

ELECTRICAL ENGINEERING

Speed Control of a Three Phase Induction Motor Fed from a Single Phase Supply

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(Received 4/7/1991; accepted for publication 7/6/1993)

Abstract. This paper describes a novel and cost effective arrangement of controlling the speed of a three phase induction motor fed from a single phase supply. Theoretical analysis is supported by experimental results. Speed control of a three phase induction motor by stator voltage variation is straightforward, simple, economical and as such increasingly employed in low to medium power applications, especially where the load torque varies as the square of the motor speed. This paper describes an alternative arrangement for the conventional scheme used to implement this technique of speed control. The proposed scheme requires single phase supply and two thyristors as opposed to the three phase supply and six thyristors in the conventional scheme. A single capacitor-phase balancer is used to adapt the motor to the supply. An optimum value for the capacitance of the balancer is chosen to give nearly minimum unbalance between the motor voltages. Experimental results confirm the superiority of the proposed scheme.

List of Symbols

v_s, f, ω	Supply voltage in volt and frequency in (Hz) and (rad/sec), respectively.
2P	no. of poles of the motor.
R_s, R_r	Stator and rotor (referred to stator) resistances, respectively.
X_s, X_r, X_m	Stator and rotor (referred to stator) leakage and magnetizing reactances at fundamental frequency, respectively.
C, X_c	Capacitance of phase balancer and its reactance at fundamental frequency, respectively.
Z	Impedance ($Z = R + jX$).
Y	$1/Z$.
z, y	$ Z $ and $ Y $, respectively.
n_s, n	Synchronous and rotor speeds, respectively.
s	Slip.

P, T	Power and torque, respectively.
T_{rot}	Torque due to rotational losses.
v, i	Phasor voltage and current, respectively.
V, I	rms value of voltage and current, respectively.
Γ, j	$1/120^\circ$ and $1/90^\circ$, respectively.
k	Harmonic order.
α	Firing angle of the thyristors.
β	The extinction angle.
ϕ	Angle of the input current of the motor under sinusoidal supply.
θ	Angle of Z

Subscript

p, n, 0	positive, negative and zero sequences, respectively.
i, o	input and output, respectively.
a, b, c	stator phases.

Doublesubscript

1	Fundamental.
k	Harmonic order.

Introduction

Speed control of a three phase induction motor by stator voltage variation is straightforward, simple, economical and as such increasingly employed in low to medium power applications, especially where the load torque varies as the square of the motor speed [1;2]. The scheme is depicted in Fig. 1. In this scheme, the stator voltage is controlled smoothly at line frequency between zero and full value by symmetrically controlling the firing angle of thyristor devices. In the recent past, considerable engineering efforts have been invested in operating a three phase induction motor from a single phase supply using a phase balancer capacitor. The scheme is illustrated in Fig. 2. When the three phase motor is connected in this fashion (Fig. 2), its performance is effected by the value of the balancer capacitor, which is selected in practice according to several criteria, such as [3;4]:

- i. zero on minimum negative sequence voltage,
- ii. minimum unbalance ratio,
- iii. maximum torque,
- iv. maximum power factor,

- v. maximum efficiency,
- vi. minimum voltage or rating of balancer capacitor.

The performance of the three phase induction motor connected as in Fig. 2 is well documented in the literature [4-7]. However, to the author's knowledge and information, no complete work [8] has been reported so far to control the speed of three phase motor configured as in Fig. 2. This paper, therefore, deals with the controlling of the speed of a three phase motor operated from a single phase supply. Performance characteristics are supported by experimental results.

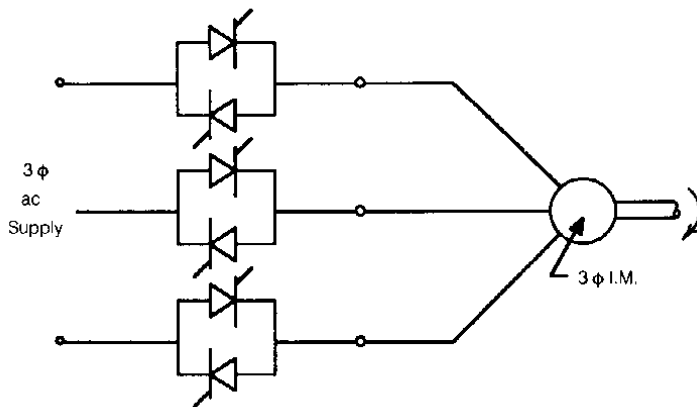


Fig. 1. Conventional stator voltage control circuit for a three phase induction motor.

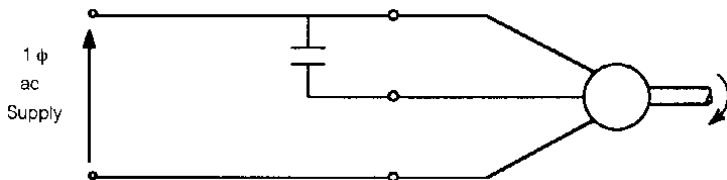


Fig. 2. Feeding a three phase induction motor from a single phase AC supply via a single capacitor phase balancer.

The Proposed Scheme and Analysis

The proposed scheme is shown in Fig. 3. The speed control of the motor is achieved by firing the thyristors in the range of $\phi \leq \alpha \leq \alpha_{\max}$. For the range $\alpha > \alpha_{\max}$, the input power to the motor is less than the power required and consequently the motor will stop. The value of α_{\max} is a function of the motor parameters, input voltage and load. For $\alpha > \phi$, the input voltage to the motor is not sinusoidal and hence may be expressed as the sum of fundamental and odd order harmonic components, as given in Appendix A [9]. The positive and negative sequence equivalent circuits of the motor for the fundamental component are shown in Figs. (4a) and (4b). Fig. 4(c) illustrates the equivalent circuit for the harmonic components. The value of the slip for each harmonic component is:

$$\begin{aligned} s_{pk} &= (kn_s - n) / kn_s \\ s_{nk} &= (kn_s - n) / kn_s \end{aligned} \quad (1)$$

Generally $kn_s \gg n$, which leads to:

$$s_{pk} = s_{nk} \approx 1$$

Accordingly and as $k X_m \gg (R_r + jkX_r)$, the circuit of Fig. 4(c) can be simplified as shown in Fig. 4(d).

For each component and for a star connected motor as shown in Fig. 5, loop equations are:

$$v_i - v_a + v_b = 0 \quad (2)$$

$$v_a + j i_c X_c - v_c = 0 \quad (3)$$

The three phase symmetrical components of stator voltages and currents are given as:

$$\begin{pmatrix} g_n \\ g_b \\ g_c \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ \Gamma^2 & \Gamma & 1 \\ \Gamma & \Gamma^2 & 1 \end{pmatrix} \begin{pmatrix} g_p \\ g_n \\ g_0 \end{pmatrix} \quad (4)$$

where g stands for phasor voltage or current.

Moreover, for each sequence component, voltage and current are related as:

$$v = iZ = i/Y \quad (5)$$

Due to the absence of a neutral connection, zero sequence current, and consequently zero sequence voltage are zero.

Solving eqns. (2-5) yields:

$$v_p = \frac{v_1 (e^{j30} + X_c Y_n e^{-j30} / \sqrt{3})}{3j + X_c (Y_p + Y_n)} \quad (6)$$

$$v_n = \frac{-v_1 (e^{-j30} - X_c Y_p e^{j30} / \sqrt{3})}{3j + X_c (Y_p + Y_n)} \quad (7)$$

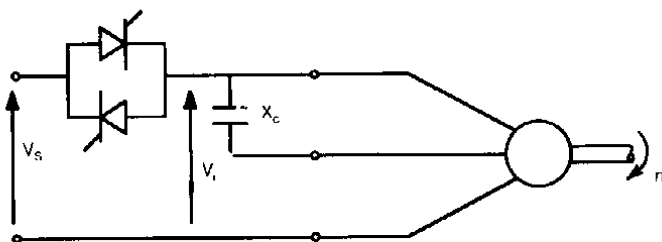


Fig. 3. Proposed circuit arrangement for speed control of three phase induction motor.

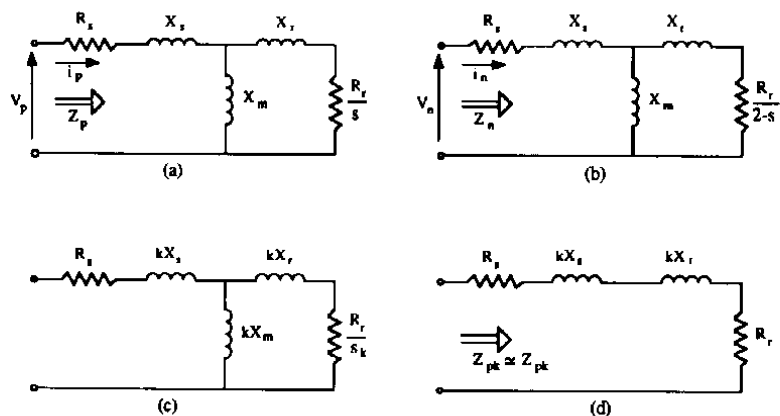


Fig. 4. Equivalent circuits of three phase induction motor

- (a) positive sequence circuit for fundamental component
- (b) negative sequence circuit for fundamental component
- (c) sequence circuit for harmonic components
- (d) simplified circuit of (c).

Optimum Value for Balancer Capacitor

The choice of the value of the balancer capacitor is governed by steady state as well as the dynamic performance of the motor. Among the different criteria mentioned in section 1, above, for selecting the capacitor value, it is verified that the value which yields minimum unbalance also yields maximum efficiency and minimum torque oscillations [6]. Accordingly, this is the optimum value which is favored in the light of steady state as well as transient performances. This value is obtained when (V_p / V_n) is maximum. So by dividing eqn. (6) by eqn. (7) first, then differentiating the magnitude of the resulting equation with respect to X_c and equating it to zero yields [3]:

$$E_2 X_c^2 + E_1 X_c + E_0 = 0 \quad (8)$$

where

$$E_0 = 3 (y_p H_1 - y_n H_2)$$

$$E_1 = 6 (y_p^2 - y_n^2)$$

$$E_2 = u_p y_n (y_p H_2 - y_n H_1)$$

$$H_1 = -2\sqrt{3} \cos(\theta_p - 60)$$

$$H_2 = 2\sqrt{3} \cos(\theta_n + 60)$$

The required value of capacitance is obtained from the maximum real root of eqn. (8). The parameters for the test motor are given in Appendix B. Fig. 6 shows the variation of the capacitor versus the slip. From Fig. 6 and eqn. (8), it is clear that the value of the capacitor which yields minimum unbalance varies with the slip. A three phase induction motor with variable stator voltage usually operates in the range of $0 \leq s \leq s_m$, where s_m is the slip at which maximum torque occurs. For balanced operation, s_m is given by [9]:

$$s_m \approx \frac{R_r}{X_s + X_r} \quad (9)$$

For the test motor, $s_m \approx 0.4$.

The capacitance value for minimum unbalance must be varied with the slip to keep minimum unbalance over the full range of operation. Alternatively, an average value of the capacitor may be used to simplify the system. This value may be taken as the value which corresponds to the value of the slip of $0.5 s_m$. For the test motor, this value is $17.5 \mu\text{F}$. Figure 7 shows the variation of (V_p/V_n) with the slip for different values of the balancer capacitor. Figure 7 shows that $17.5 \mu\text{F}$ is a suitable average value for the capacitor. It should be noted that this value does not yield minimum unbalance at starting. Accordingly, two values of the capacitor may be used, one for starting and the other for running conditions.

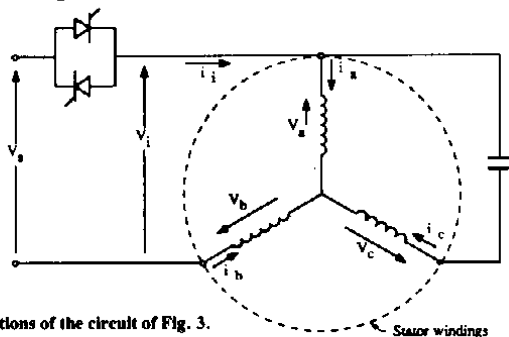


Fig. 5. Voltage and current directions of the circuit of Fig. 3.

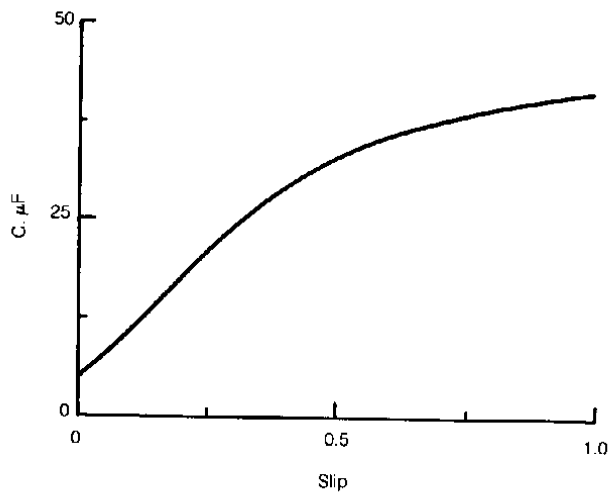


Fig. 6. Variation of the capacitance for minimum unbalance versus slip.

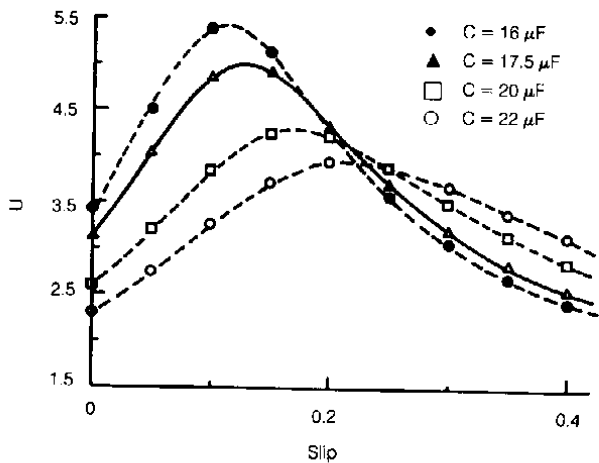


Fig. 7. V_p/V_n versus slip for different values of capacitors.

Performance of the Motor

The performance of the motor under the proposed system can be computed as shown in the flow chart of Fig. 8. For the range $\alpha \leq \phi$, v_i is sinusoidal, constant and equal to the supply voltage. For $\alpha \leq \phi$ the waveform of v_i distorts and decreases as α increases which leads to decreasing the speed. The value of each harmonic component of v_i is given by [9]:

$$v_{ik} = \sqrt{b_k^2 + d_k^2} \psi_k \quad (10)$$

where b_k , d_k and ϕ_k are as given in Appendix A.

The value of v_{pk} and v_{nk} is found using eqns. (6) and (7) then other performance values of the motor are consequently calculated as follows:

$$i_{pk} = v_{pk} / Z_{pk}$$

$$i_{nk} = v_{nk} / Z_{nk} \quad (11)$$

$$i_i = - \sum_{k=1}^{\infty} i_{bk} \quad (12)$$

where $i_{bk} = \Gamma^2 i_{pk} + \Gamma i_{nk}$

The input power to the motor is:

$$P_i = 3 (V_{p1} I_{p1} \cos \theta_{p1} + V_{n1} I_{n1} \cos \theta_{n1}) + \sum_{k=3}^{\infty} P_{ik} \quad (13)$$

where $P_{ik} = 3 (V_{pk} I_{pk} \cos \theta_{pk} + V_{nk} I_{nk} \cos \theta_{nk})$

From Fig. 4, it is realized that as k increases, the value of θ_{pk} and θ_{nk} approaches 90° which makes the value of P_{ik} very small and negligible. This means that the harmonics do not contribute significantly to the developed power and consequently the torque. Accordingly, the output torque may be expressed as:

$$T_0 = \frac{3(2P)}{4 \pi f} [I_{p1}^2 (R_p - R_s) - I_{n1}^2 (R_n - R_s)] - T_{rot} \quad (14)$$

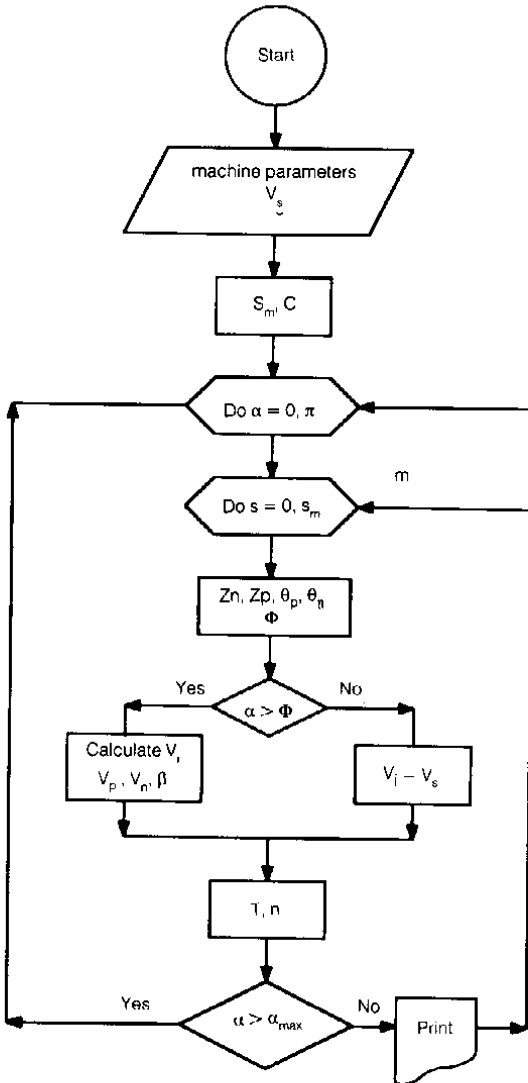


Fig. 8. Flow chart to compute the motor performances.

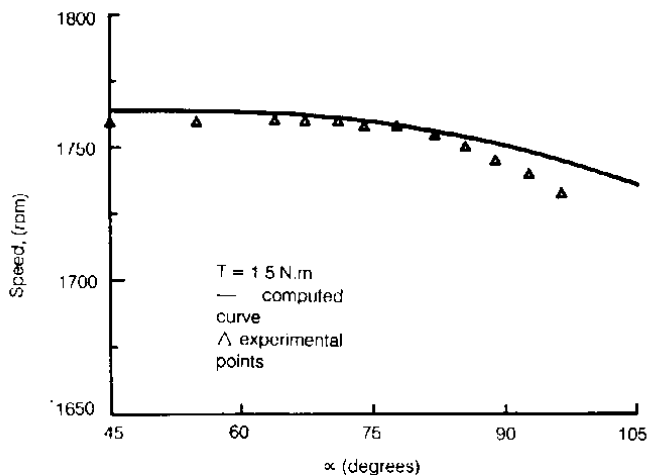


Fig. 9. Motor speed versus value of α under a load torque of 1.5 N.m.

Experimental Results

The performance of the motor under the proposed scheme of Fig. 3 was experimentally studied at a supply voltage of 380 V under different loads. Figure 9 shows the computed and measured experimental variation of the motor speed versus the firing angle ' α ' under a load of 1.5 N.m. Figures 10(a), (b) and (c) show the recorded waveforms of input voltage ' v_i ' and input current ' i_i ' when the motor torque was 1.0 N.m. In Fig. 10(a), the firing angle was 0° . When the firing angle was gradually increased, the waveform of v_i remained unchanged as that of Fig. 10(a) and accordingly the speed also remained constant. When α exceeded 59° , v_i started to distort. The recorded waveforms of v_i and i_i at firing angles of 67° and 100° , are shown in Fig. 10(b) and (c), respectively. For the range of $\alpha \leq 65^\circ$ in Fig. 9 and $\alpha \leq 59^\circ$ in Fig. 10(a), α was less than ϕ , the input voltage ' v_i ' was sinusoidal and constant and consequently the speed was also constant. The results of Figs. 9 and 10 correlate with the expected and computed results.

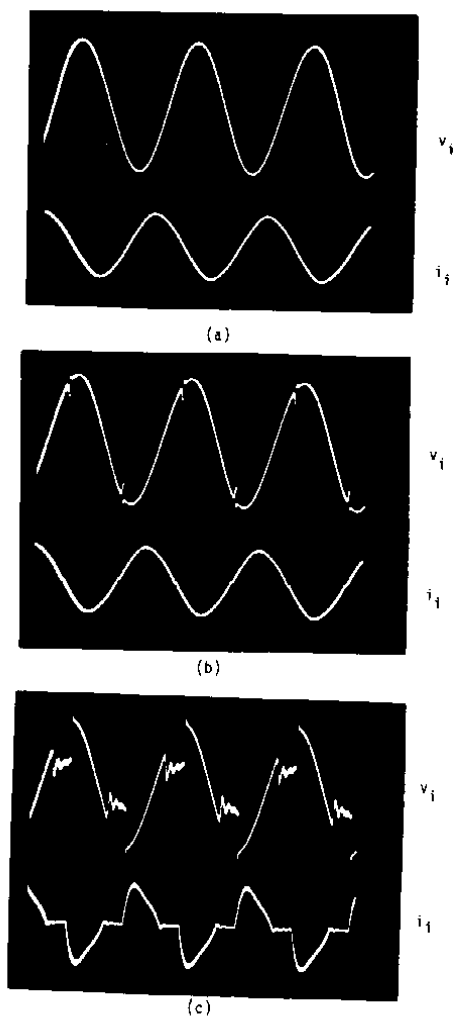


Fig. 10. Waveforms of v_i (upper) and i_i (lower) under a load torque of 1.0 N.m (time: 5 msec/div, voltage scale: uncalibrated)

(a) at $\alpha = 0^\circ$ and 59°

(b) at $\alpha \approx 67^\circ$

(c) at $\alpha = 100^\circ$

Conclusion

A new and cost effective scheme for the variable speed three phase induction motor fed from a single phase supply has been presented. The proposed scheme requires single phase supply and two thyristors as opposed to the three phase supply and six thyristors in the conventional scheme. A single capacitor phase balancer has been used in the new scheme. An optimum value of the balancing capacitor is chosen to give nearly minimum unbalance between the stator voltages in order to achieve better performance both in the steady state and transient conditions. Experimental results confirm the superiority of the proposed scheme. It should be noted that the range of speed variation achieved by this scheme is narrow for constant torque loads. Accordingly, this scheme is mainly recommended to be used for the loads where the torque varies as the square of the speed.

Acknowledgement. The author would like to thank Mr. H. Al-Ghalban, the Engineer of the Electrical Machines Laboratory, College of Engineering, King Saud University, for his help in carrying out the experimental tests.

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Appendix A

When v_i is distorted it can be expressed by the following Fourier series:

$$v_i(\omega t) = \sum_{k=1}^{\infty} \sqrt{b_k^2 + d_k^2} \sin(k\omega t + \psi_k)$$

where

$$b_1 = \frac{V}{2\pi} (\cos 2\alpha - \cos 2\beta)$$

$$d_1 = \frac{V}{2\pi} [2(\beta - \alpha) - \sin 2\beta + \sin 2\alpha]$$

$$\psi_1 = \tan^{-1}(b_1/d_1)$$

$$b_k = \frac{V}{2\pi} \left[\frac{2}{k+1} \{ \cos(k+1)\alpha - \cos(k+1)\beta \} - \frac{2}{k-1} \{ \cos(k-1)\alpha - \cos(k-1)\beta \} \right]$$

$$d_k = \frac{V}{2\pi} \left[\frac{2}{k+1} \{ \sin(k+1)\alpha - \sin(k+1)\beta \} - \frac{2}{k-1} \{ \sin(k-1)\alpha - \sin(k-1)\beta \} \right]$$

$$\psi_k = \tan^{-1}(b_k/d_k)$$

where the extinction angle ' β ' is the solution of the following transcendental equation:

$$\sin(\beta - \phi) - \sin(\alpha - \phi) e^{(\beta - \alpha) \cot \phi} = 0$$

Appendix B

The ratings and parameters of the test motor are given as follows: 380 V, 2.31 A, 0.75 kW, 60 Hz, $2P = 4$, $T_{rot} = 0.4 \text{ N}\cdot\text{m}$, $R_s = 10.5\Omega$, $R_r = 12.57\Omega$, $X_r = X_s = 15\Omega$ and $X_m = 251\Omega$.

التحكم في سرعة المحرك الحثي الثلاثي الطور المغذى من مصدر أحادي الطور

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ملخص البحث. إن التحكم في سرعة المحرك الحثي الثلاثي الطور بواسطة تغيير قيمة جهد الجزء الثابت منه، يعتبر أسلوباً سهلاً واقتصادياً ولهذا أصبح اتباع هذا الأسلوب يتزايد مع مرور الوقت للتطبيقات ذات القدرة الصغيرة والمتوسطة خاصة إذا كان عزم الحمل يتناسب طردياً مع مربع السرعة. والدائرة المستعملة للحصول على التحكم المذكور تستلزم التحكم في ستة من «الثايرستورات» الموصولة بين المحرك والمصدر الثلاثي الطور. ويتم الحصول على تغيير الجهد المطلوب بواسطة التحكم في زاوية قرح «الثايرستورات» بدون تغيير قيمة تردد المصدر.

ويهدف هذا البحث إلى عرض دائرة مقترحة تهدف إلى تبسيط التحكم المذكور أعلاه وذلك بالاستعاضة عن المصدر الثلاثي الطور بمصدر أحادي فقط والسنة ثايرستورات بلأثنين منها فقط. ويتم موازنة جهود المحرك بتوصيل مواسع بين المصدر والطرف الثالث للمحرك. ويعرض البحث تحليلاً للدائرة المقترحة وبيانا لأداء المحرك مع إعطاء بعض النتائج العملية لدعم الجزء النظري من البحث.