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Optimum Manpower Distribution in Airlines Daily Maintenance Shifts

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The main purpose of this paper is to obtain an optimal distribution for the engineers and technicians in every shift of the shifts working in daily maintenance sections of airlines. The problem is considered a queueing one. Arrivals are considered to follow Poisson process and service times are assumed to be generally distributed. A model based on the above mentioned statements is constructed then a numerical application from Egypt Air is given a case study.

Introduction

Most of the airlines are working 24 hours per day in the system of shifts. Each shift is responsible of receiving, servicing, clearing defects and departure of all aircrafts that arrive or depart during its working period. The shifts consists of a group of engineers and technicians in a number of branches of the technical work. In each branch, the engineers and technicians are grouped in a number of teams S, each team is responsible for a number of arrivals. One may find an engineer or a foreman in two or more of these teams.

The problem is to determine the optimum number of teams in each of these branches given that the construction of each of these teams is predetermined by the relevant technical department. For this, the aircrafts arrivals are considered as an input to a queueing system with arrival rate λ and the service is carried out by a number of servers S with a service mean b.

The optimization is searched in the sense of minimizing the total costs of the system i.e. the problem is to determine the number of maintenance teams S that minimizes the total costs of the system under the given conditions and requirements from the technical view points.

Assumptions

The following are the assumptions and all considerations that are related to the problem. These assumptions deal with both technical and statistical view points.

Technical Assumptions

1. In each of the branches of technical work on the aircraft, the size of each team is exactly determined by the relevant technical department.

2. The effort of an engineer or a supervisor may be divided to supervise two or more teams so a one third of engineer will mean that the engineer supervises three teams at the same time.

3. All teams are ready to work on any type of aircrafts that are flying on the airlines.

4. Each team is provided with all technical requirements for his work without any common use of facilities or equipment of another team.

5. Two or more teams of different technical classes may work on the same aircraft at the same time.

6. The service time is that time which starts from aircraft arrival to the base and ends when the aircraft is technically ready for departure from the point of view of the working team.

7. Major defects and failures which lead to aircraft towing from maintenance lines are not taken into consideration where it is repaired by a special group of engineers and technicians.

Statistical Assumptions

1. The aircrafts arrive to the maintenance line in a Poisson process.

2. The arrival means all aircrafts that need technical work by any of the teams of the shift.

3. The parameter of the arrival process is not constant throughout the shifts of the day, moreover, it is not constant throughout one shift.

4. The service time is a general random variable. It is studies carefully to determine its statistical parameters.

5. The system is a queueing one. The queueing discipline is determined according to the rule: First come first serviced.

6. Any delayed aircraft costs C_1 in monetary units per unit time.

7. The total costs of a team in the shift is C_2 in monetary units per unit time in a certain branch of technical work.

The Model

Let us have an airline which is operating in the system of shifts, three shifts perday. Five branches of technical work are assumed to be working on the aircraft, these are: airframe — engines — radio and radar systems — electric systems — instruments and electronic systems. In each shift, the five branches of the technical work service the aircraft arrivals. The optimal number of teams is studied in each branch, then the optimum total number of teams in the shift will be the sum of the optimum number of teams in each branch.

Let the working period of a shift be divided to intervals of statistical equilibrium i.e. periods of homogeneous arrival and service rates. During a period of statistical equilibrium, let us assume aircrafts arrive in a Poisson process with parameter λ and they are serviced by a single server. Let the service times of these aircrafts be independent and identically distributed random variables so, let:

λ = rate of arrivals of aircrafts per unit time.

V = service time, let it be a random variable with $P(V_n \leqslant x) = \beta(x)$

$E(V) =$ b = mean value of the service time ....(1)

b_1 = the second moment of service time.

$\delta 2_V$ = Variance of the service time, so $b_1 = b^2 + \delta\, ^2V$ .... (2)

$Q(t)$ = number of aircrafts in the system at time t

$R(t)$ = the remaining time of the aircraft currently being serviced.

Q_n = number of aircrafts in the system soon after the n^{th} aircraft being completely serviced.

V_n = service time of the n^{th} aircraft.

X_n = number of aircrafts arriving during V_n with Poisson assumption for the arrival process. One so have:

$$K_j = p(X_n = j) = \int_o^\infty p(X_n = j \mid V_n\, dX$$

$$p(t < V_n \leqslant t + dt) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$= \int_o^\infty \frac{\bar{e}^{\lambda t} (\lambda t)^j}{j!} d\beta(t); \; j = 0, 1, 2 \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Define the following two functions:

$$\psi \quad (\theta) = \int_o^\infty \bar{e}^{\theta t} d\beta(t)$$

= Laplace - Stieltjes transformation of the distribution $\beta(X)$. . (5)

$$K(Z) = \sum_{j=o}^{\infty} K_j \quad Z^j \qquad |Z| \leqslant 1$$

= Generating function of the probability distribution . (6)

From the properties of both, laplace - Stieltjes transforms and probability generating functions we have:

$$\left.\begin{aligned} b &= E(X_n) = -\psi'(0) \\ & E(V^2_n) = \psi''(0) \\ & E(X_n) = K'(1) \\ & E(X^2_n) = K''(1) + K'(1) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

So from (4):

$$K(Z) = \sum_{j=o}^{\infty} K_j Z^j$$

$$= \sum_{j=o}^{\infty} Z^j \int_o^{\infty} \bar{e}^{\lambda t} \frac{(\lambda t)^j}{j!} \, d\beta(t)$$

$$= \int_o^{\infty} \bar{e}^{\lambda t} \sum_{j=o}^{\infty} \frac{(\lambda t \; Z)^j}{j!} \, d\beta(t)$$

$$= \int_o^{\infty} \bar{e}^{\lambda t} \; e^{\lambda t Z} \, d\beta(t)$$

$$= \int_o^{\infty} \bar{e}^{\,t} \; (\lambda - \lambda Z) \, d\beta(t)$$

$$= \psi(\lambda - \lambda Z) \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$K(Z) = \frac{\partial K}{\partial Z} = \frac{\partial \psi(\lambda - \lambda Z)}{\partial Z}$$

$$= -\lambda \psi'(\lambda - \lambda Z)$$

$$K''(Z) = \frac{\partial^2 K}{\partial Z^2} = \frac{\partial^2 \psi(\lambda - \lambda Z)}{\partial Z^2}$$

$$= \lambda^2 \psi''(\lambda - \lambda Z)$$

$$\therefore \; E(X_n) = K^1(1) = -\lambda \psi (0) = \lambda b = \rho \quad \ldots\ldots\ldots\ldots \textbf{(9)}$$

$$E(X^2_n) = \lambda^2 E(V^2_n) + \rho \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Consider now the relation between Q_n and Q_{n+1}

$$Q_{n+1} = \begin{cases} Q_n - 1 + X_{n+1} & ; \text{ if } Q_n > 0 \\ X_{n+1} & ; \text{ if } Q_n = 0 \end{cases} \quad \ldots\ldots (11)$$

For convenience define the Heaviside function H(x) as :

$$H(x) = \begin{matrix} 1 & ; \text{ if } X > 0 \\ 0 & ; \text{ if } X \leqslant 0 \end{matrix} \quad \ldots\ldots\ldots\ldots\ldots (12)$$

The function (12) has the following two properties:

$$\left. \begin{matrix} H^2(X) = H(X) \\ XH(x) = X \end{matrix} \right\} \; ; \text{ if } X \geqslant 0 \quad \ldots\ldots\ldots\ldots\ldots (13)$$

One may write equation (11) in the form:

$$Q_{n+1} = Q_n - H(Q_n) + X_{n+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and

$$Q^2_{n+1} = Q^2_n H^2(Q_n) + X^2_{n+1} - 2 Q_n H(Q_n) -$$

$$2X_{n+1} H(Q_n) + 2 Q_n X_{n+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Taking the expectation of both sides of equations (14) and (15) as $n \longrightarrow \infty$ and dropping the suffes n, one can show that

$$E(Q) = \rho + \frac{\rho^2)}{2(1-\rho)} + \frac{\lambda^2 + \delta^2_V}{2(1-\rho)} \quad \ldots\ldots (16)$$

From the known relation :

$$E(Q) = \lambda \; E(W + V)$$

Where E(W+V) is the expected stay, one can deduce that:

$$E(V) = \frac{\lambda (b^2 + \delta^2 V)}{2(1 - \rho)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Remember that we assume in the above derivation that we have a case of single server; let us have now :

S = Number of servers (maintenance teams) in the system.

C_1 = Cost of delay of an aircraft per unit time.

C_2 = Cost of a new team per unit time.

W_s = The long run waiting time (delay) of an aircraft.

In this case, the system is considered as to be the derived one except that the arrival rate will be λ/S while the service rate still $1/b$. One therefore have:

$$E(W_s) = \frac{(\lambda/S)(b^2 + \delta_v^2)}{2(1 - \lambda b/S)}$$

$$= \frac{\lambda (b^2 + \delta_v^2)}{2(S - \lambda b)}$$

If C is the total costs of the system, then:

$$E(C) = \frac{C_1 \lambda (b^2 + \delta_v^2)}{2(S - b)} + C_2 S, \quad \ldots\ldots\ldots (18)$$

differentiating with respect to S and equating to zero yields the optimum number of teams S_o to be :

$$S_o = \lambda b + \sqrt{\frac{\lambda C_1 (b^2 + \delta_v^2}{2 C_2}} \quad \ldots\ldots\ldots\ldots (19)$$

Since S_o should be integer valued, the optimum number of teams will be given by:

$$(S_o) \quad \text{if} \quad E(C) \Big| S_o < E(C) \Big| (S_o) + 1$$

$(S_o + 1)$ if $E(C) \mid (S_o > E(C) \mid (S_o) + 1$

Where (x) denotes the greatest integer contained in x.

Sensitivity Analysis

The following is a study for the sensitivity of S_o with respect to all parameters that affect its value:

Sensitivity of S_o with Respect to

The partial derivative $\frac{\partial S_o}{\partial \lambda}$ is considered good representation for the sensitivity of S_o w.r.t. λ assuming b and δ_v to be constant so the sensitivity of S_o with respect to λ will be:

$$\frac{\partial S_o}{\partial \lambda} = b + ½ \sqrt{\frac{C_1 \ (b^2 + \delta_v^2}{2 C_2}} \qquad \text{.........(20)}$$

Sensitivity of S_o With Respect to b

Also the sensitivity of S_o with respect to b will be given by:

$$\frac{\partial S_o}{\partial b} = \lambda + \sqrt{\frac{C_1 \ \lambda b}{2 C_2 (b^2 + \delta_v^2)}} \qquad \text{............(21)}$$

Sensitivity of S_o with Respect to δ V

Using the same approach, one can show that the sensitivity of S_o w.r.t. δ_v is given by:

$$\frac{\partial S_o}{\partial \delta_v} = \sqrt{\frac{\lambda C_1 \ \delta_v}{2 C_2 \ (b^2 + \delta_v^2)}} \qquad \text{............(22)}$$

The Case Study

Maintenance lines are working in Egyptair, as well as many airlines; in the system of shifts. Three shifts are working throughout the 24 hours of a day.

The first shift works from 6.30 morning up to 14.30. The Second shift works from 14.30 up to 22.30. The third shift works from 22.30 up to 6.30.

The given data shows that the day may be divided to the following periods of statistical equilibrium with corresponding number of arrivals per hour.

First Shift:

period	1	6.30	—	9.30	with	λ	=	3.5
period	2	9.30	—	12.00	with	λ	=	2.67
period	3	12.00	—	14.30	with	λ	=	2.80

Second Shift:

period	4	14.30	—	18.00	with	λ	=	1.50
period	5	18.00	—	22.30	with	λ	=	1.25

Third Shift:

period	6	22.30	—	3.00	with	λ	=	2.33
period	7	3.00	—	6.30	with	λ	=	0.80

A study of the mean value and variance of the service time yields the following results in the five branches of technical work:

Branch	Mean service time (hours)	Standard deviation of service time
Air frame	0.883	0.295
Engines	0.500	0.178
Electric systems	0.600	0.200

Instruments & electronic systems	0.500	0.190
Radio and radar systems	0.350	0.117

Using these results and the derived results of the model, the optimum no. of teams in each period for all branches can be found to be as given in the following table:

Period \ Branch	Air frame	Engines	Electric systems	Instruments & electronic systems	Radio & Radar
1	9	6	7	6	4
2	7	5	6	5	4
3	8	5	6	5	4
4	5	3	4	3	3
5	5	3	4	3	2
6	7	5	5	4	3
7	4	2	3	2	2

To perform a sensitivity analysis for the case study, one may start with sensitivity of S_O with respect to λ where it is $\frac{\partial S_O}{\partial \lambda}$ as given by equation (20).

The following is the table for the values of $\frac{\partial S_O}{\partial}$ at different periods in all branches:

Period \ Branch	Air frame	Engines	Electric systems	Instruments & electronics	Radio & Radar systems
1	1.760	1.077	1.268	1.033	0.799
2	1.887	1.161	1.365	1.110	0.864
3	1.863	1.145	1.347	1.096	0.852

4	2.222	1.382	1.621	1.314	1.036
5	2.350	1.466	1.718	1.392	1.101
6	1.957	1.208	1.419	1.153	0.900
7	2.717	1.707	1.998	1.615	1.289

From the table, it is obvious that the sensitivity of S_o with respect to λ decreases as λ increases. This means that at high arrival rates, the optimum number of teams will be more stable and insensitive w.r.t. the variations of λ. Also at lower rate of arrivals, the optimum number of teams will be more sensitive w.r.t. the variations of λ.

The sensitivity of S_o w.r.t. b may be obtained using equation (21) and the sensitivity of S_o w.r.t. δ_v may be obtained using equation (22).

From the sensitivity analysis of S_o w.r.t. both b and δ_v, one can deduce that the aging of the aircraft yields an increase in the number of teams required to fix it.

Summary

The paper is based mainly on a queueing system with Poisson arrivals and a general service time distribution with multiserver system. The optimization process is a minimization one for the costs of the system. The two main cost parameters for the optimum policy are, C_1 and C_2. The paper gives the manpower required for every branch of technical work on the air craft through the shifts of the day. It also shows the sensitivity of the optimum solution S_o w.r.t. arrival rate λ, the main service time b and the standard deviation of the service time. The study shows that the optimum solution is sensitive for the variations of λ at its lower values while S_o is insensitive for the variations of λ as long as the values of λ are high. Also it is seen that the aircraft gets old, the number of teams required to fix it becomes more and respond with high sensitivity. A Fortran IV computer program is programmed for the problem.

Although the Poisson arrival distribution and the general service time distributions are the best representation for the case under study, the field is opened for more studies and further assumptions concerning both the arrival and service time distributions.

COMPUTER PROGRAM

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LEVEL 2.1 (JAN 75)     MAIN     05/360 FORTRAN N EXTENDED     DATE 79-144/15.37.47

                                      /STRUCTURED SOURCE LISTING/
                              C    PROGRAM TO COMPUTE SIJ
(004  ISW  0002                    DIMENSION X(7), 8(5), C(5), D(5), LIJ(7.5)
      ISW  0003          10        FORMAT (7 (2 X, F6.3) )
      ISW  0004          20        FORMAT (5 (2X, F6.3)
      ISW  0005          30        FORMAT (214)
      ISW  0006          40        FORMAT (+X, 11,5(5X,14)
      ISW  0007         101        FORMAT (13X,                TEANS,95X//564
                                   1       ,76X//54HPERIOD   AIR FRAME    ENGIN
                                   ZES            ELEC.        INST.       RADIO,76X//56H
                                   3                       ,76X
      ISW  0008                    READ (1, 10)M,N
      ISW  0009                    READ (1,10)(X(I),1=1=M)
(003  ISW  0010                    READ (1,20) (B(J) =J=I,M)
      ISW  0011                    READ (1, 20)(C(J), J=I,W)
      ISW  0012                    READ (1,20)(D(J),J=1,N)
      ISW  0013                    DO  100 J=1,M
      ISM  0014                    DO 100 j=1,M)
(001  ISW  0015                    IS=0
      ISW  0016                    ISI =0
      ISW  0017                    IS   =X(I)=8(J)+(X(I)+300+(B)(J)+2)/(2+D(J)))**.5
      ISW  0018                    ISI=IS +I
      ISW  0019                    EIS =IS*D(J) +500*X(I)*(B(J)**2+(**2)/(2*(15-X)*5(J)))
      ISW  0020                    EIST  =ISIRD(J)+500**(I)(S)(J)**2+(C(J)**2/**2)/(2*(ISI-X(I)*B(J)))
      ISW  0021                    IF (EIST -EIS)60,70,70
      ISW  0022          60           LIJOI, J) = ISI
      ISW  0023                       60   70    180
      ISW  0024          70       (IJ(1,J) = 15
      ISW  0025         100        CONTINUE
001)                               C
003)                               C
      ISW  0026                    WRITE  (2,101)
      ISW  0027                    DO    110  1=1=M
(002  ISW  0028         110        WRITE  (2,40)(1=(LIJ(1,J),M)
002)                               C
      ISW  0029                    STOP
004)                               C
      ISW  0030                    END
```

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التوزيع الأمثل للعمالة في نوبات الصيانة اليومية بشركات الطيران

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يهتم هذا البحث بدراسة مشكلة تحديد العدد الامثل من العمال والفنيين والمهندسين اللازمة لاجراء عمليات الصيانة اليومية على الطائرات في شركات الطيران.

وقد عالج البحث المشكلة في خلال نوبة واحدة في تخصص معين من التخصصات التي تعمل على الطائرة (وهي عادة: الهيكل ـ المحرك العدادات والاجهزة الالكترونية ـ الاجهزة الكهربائية ـ الراديو والاجهزة اللاسلكية) وذلك بدراسة معدلات الوصول والخدمة للطائرات في هذا التخصص خلال تلك النوبة ثم عمل نموذج صفوف له دالة تكاليف تتكون من شقين احدهما ناتج عن تأخير الطائرة عن رحلتها بسبب عدم سرعة الخدمة والثاني ناتج عن تكاليف اطقم الصيانة العاملة بمحطة الخدمة وبمعالجة هذه الدالة رياضيا ينتج العدد الامثل من هذا الاطقم ـ خلال النوبة وفي التخصص محل البحث ـ والذي يجعل دالة تكاليف أقل ما يمكن وباعادة هذه العملية لكل النوبات في كل التخصصات يمكن تحديد حجم العمالة في محطات الخدمة على مدار اليوم في كل التخصصات. وقد تم عمل تحليل لحساسية الحل الامثل بالنسبة الى المتغيرات الاساسية التي تؤثر فيه وهي معدل الوصول ومتوسط وتباين معدل الخدمة ثم طبق النموذج على حالة دراسية من شركة مصر للطيران وتمت حسابات استخدام برنامج حاسبات الكترونية صمم خصيصا لهذا الغرض ونفذ على حاسب الكتروني طراز آ . ب . م ـ ٣٧٠ بلغة فورتران ـ ٤ .