop a mathematical model that will help the company's managers in their decision making. The managerial decisions concerning the purchasing, production, and inventory functions require answers to the following questions:

- From where should each raw material be supplied?
- What quantity of each raw material should be ordered from each supplier during each period?
- What quantity of hot-metal should be produced by each potline during each period?
- What quantity of hot-metal should be cast, and what quantity should be shipped to internal and external customers during each period?
- What quantity of each finished product should be produced during each period?
- What quantity of each raw material and each final product should be carried in stock at the end of each period?

Because of the size of the problem (number of raw materials, number of final products, number of periods, ...) and the complexity of the company organization (interaction between the different functions of the company), an adequate mathematical model is required for the answers to the above questions.

The literature of production planning [1-3] contains a large number of linear programming applications. We believe that linear programming is the most alluring field in Operations Research and has a wide applicability in other areas of Industrial Engineering. The solvability and the understandability of linear programming lead us to use this mathematical tool for our model. Moreover, the linear programming model will allow us to incorporate all the decision variables of the company from the purchase of raw materials to the casting process. Finally, the output of the linear program will provide the ranges of the different parameters of the model over which they can fluctuate without changing the optimal solutions. These ranges are of vital importance since most of the parameters are based on management estimation.

The derived mathematical formulation can be fed into the *TURBOSIMPLEX* software [4] to obtain optimal values of the decision variables (quantity of raw materials to be purchased, quantity of hot-metal to be produced, quantity of aluminum to be sold, ... etc.) that achieve the lowest procurement of raw materials and addition elements, inventory investment at the seaport and the company's local warehouses, reduction, transportation, and casting costs. Profit was not considered as an objective funiton for our linear program because we are assuming that the company is meeting all demand requirements of the final products. This assumption will continue to be valid as long as the company is negotiating the customer orders on a contract basis. Under this contract, the company considers firm orders for which the due dates and the quantity can not be changed at any time. Therefore, the profit maximization is equivalent to cost minimization. Next, we will follow the general steps taken in model formulation of a pratical problem.

Selection of a planning horizon

One of the most important decisions that has to be made before the formulation is the time horizon covered by the model and the number of time periods into which that time horizon is divided. In order to make our problem as simple as possible, we can assume a planning horizon of half a year with three periods of two months each. However, the model can be easily modified to cover more periods (6 periods, 12 periods, 52 periods, ... etc) with a longer planning horizon. It should also be noted, as we shall see below, that the choice of the length of the planning horizon will have an effect of the size of the model. In practice, this large scale production planning model is implemented using the rolling horizon strategy. Under this strategy, the model is repeatedly solved for short time horizon (3 to 6 months) as new information becomes available and is used to update the model parameters.

Selection of decision variables and parameters

The notation, which is used in the linear programming formulation, is described in Appendix 1:

Decision variables

The decision variables consist of the information necessary for the different managers of the company to answer the above managerial questions. The decision variables, which were defined for this purpose, are presented in Appendix 2.

Parameters

The parameters of the linear programming model are presented in Appendix 3.

Definition of the objective function

The following expression representing the total cost incurred during the planning horizon is used as the objective function for the linear program.

$$Z = \sum_{j} \sum_{i=1}^{n_j} \sum_{t=1}^{TH} \left(P_{ji} + T_{ji} + TR_{ji}\right) X_{jit} \tag{1}$$

$$+ \sum_{t=1}^{TH} h_{Cmt} I_{Cmt} + h_{Am} I_{Amt} \tag{2}$$

$$+ \sum_{k} \sum_{t=1}^{TH} TC_{kt} X_{kt} \tag{3}$$

$$+ \sum_{k} \sum_{t=1}^{TH} h_{j1} I_{j1t} + \sum_{k} \sum_{t=1}^{TH} h_{k1} I_{k1t} \tag{4}$$

$$+ TN_{F1} \sum_{i=1}^{SF} \sum_{t=1}^{TH} X_{Fit} + TN_{Y1} \sum_{i=1}^{SY} \sum_{t=1}^{TH} X_{Yit} + TN_{p1} \sum_{i=1}^{SP} \sum_{t=1}^{TH} X_{Pit} \tag{5}$$

$$+ \sum_{t=1}^{TH} \sum_{L=1}^{3} TRC_{Lt} HM_{Lt} \tag{6}$$

$$+ \sum_{t=1}^{TH} \sum_{u=1}^{s} TRHS_{u} HMS_{u} \tag{7}$$

$$+ \sum_{t=1}^{TH} \sum_{f} CS_{ft} X_{ft} \tag{8}$$

$$+ \sum_{t=1}^{TH} \sum_{f} h_{ft} I_{ft} \tag{9}$$

In the following, we will describe all the terms in the objective function.

The first term is the total procurement cost for raw materials.
The second term is the marine's warehouse inventory cost for coke and alumina.
The third term is the total procurement cost for addition elements.

The fourth term is the inventory cost for raw materials and addition elements which are left in the plant's warehouse.
The fifth term is the transportation cost from the seaport to the plant's warehouse for fluoride, cryolite, and pitch.
The sixth term is the total reduction process cost.
The seventh term is the transportation cost for the quantity of hot metal shipped to internal consumers.
The eighth term is the total casting cost for all finished products.
The ninth term is the inventory cost for all finished products left in the plant storage area.

Definition of the constraints

Suppliers limitation constraints

$$1.\quad \sum_{t=1}^{TH} X_{jit} \le MQ_{ji};\ \ j = C, A, F, T, P; i = 1, \ldots, n_j,$$

These constraints state that the quantity of each raw material that can be ordered from each supplier over the planning horizon should be less or equal to maximum allowable import quantity. This restriction is imposed by the company's country Central Bank to control the outflow of foreign currency.

Seaport-plant warehouses transfer quantity constraints

$$2.\quad \sum_{i=1}^{n_j} X_{jit} + I_{jm(t-1)} - I_{jmt} = QP_{mj}; J = C, A; t = 1, \ldots, TH.$$

These equations state that the quantity of coke or alumina transferred from the seaport's warehouse to the plant's warehouse through the ropeway should be equal to the quantity of coke or alumina received from all suppliers during period t plus the beginning inventory and minus the ending inventory at the same period.

Storage capacity constraints for the seaport's warehouse

$$3.\quad I_{jmt} \le C_{jm} - QP_{mj}; j = C, A; t = 1, \ldots, TH.$$

Constraints (3) restrict the maximum inventory of coke and alumina to be less than the storage capacity of the seaport's warehouse. The maximum inventory is attained at the beginning of the period since the inventory of each material is depleting linearly with respect to the usage rate of the same material. The amount of material j held in stock at the beginning of period t is given by:

$$\sum_{j=1}^{n_j} X_{jit} + I_{jm(t-1)} \quad \text{for} \quad j = C, A.$$

Therefore, constraints (3) can be obtained by using constraints (2) and the above expression for the beginning inventory.

Plant's warehouse capacity constraints for raw materials

$$4. \quad XD_{jt} \geq Qd_{mj}; \; j = C, A.; t = 1, \ldots, TH.$$

Constraints (4) states that the daily transfer quantity from the plant's warehouse to the potrooms should be at least the daily quantity which is delivered to the plant's warehouse from the seaport's warehouse through the ropeway. This constraint is introduced to ensure that the maximum inventory is always attained at the beginning of the period (day 1). To see this, the beginning inventory at day d (d=1, ..., 60) is $(I_{jl(t-1)} + (d-1)(Qd_{mj} - XD_{jt}))$. Therefore, the inventory will not increase only if $XD_{jt} \geq Qd_{mj}$

$$5. \quad I_{jl(t-1)} \leq C_{jl} - Qd_{mj}; \; j = C, A; \; t - 1, \ldots, TH.$$

Clearly, if $XD_{jt} \geq Qd_{mj}$ by constraint 4, then the maximum inventory will occur after the receipt of the first Qd_{m_j} mt. Hence, the maximum inventory is $I_{j(t-1)} + Qd_{mj}$ which should be smaller than the plant's warehouse space allocated to material j.

$$6. \quad \sum_{i=1}^{n_j} X_{jit} + I_{jl(t-1)} \leq C_{jl}; \; j = F, Y, P; \; t = 1, \ldots, TH.$$

These constraints show the limitation on the capacity of the plant's warehouse for fluoride, cryolite, and pitch at any time during period t. Since we assume that the raw materials are supplied at the beginning of the period, then the maximum inventory is always reached at the delivery time.

Plant's warehouse capacity constraints for addition elements

$$7. \quad X_{kt} + I_{kl(t-1)} \leq C_{kl}; \; k = S, M, R, I, T, B; \; t = 1, \ldots, TH.$$

Constraints (7) show the restriction on the capacity of the plant's warehouse for addition elements at any time during period t. The left hand side of the constraints is the maximum inventory which is attained at the beginning of the period.

Storage capacity constraints for finished products

8. $$I_{ft} + I_{f(t-1)} \leq 2S_{ft};\ f = ST,\ TI,\ RI,\ EX,\ BS;\ t = 1,\ldots,TH.$$

These constraints state that the average quantity of each finished product stored in the aluminum storage area should be less or equal to the maximum storage capacity available for each type of product. Since the finished products are transferred to the warehouse as soon as they are produced the average inventory is an adequate measure for such constraints.

Plant's warehouse–potrooms transfer quantity constraints

9. $$60XD_{jt} - I_{jl(t-1)} \leq QP_{mj};\ j = C,A;\ t = 1,\ldots,TH.$$

At end of day d the inventory of raw material j (j=C,A) is $I_{jl(t-1)} + d(Qd_{mj}\text{-}XD_{jt})$. The ending inventory will not increase since $XD_{jt} \geqslant Qd_{mj}$. However, we should always carry enough coke and alumina to send to the potrooms. In other words, at any day of period t the beginning inventory plus the quantity received through the ropeway should be larger than the quantity shipped to the potline, that is,

$$I_{jl(t-1)} + dQd_{mj} - (d-1)Xd_{jt} \geq XD_{jt},$$

or,

$$\left(XD_{jt} - Qd_{mj}\right)d \leq I_{jl(t-1)}.$$

For example,

at day 1 $$\left(XD_{jt} - Qd_{mj}\right) \leq I_{jl(t-1)}.$$

at day 2 $$2\left(XD_{jt} - Qd_{mj}\right) \leq I_{jl(t-1)}.$$

at day 60 $$60\left(XD_{jt} - Qd_{mj}\right) \leq I_{jl(t-1)}.$$

or $$60XD_{jt} - I_{jl(t-1)} \leq Qp_{mj};\ t = 1,2,3,\ldots,TH.$$

From the above example it is clear that if the restriction is satisfied for day 60, then it is also satisfied for the remaining days.

$$10.\quad 60XD_{jt} + I_{jl(t-1)} - I_{jlt} = Qp_{mj};\ \ j = C, A;\ \ t = 1, \ldots, TH.$$

Equations (10) are the balance equations for the quantities of coke and alumina transferred to the plant's warehouse during period t.

Production process constraints

$$11.\quad -B_j HM_t + 60XD_{jt} = 0;\ \ j = C, A;\ \ t = 1, \ldots, TH.$$

$$12.\quad -B_j HM_t + \sum_{i=1}^{n_j} X_{jit} + I_{jl(t-1)} - I_{jit} = 0;\ \ j = F, Y, P;\ \ t = 1, \ldots, TH.$$

Constraints (11) and (12) state that B_j metric tones (mt) of raw material j are required in order to produce one mt of hot metal of aluminum. They are balance equations since the quantity of each raw material consumed in the production of HM_t mt of hot metal should be equal to the quantity available for these raw material.

Potline balance equations

$$13.\quad -HM_t + \sum_{L=1}^{3} HM_{lt} = 0;\ \ t = 1, \ldots, TH.$$

These equations indicate that the total quantity of hot metal produced during period t is equal to the total quantity of hot metal produced by all potlines at period t.

Hot metal demand constraints

$$14.\qquad HM_t \geq DHM_t;\ \ t = 1, \ldots, TH.$$

These constraints state that the total hot metal produced during period t should be greater than or equal to the demand of hot metal.

Potline capacity constraints

$$15.\qquad HM_{Lt} \leq CAP_{Lt};\ \ t = 1, \ldots, TH.$$

Constraints (15) restrict the hot metal produced by each potline to be less than or equal to the production capacity.

Hot metal balance equations

$$16.\quad -HM_t + \sum_{u=1}^{2} HMS_{ut} + HMC_t = 0;\ t = 1,\ldots,TH.$$

These constraints indicate that the total quantity of hot metal produced by all potlines should be equal to the quantity of hot metal required to supply all internal consumers and the quantity delivered to the casthouse.

Internal consumer demand constraints

$$17.\quad HMS_{ut} \geq DHMS_{ut};\ u = 1,2;\ t = 1,\ldots,TH.$$

These constraints state that the quantity of hot metal supplied to the internal consumer must be is enough to satisfy its demand.

Casting balance equations

$$18.\quad X_{kt} + I_{k1(t-1)} - I_{k1t} - V_k HMC_t = 0;\ K = S,M,R,I,T,B;\ t = 1,\ldots,TH.$$

These equations state that the available quantity of each of the six addition elements is enough to meet the required alloy specifications of the different types of finished products.

$$19.\quad HMC_t - \sum_{f} X_{ft} + \sum_{k} X_{kt} + I_{k1(t-1)} - I_{k1t} = 0;\ t = 1,\ldots,TH.$$

This is the balance equations for the casting operation (input to the casthouse is equal to the output of the casthouse).

Casting capacity constraints

$$20.\quad \sum_{t=1}^{TH} X_{ft} \leq MC_{ft}; f = ST,TI,RI,EX,BS;\ t = 1,\ldots,TH.$$

These constraints state that the total quantity of each finished products produced during the planning horizon is less than or equal to the company's casting capacity of these types.

Final product demand constraints

$$21.\quad X_{ft} + I_{f(t-1)} - I_{ft} = D_{ft}; f = ST,TI,RI,EX;\ t = 1,\ldots,TH.$$

These equations state that the quantity of each finished product (excluding bus bar which is produced for local consumption) available for shipping is equal to the quantity demanded by internal and external customers.

As it can be noticed, the above formulated model is a large scale linear program with ((35+SC+SA+SF+SY+SP) TH) decision variables and ((SC+SA+SF+SY+SP)+45 TH) constraints excluding the nonnegativity constraints. This confirms our earlier statement that the length of the planning horizon increases the size of the model as it gets increased.

Data Gathering

Having formulated the company's production planning model as a linear program, the next step is to gather the data required to define the parameters introduced above. This step was the most time consuming and required several trips to the company.

In the computation of the inventory holding cost, we assume an inventory carrying charge of 10%. Also, we assume that the initial inventories for major raw materials (coke, alumina, pitch, cryolite, fluoride) are equal to one month consumption. For the remaining raw material, the initial inventories are set to zero.

Raw materials data

Table 1 reports the data needed for the five major raw materials.

Addition elements data

Table 2 shows the data that define the parameters related to the addition elements.

Potline reduction cost

In our formulation, we assumed that the company's potlines have different hot metal production capacities. We also assumed that the total reduction process cost incurred in each potroom is defined as the sum of the electrical reduction cost and the cost of mixing raw material to produce one mt of hot metal. The electrical reduction cost is calculated as the product of the energy required and the cost of 1 KWh of electricity. However, we should be aware of the fact that the energy required differs from one potline to the other. The total reduction process cost (TRC) for each potline is

Table 1. Raw material data

Data	Raw materials						
	Coke	Alumina	Fluoride		Cryolite		Pitch
Source(s)	USA	Australia	Jordan	Others	Italy	Japan	Germany
Transportation cost $/mt	50	17	20	75	80	70	70
Import quantity, mt/period	100000	20000	2000	3000	1000	700	
Storage space in seaport 1, mt	16,500	80,000	–		–		–
Storage space in seaport 2, mt	8000	50000	6000		2000		6000
Transit cost, $/mt	–	–	12		12		12
Seaport 1 – Plant Transportation cost, $/mt	4.8	4.8	–		–		–
Seaport 2 – Plant Transportation cost, $/mt	–	–	3.5		3.5		3.5
Marine holding cost $/mt	20	25	–		–		–
Local holding cost $/mt	25.48	27.18	126.3		99		30.55
Quantity required to produce one mt of hot metal, mt	0.34	1.89	0.04		0.12		0.09
Initial inventory, mt	5700	31500	700		200		1500

– not applicable.

$$TRC_1 = 511.62 \text{ \$/mt}$$

$$TRC_2 = 514.29 \text{ \$/mt}$$

$$TRC_3 = 5495.60 \text{ \$/mt}$$

Potline capacities and hot metal demands

The production capacities of hot metal for each potline during any period t are as follows:

CAP_{1t} = 14000 mt/period, CAP_{2t} = 12000 mt/period, CAP_{3t} = 18000 mt/period.

Table 2. Addition elements data

Addition element	Total purchasing cost of one mt	Storage space allocated, mt	Holding cost, $/mt	Blending quantity with one mt of hot metal
Silicon S	1,830	1,100	183	0.002
Magnesium M	4,050	806	405	0.003
Copper R	450	65	45	0.001
Iron I	550	403	55	0.001
Titanium T	1,000	237	100	0.0001
Boron B	880	40	88	0.001

The forecasted demands (quantity to deliver to the casthouse and to the internal customers) for hot metal over the coming three periods are: 40000, 38500, 38000.

The predicted demands for hot metal from internal consumers over the coming three periods are:

Internal consumer 1: 5030, 4915, 3810.

Internal consumer 2: 960, 940, 920.

Finished products data

Table 3 displays the finished product data.

Output Analysis

The analysis of the output is the most important stage in a practical application of linear programming. The analysis serves to help the managers to assess the consequences of their decisions (restriction on materials import, increase in production capacity, restriction on storage capacity, change in material prices, etc.) before implementing the linear program solution. Moreover, the analysis of the output

Table 3. Finished products data

Data		Type of ingots				
		Standard ingot	T-ingot	Extrusion billet	Rolling ingot	Bus bar
Storage capacity, mt		2,600	1,000	11,500	8,000	30
Casting cost, $/mt		27	37.5	56	59	25
Holding cost, $/mt		226.5	226.5	237	236	220
Production capacity, mt/period		10000	5000	30000	35000	30
Forecast	t = 1	9,630	230	7,170	11,550	3
	t = 2	9,460	225	7,010	11,280	3
	t = 3	9,240	220	6,860	11,045	3

should be carried out because most of the data gathered are based on managers' estimations and forecasting (future demand, cost data, hot metal and finished products specifications, etc.). Therefore, it is of vital importance to study the sensitivity of the linear program solution to the change in the data input.

The examination of the output file related to the optimal values of the decision variables revealed that the recommended purchasing and production schedules should be as shown in Table 4 and Table 5, respectively.

The distribution of the quantity of hot metal produced among the casthouse and internal customers is shown in Table 6.

The optimal solution of the linear program suggested that no inventory of coke and alumina should be held in the marine warehouse at the end of each period. However, the ending inventories in the local warehouse for the raw materials and in the Aluminum warehouse for the finished products are as reported in Table 7.

Finally, the optimal values for the daily transfer of coke and alumina from the local warehouse to the potrooms are summarized in Table 8.

Using the above optimal solution, the total cost of purchasing, inventory, reduction, and casting will be $ 140,945,596/six month.

Table 4. Raw materials purchasing schedule

Raw material mt/period	Period		
	1	2	3
Coke	11520	11520	11520
Alumina	70860	70860	70860
Fluoride	900	1540	1520
Cryolite	280	462	456
Pitch	2100	3465	3420
Silicon	56.8257	55.5064	54.2962
Magnisium	85.2385	82.25	81.4443
Copper	28.4129	27.7532	27.1481
Iron	28.4129	27.7532	27.1481
Titanium	2.8413	2.7753	2.7148
Boron	28.4129	27.7532	27.1481

Table 5. Production schedule

Product mt/period	Period		
	1	2	3
Hot-metal	40000	38500	38000
ST-ingot	9690	9460	9240
T-ingot	230	225	220
R-ingot	7170	7010	6860
EX-ingot	11550	11280	11045
B-bar	3	3	3

Table 6. Hot metal distribution

Product mt/period	1	2	3
Cast house	28412.86	27753.20	27148.1
Consumer 1	10627.14	9806.8	9931.9
Consumer 2	960	940	920

Table 7. Ending inventory output

Product, mt	1	2	3
Coke	2620	2050	650
Alumina	26760	24855	23895
Fluorid	0	0	0
Cryolite	0	0	0
Pitch	0	0	0
Silicon	0	0	0
Magnm.	0	0	0
Copper	0	0	0
Iron	0	0	0
Titanium	0	0	0
Boron	0	0	0
ST-ingot	0	0	0
T-ingot	0	0	0
R-ingot	0	0	0
EX-ingot	0	0	0
B-bar	0	0	0

Table 8. Daily transfer of coke and alumina

Product, mt/day	1	2	3
Coke	226.67	218.17	215.33
Alumina	1260.0	1212.75	1197.0

After examining the reduced cost of each of the decision variables, the shadow price of each constraint, the ranges of the objective coefficients, and the range of the constraint right-hand-sides, it is worth mentioning the following points:

- The purchase of some quantity of fluoride from supplier 1 instead of purchasing it from others during period 1 will not change the optimal cost since the reduced cost of the corresponding decision variable is zero.

- The same remark holds for the purchase of fluoride from supplier 2 during period 3 and for the purchase of cryolite from supplier 1 during period 1 and from supplier 2 during period 3.

- Any additional mt of coke left in the marine warehouse by the end of period 1, 2, and 3 will increase the cost by \$ 20, \$ 20, and \$ 270, respectively. This increase represents the cost of holding an additional mt storage at the end of period 1 and 2, and the cost of purchasing and holding an additional mt by the end of period 3.

- The previous remark can also be used to explain the reduced cost of the ending inventory variables corresponding to other raw materials and finished products.

- If the import quota of fluoride is increased by one mt the cost will decrease by \$55. The net change in the cost function is valid as long as the quantity of fluoride to be imported is in the range [1520, 3060]. Outside this range, the optimal solution will be different, and we can only give a bound on the change in the cost function which is equal to the change in the allowable quantity times 55. To obtain the new optimal solution the linear program has to be resolved.

- The same remark holds for all constraints with negative shadow prices.

- If the quantity of coke that can be transferred through the ropeway from the marine warehouse to the local warehouse is increased by one mt, then the total cost will increase by \$250. It is clear that an increase in the ropeway transfer capacity has an effect on the finished products quantities and on the amounts of coke and casting products that are held in stock. Therefore, the \$250 increase in the total cost is due to the increase in inventory, transportation, reduction, and casting costs. The company will increase the ropeway transfer quantity if it can find new markets for the supplement quantities of finished products and only when the unit revenue resulting from this increase is larger than \$250. The net change of the total cost is valid as long as the transfer quantity is in the range [0 , 20000]. Outside this range, the problem was resolved for different values of the ropeway transfer quantity. It was observed that the change in the total cost does not longer vary linearly as a function of the increase in the ropeway transfer quantity.

- An increase in the capacity of potline 1 by one mt will reduce the cost by \$2.67. The capacity can be increased to 22,000 without changing the optimal solution and cost will decrease by $2.67 \times 8000 =$ \$21,360.

- The increase of potline 2 capacity will not have any effect on the optimal solution since it is not used to full capacity.

- The increase in potline 3 capacity will decrease the cost by \$18.69 for each additional one mt.

Currently, the company has over-capacity for the produciton of finished products since the casthouse capacity is not fully utilized.

The optimal solution will not change as long as the cost of procuring (purchasing cost + transportation cost + transit cost) one mt of coke is in the range of [230, infinity]. The total cost changes linearly in this range with respect to the optimal value of the quantity of coke purchased; that is, the total cost change is equal to $11{,}520 \times \delta$, where δ is the net change in the cost coefficient of coke purchased.

- The range of the other objective function coefficients can be explained as in the above remark.

Conclusion

In this paper, we developed a production planning model that will help the aluminum company to minimize its total cost of aluminum production. The model was formulated as a linear program that is considered as one of the most practical production planning tools. The model takes into account the company flow of materials from the purchasing of raw materials to the reduction and casting stages. The linear program formulated has 135 variables and 160 constraints.

The results obtained will assist the company in its decision making with respect to out-of-hand parameter changes (demand, raw material prices, ...) and any modification in its production policy (production quantity, distribution, ...). The problem of a sudden change in parameters can be tackled by sensitivity analysis, that is, by looking over the range of each parameter and the net change resulting by its alteration.

Finally, the research study presented in this paper is not the final step in a practical applicaiton of Operations Research techniques. The implementation of the model is the next phase in such type of study. In fact, the obtained output should be tested to ensure that the model represents the real situation. In case the output is unacceptable the model has to be refined to include the missing information and tested again until the model becomes adequate.

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Appendix 1

Notation

The index j is used to represent the five major raw material, j = C,A,F,Y,P

The index i is used to denote the location of supplier sources, $i=1,\ldots,n_j$, where n_j is the number of available sources for raw material j. n_j is equal to SC for pitch coke, SA for alumina, SF for fluoride, SY for cryolite, and SP for pitch.

The index l is used to represent the plant's warehouse.

The index m is used ot represent the seaport (marine) warehouse.

The index k is used to represent the different types of addition elements. k = S, M, R, I, T, B, where S stands for silicon, M for magnesium, R for copper, I for titanium, and B for boron.

The index L is used to represent the potline.

The index t is used to represent the time period, t = 1,..., TH, where TH is the number of periods in the time horizon.

The index u is used to represent the internal consumers for hot metal.

The index f is used to denote the five finished products, f = SI, TI, EX, RI, BS.

Appendix 2

Model's Decision Variables

All the decision variables, defined below, are expressed in metric tones per period.

X_{ijt} = Quantity of raw material j, imported from source i at the beginning of period t, where i= 1, 2, 3,...,n_j, j= C, A, F, Y, P, and t= 1, 2, 3,...,TH.

I_{jmt} = Amount of raw material j, left in the marine's warehouse at the end of period t, where j= C, A.

I_{jlm} = Amount of raw material j, left in the plant's warehouse at the end of period t, where j= C, A, F, Y, P.

X_{kt} = Quantity of imported addition element k, at the beginning of period t, where k= S, M, R, I, T, B.

I_{klt} = Amount of addition element k, left in the plant's warehouse at the end of period t, where k= S, M, R, I, T, B.

XD_{jt} = Daily transfer of raw material j from the plant's warehouse to the potlines, where j= C,A.

HM_{Lt} = Quantity of hot metal produced by potline L during period t, where L= 1, 2, 3.

HM_t = Total quantity of hot metal produced during period t.

X_{ft} = Quantity of final product f, produced during period t, where f = ST, TI, RI, EX, BS, and t = 1, ..., TH.

I_{ft} = Amount of final product f left in stock at end of period, t, where f = ST, TI, RI, EX, BS, and t = 1,..., TH.

HMS_{ut} = Quantity of hot metal delivered to the internal consumer u during period t, where u= 1,2.

HMC_t = Quantity of hot metal supplied to casthouse during period t.

Appendix 3

Model's Parameters

P_{ji} = Purchasing price of raw material j from source i, in $/mt, where j= C, A, F, Y, P.

T_{ji} = Transportation cost of raw material j from source i to the seaports for coke, alumina, fluoride, cryolite, and pitch, in $/mt.

TR_j = Transit cost of raw material j, in $/mt, where j= F, Y, P. The transit costs are incurred for the raw materials that are unloaded in the seaport.

h_{jm} = Inventory holding cost for each mt of raw material j left in the marine's warehouse for one period, in \$/mt/period, where j= C,A.

TC_{kt} = Total purchasing and transportation cost of addition element k from source to the plant's warehouse during period t, in \$/mt.

TN_{jl} = Transportation cost of raw material j in \$/mt, from the seaport to the plant's local warehouse l, where j= F, Y, P.

Qd_{mj} = Daily quantity of raw material j carried from the marine's warehouse to the plant's warehouse, in mt.

Qp_{mj} = Quantity of raw material j carried from the marine's warehouse to the plant's warehouse during the period, in mt.

C_{jm} = Storage capacity for raw material j in the marine's wharehouse, in mt, where j = C,A.

C_{jl} = Storage capacity in the plant's warehouse for raw material j, in mt, where j= C, A, F, Y, P.

h_{jl} = Inventory holding cost for each mt of raw material j left in the plant's warehouse for one period, in \$/mt/period, where j= C, A, F, Y, P.

h_{kl} = Inventory holding cost for each mt of addition element k left in the plant's warehouse for one period, in \$/mt/period, where k= S, M,..., B.

MQ_{ji} = Maximum quantity of raw material j that can be imported from supplier i over the planning horizon, in mt.

CS_{ft} = Casting cost of final product f during period t, in \$/mt, where f = ST, TI, RI, EX, BS.

MC_{ft} = Maximum capacity of the plant for final product f during period t, in mt, where f= ST, TI, RI, EX, BS.

D_{ft} = Demand for final product f from external and internal customers, where f = ST, TI, RI, EX, BS.

$TRHS_u$ = Transportation cost of hot metal from the plant's casthouse to consumer u, in \$/mt.

h_{ft} = Inventory holding cost of final product f left in storage (Aluminum storage area) for one period, in \$/mt/period, where f = ST, TI, RI, EX, BS.

C_{kl} = Capacity of the plant's warehouse for addition element k, in mt.

B_j = Quantity of raw material j, in mt, required to produce one mt of hot metal aluminum.

TRC_{Lt} = Total reduction cost of potline L during period t, in \$/mt.

S_{ft} = Capacity of storage (Aluminum storage area) for final product f, in mt, where f = ST, TI, RI, EX, BS.

DHM_t = Demand of hot metal during period t, in mt.

CAP_{Lt} = Production capacity of potline L, in mt, during period t, in mt.

$DHMS_{us}$= Demand of hot metal from consumer u during period t, in mt.

V_k = Quantity of addition element k to blend with the hot metal in order to produce the specific alloy for the different type of finished products, in mt.

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نموذج تخطيط لشركة إنتاج ألومنيوم

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ملخص البحث. نقدم في هذا البحث دراسة واقعية لتصميم نموذج لتخطيط المنتجات متعددة المراحل والفترات لشركة إنتاج الومنيوم. ويهدف النموذج إلى الحصول على أمثل سياسة تشغيل، والتي تقلل من التكلفة الكليّة للألومنيوم، وقد تم تصميم النموذج بأسلوب البرمجة الخطية والذي يأخذ في اعتباره جميع مراحل الإنتاج من شراء المواد الخام إلى الحصول على المنتج النهائي. وستساعد نتائج هذا النموذج الشركة في اتخاذ القرارات وذلك باعتبار تغير العوامل الخارجة عن إرادتها في حالة أي تعديلات في سياسة الإنتاج.