J. King Saud Univ. Vol. 11 Science (2), pp. 107-121 (A.H. 1419/1999)

A Test for Exponentiality against New Better than Used Average

M.I. Hendi, H. Al-Nachawati and A.S. Al-Ruzaiza

Department of Statistics, College of Science, P.O.Box 2455, King Saud University, Riyadh 11451, Saudi Arabia

(Received 1/6/1998; accepted for publication 2/12/1998)

Abstract. A U-statistic is derived for testing exponentiality against new better (worse) than used average NBUA (NWUA). This class NBUA is an intermediate class between NBU and NBUE and is more appealing in practice than either the NBU or NBUE classes. The asymptotic relative efficiency with respect to the Hollander-Proschan statistics is studied. Selected critical values are tabulated for sample size n= 5(1) 50. Some sets of real data are used as examples to elucide the use of the proposed test statistic for practical reliability analysis. The problem when right censored data is available is also handled.

Keywords: New better than used average (NBUA), U-test, Reliability, life testing, exponential distribution, hypothesis testing.

Introduction

In many reliability applications, various classes of life distributions and their duals have been introduced to describe several types of deterioration or improvement that accompany aging. Among the well-known families are the classes of increasing failure rate (IFR), increasing failure average (IFRA), new better than used (NBU), new better than used in expectation (NBUE) and harmonic new better than used in expectation (HNBUE). The implication among these classes is such that:

$$\text{IFR} \Rightarrow \text{IFRA} \Rightarrow \text{NBU} \Rightarrow \text{NBUE} \Rightarrow \text{HNBUE}$$

Definition 1.1: Let X be a nonnegative random variable representing the life of an item with distribution function $F(x)$ $(x > 0)$. Let $\overline{F}(x) = 1 - F(x)$. The residual life X_t at age $t \geq 0$ is the random variable with distribution function $F_t(x)$ and survival function.

$$\bar{F}_t(x)=\begin{cases}\dfrac{\bar{F}(t+x)}{\bar{F}(t)} & \bar{F}(t)>0\\ 0, & \bar{F}(t)=0.\end{cases} \tag{1.1}$$

It is well known that F belongs to IFR(DFR) class iff X_t is decreasing (increasing) in $t\geq 0$ on stochastic sense, F belongs to NBU(NWU) class if X_t is smaller (larger) than X for any $t\geq 0$ in convex ordering [1].

Definition 1.2: Let X and Y be two random variables with (marginal) df's F and G, respectively. We say that X is less variable than Y (or X is smaller in weak stochastic ordering (wst) thanY), and write $X\leq_{wst} Y$ iff $\int_0^x \bar{F}(u)du\leq\int_0^x \bar{G}(u)du$.

The weak stochastic ordering (wst) is related to convex ordering (c) [1;2] as seen from the following theorem.

Theorem 1.1: Let X and Y be two random variables with distribution F and G, respectively with $F(0-)=G(0-)=0$ and $\int_0^\infty \bar{F}(u)du=\int_0^\infty \bar{G}(u)du$ (i.e, F and G have the same mean). Then

$$X\leq_{wst} Y \Leftrightarrow X\leq_c Y \text{ or } F\leq_{wst} G\Leftrightarrow F\leq_c G. \tag{1.2}$$

Proof: Since F and G have the same mean,

$$\int_0^\infty \bar{F}(u)du=\int_0^\infty \bar{G}(u)du \Leftrightarrow \int_0^X \bar{F}(u)du+\int_X^\infty \bar{F}(u)$$
$$=\int_0^X \bar{G}(u)du+\int_X^\infty \bar{G}(u)du$$

It follows that

$$\int_0^X \bar{F}(u)du\geq\int_0^X \bar{G}(u)du \Leftrightarrow \int_X^\infty \bar{F}(u)du\leq\int_X^\infty \bar{G}(u)du \tag{1.3}$$

establishing the result.

Substituting weak stochastic ordering (wst) for stochastic ordering (st) [2], we have the following definition:

Definition 1.3: The non-negative random variable X with distribution F is said to be new better than used in average ordering (NBUA) if $X_t \le_{wst} X$ for all $t \ge 0$. In such a case we write $X \in$ NBUA (or $F \in$ NBUA). Its dual class is new worse than used in average ordering (NWUA) which is defined by $X_t \ge_{wst} X$ for all $t \ge 0$. The above inequalities are equivalent to

$$\int_0^x \overline{F}_t(u)du \le (\ge) \int_0^x \overline{F}(u)du; \quad t \ge 0, x > 0 \tag{1.4}$$

or

$$\int_0^x \overline{F}(u+t)du \le (\ge) \overline{F}(t) \int_0^x \overline{F}(u)du; \quad t \ge 0, x > 0 \tag{1.5}$$

Example 1.1: Consider the survival function $\overline{F}(x)$ given by

$$\overline{F}(x) = \begin{cases} 1 & \forall\, 0 \le x \le 3 \\ e^{6-2x} & \forall\, 3 \le x \le 4 \\ e^{2-x} & \forall\, 4 \le x. \end{cases}$$

It is easy to prove $\overline{F} \in$ NBUA.

The class of new better than used in average (NBUA) is an intermediate class between NBU and NBUE. One might look at the NBUA property as comparing the average performance of corresponding used units, but the NBU notion of aging compares a new unit with used units of all possible ages and NBUE concept does the same for all available new units of all possible ages.

The NBUA class was defined by several authors Deshpande *et al* [3] call it NBU (2) among sets of classes in terms of stochastic dominance. Abouammoh and Ahmed [4] call it NBUAS when they studied some reliability operations such as convolutions, mixtures, coherent systems, Poisson shock models and TTT-test Statistic is proposed. Further properties of this class were investigated by Hendi and Rady [5] such as closure properties under parallel systems, nonhomogeneous Poisson shock models and the Laplace transforms characterization for this class. They show that NBU $\not\rightleftarrows$ NBUA $\not\rightleftarrows$ NBUA.

This paper is organized as follows: In section 2, we present a test statistic based on a U-statistic for testing H_0: F is exponential (μ) vs. H_1: F is NBUA and not exponential, where $\mu = \int_0^\infty \overline{F}(x)\,dx$. The Pitman asymptotic relative efficiency for several alternatives, including three alternatives given by Hollander and Proschan [6] are also given. Monte Carlo null distribution critical points are obtained for sample sizes n=5(1)50. An example using real data representing 40 patients suffering from blood cancer from one of Ministry of Health hospitals in Saudi Arabia is given as an application. In Section 3 we consider the problem of dealing with right censored data and selected critical values are tabulated, an example also using data from Susarla and Vanryzin [7] is used as an application in medial sciences in both complete and incomplete data. Finally, Section 4 contains some concluding remarks.

On Testing Exponentiality against NBUA Alternatives in the Noncensored Case Introduction

The U-statistic test

The test presented here depends on a sample $X_1, \ldots\ldots, X_n$ from F. we wish to test null hypothesis H_0: $\overline{F}(x) = \exp(-x/\mu)$, $x \geq 0, \mu > 0$ is unknown versus H_1: $\overline{F}$ is NBUA means $X_t \leq_{wst} X$ or equivalently $\int_0^x \overline{F}(u+t)du \leq \overline{F}(t)\int_0^x \overline{F}(u)du$ for all $x, t \geq 0$, set $v(x) = \int_0^x \overline{F}(u)du$. Thus F is NBUA iff.

$$v(x+t) - v(t) \leq \overline{F}(t)v(x), \qquad x, t \geq 0. \tag{2.1}$$

Lemma 2.1: If F is NBUA then a measure of the deviation from the null hypothesis H_0 is $\Delta_F > 0$, where

$$\Delta_F = 3\int_0^\infty v(x)dF(x) - 2\int_0^\infty\int_0^\infty v(x+t)dF(x)\,dF(t) \tag{2.2}$$

Proof: Clearly from (2.1) F is NBUA iff.

$$\overline{F}(t)v(x) + v(t) - v(x+t) \geq 0. \tag{2.3}$$

Take the integral with respect to $F(x)$ and $F(t)$. result follows □

Let $X_1, \ldots.. X_n$, be a random sample from F, let $F_n(x)$ denote the empirical distribution, and estimate $v(x)$ by

$$\hat{v}_n(x)=\overline{X}-n^{-1}\sum_{i=1}^{n}(X_i-x)I(X_i-x), \tag{2.4}$$

where

$$I(\omega)=\begin{cases}0 & \omega\le 0\\ 1 & \omega>0\end{cases}$$

and $\overline{X}$ is the sample mean. We can write $\overline{v}_n$ in (2.4) as follows:

$$\hat{v}_n(x)=n^{-1}\sum_{i=1}^{n}\{X_i\,I(x\ge X_i)+xI(X_i\ge x)\}. \tag{2.5}$$

Let $\hat{v}_n(x)=n^{-1}\sum_{i=1}^{n}\xi(X_i,x)$, where $\xi(X_i,x)=X_i I(x\ge X_i)+x\,I(X_i\ge x)$. We estimate Δ_F in (2.2) by

$$\hat{\Delta}_{F_n}=3\int_0^\infty \hat{v}_n(x)dF_n(x)-2\int_0^\infty\int_0^\infty \hat{v}_n(x+y)dF_n(x)dF_n(y)$$

i.e.

$$\hat{\Delta}_{F_n}=\frac{3}{n^2}\sum_{i-j}^{n}\sum_{j=1}^{n}\phi(X_i,X_j)-\frac{2}{n^3}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\phi(X_i,X_j+X_k).$$

i.e.

$$\hat{\Delta}_{F_n}=n^{-3}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\{3\phi(X_i,X_j)-2\phi(X_i,X_j+X_k)\} \tag{2.6}$$

i.e.

$$\hat{\Delta}_{F_n}=n^{-3}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\{3X_i\,I(X_j\ge X_i)+3X_j\,I(X_i\ge X_j)$$

$$-2X_i\,I(X_j+X_k\ge X_i)-2(X_j+X_k)I(X_i\ge X_j+X_k)\} \tag{2.7}$$

If we define $\phi(X_1,X_2,X_3)=3X_1\,I(X_2\ge X_1)+3\,X_2\,I\,(X_1\ge X_2)-2\,X_1\,(X_2+X_3\ge X_1)-2\,(X_2+X_3)\,I\,(X_1\ge X_2+X_3)$ and define the symmetric kernel

$$\psi(X_1,X_2,X_3)=\frac{1}{3!}\sum_R\phi(X_{i1},X_{i2},X_{i3}),$$

where the sum is over all arrangements of X_1, X_2, X_3 then $\hat{\Delta}_{F_n}$ in (2.7) is equivalent to the U_n statistic given by

$$U_n = \frac{1}{\binom{n}{3}} \sum_{i<j<k} \phi\left(X_i, X_j, X_k\right) \tag{2.8}$$

The following theorem summarizes the large sample properties of $\hat{\Delta}_{F_n}$ or U_u.

Theorem 2.1: (i) As $n \to \infty$, U_n converges to Δ_F with probability one.

(ii) As $n \to \infty$, $n^{½}$ $(U_n - \Delta_F)$ is asymptotically normal with mean 0 and variance

$$\begin{aligned}\sigma^2 = Var\Big\{&3X\overline{F}(X)+6\int_0^X udF(u)-6X-\int_0^\infty udF(u)\\ &+2X\int_0^X \int_0^{X-y} dF(u)dF(y)-4\int_0^X \int_0^{X-y} udF(u)dF(y)\\ &-\int_0^\infty \int_0^{X+y} udF(u)dF(y)+4\int_0^\infty y\int_0^{X+y} dF(u)dF(y)\\ &+3\int_0^\infty \int_0^y udF(u)dF(y)-3\int_0^\infty y\int_0^y dF(u)dF(y)\\ &+4X\int_0^\infty \int_0^{X+y} dF(u)dF(y)\Big\}\end{aligned} \tag{2.9}$$

(iii) under H_0, the variance reduces to $\sigma_0^2 = \left(-\frac{1}{2}+2Xe^{-X}\right)^2 = \frac{5}{108}$.

(iv) If F is continuous NBUA, then the test is consistent.

Proof. (i) Follows from Theorem A of Serfling ([8], p. 190), while (ii) and (iii) follow from the standard theory of U-statistics cf. Lee [9], Ahmad [10] and by direct calculation respectively. To prove part (iv) frist let us write (2.2) in the following form.

$$\Delta_F = \int_0^\infty \int_0^\infty \{3v(x)-2v(x+t)\}dF(x)dF(t) \tag{2.10}$$

Let $D(x,t)=3v(x)-2v(x+t)$. Since F is not exponential then $D(x, t) > 0$ for at least one value of (x,t) call it (x_0, t_0).

Set $(x_1, t_1) = \inf\{(x,t)/x \le x_0 \text{ and } t \le t_0, \overline{F}(x)=\overline{F}(x_0) \text{ and } \overline{F}(t)=\overline{F}(t_0)\}$. Thus

$$D(x_1,t_1) = 3v(x_1)\overline{F}(t_1) - 2v(x_1+t_1) \geq 3v(x_1)\overline{F}(t_1) - 2v(x_0+t_0)$$
$$= v(x_0)\overline{F}(t_0) - v(x_0+t_0) = D(x_0,t_0) > 0 \text{ and}$$

$F(x_1 + \delta_1) - F(x_1) > 0$ and $F(x_1 + \delta_2) - F(t_1) > 0$ and since x_1 and t_1 are point of increase of F thus $\Delta_F > 0$ □.

Asymptotic Relative Efficiency

Since the above test is new and no other tests are known for NBUA class we compare our test U_n to some classes and choose the test V* & K* presented by Hollander and Proschan [6] for DMRL & NBUE classes respectively. The comparisons are achieved by using Pitman asymptotic relative efficiency, which is defined as follows. Let T_{1n} and T_{2n} be two test statistics for testing $H_0: F_\theta \in \{F_{\theta_n}\}, \theta_n = \theta + cn^{-1/2}$ with c an arbitrary constant, then the asymptotic relative efficiency of T_{1n} relative to T_{2n} is defined by

$$e(T_{1n}, T_{2n}) = \left[\mu_1'(\theta_0)/\sigma_1(\theta_0)\right] / \left[\mu_2'(\theta_0)/\sigma_2(\theta_0)\right],$$

where

$$\mu_i(\theta_0) = \left\{\lim_{n\to\infty}\left(\frac{\partial}{\partial\theta}E(T_{in})\right)\right\}_{\theta\to\theta_0} \quad \text{and} \quad \mu_i'(\theta_0) = \sigma_i^2(\theta_0) = \lim_{n\to\infty} \operatorname{Var}_0(T_{in}),$$

i=1,2 is the null variance.
We choose the following four alternatives:

(i) Weibull family: $\overline{F}_1(x) = \exp(-x^\theta), x > 0, \theta \geq 1$

(ii) Linear failure rate family : $\overline{F}_2(x) = \exp(-x - \theta x^2/2), x > 0, \theta \geq 0$

(iii) Makeham family : $\overline{F}_2(x) = \exp(-x - \theta x^2/2), x > 0, \theta \geq 0$

(iv) Gamma family: $F_4(x) = \int_x^\infty e^{-u} u^{\theta-1} du / \Gamma(\theta), x > 0, \theta \geq 0$

Note that H_0 (the exponential) is attained at $\theta = 1$, in (i) and (iv), and is attained at $\theta = 0$ in (ii) and (iii).

Direct calculations of the asymptotic efficiencies of the NBUA test above are given in Table 2.1.

Table 2.1. Asymptotic efficiencies of U_n for the four alternatives.

Efficiency	Weibull	Linear failure rate	Makeham	Gamma
$\frac{\mu_1'(\theta)}{\sigma_1(\theta_0)} = \frac{\frac{\partial \Delta_F}{\partial \theta}\Big\vert \theta=\theta_0}{\sigma_1(\theta_0)}$	5.28	0.581	0.258	4.423

In Table 2.2 we give the asymptotic relative efficiencies of the U_n test of NBUA with respect to V*, K* for DMRL and NBUE, respectively.

Table 2.2. Asymptotic relative efficiencies of U_n for Hollander and Proschan [6] tests.

Relative efficiency	Weibull	Linear failure rate	Makeham	Gamma
$E(U_n, V^*)$	6.278	0.645	1.066	11.73
$E(U_n, K^*)$	4.4	0.669	0.893	6.11

Monte Carlo Null Distribution Critical Points

In practice, simulated percentiles for small samples are commonly used by applied statisticians and reliability analysts. We have simulated the upper percentile points for %90, %95 and %99. Table 2.3 gives these percentile points of the statistic $\hat{\Delta}_{F_n}$ in (2.7) and the calculations are based on 9,000 simulated samples of sizes n=5(1)50.

Table 2.3. Critical values for percentiles of $\hat{\Delta}_{F_n}$

n	% 90	%95	%99
5	1.2230	1.4346	1.8936
6	0.9980	1.1530	1.5256
7	0.8405	0.9792	1.2595
8	0.7187	0.8261	1.0688
9	0.6334	0.7259	0.9408
10	0.5676	0.6510	0.8251
11	0.5103	0.5796	0.7515
12	0.4652	0.5310	0.6720
13	0.4264	0.4865	0.6178
14	0.3961	0.4539	0.5665
15	0.3682	0.4215	0.5307

Continued ....

Table 2.3 (Contd.).

N	% 90	% 95	% 99
16	0.3468	0.3950	0.5043
17	0.3253	0.3701	0.4619
18	0.3066	0.3481	0.4405
19	0.2903	0.3279	0.4128
20	0.2770	0.3147	0.3969
21	0.2637	0.2981	0.3804
22	0.2511	0.2855	0.3596
23	0.2427	0.2782	0.3487
24	0.2319	0.2616	0.3327
25	0.2228	0.2542	0.3168
26	0.2129	0.2426	0.3033
27	0.2051	0.2331	0.2927
28	0.2004	0.2287	0.2844
29	0.1914	0.2180	0.2751
30	0.1887	0.2138	0.2723
31	0.1810	0.2061	0.2608
32	0.1756	0.1993	0.2539
33	0.1736	0.1966	0.2462
34	0.1673	0.1904	0.2403
35	0.1623	0.1856	0.2336
36	0.1575	0.1811	0.2260
37	0.1562	0.1765	0.2226
38	0.1526	0.1737	0.2173
39	0.1469	0,1682	0.2113
40	0.1433	0.1629	0.2063
41	0.1414	0.1617	0.2001
42	0.1378	0.1569	0.1962
43	0.1340	0.1523	0.1929
44	0.1325	0.1512	0.1910
45	0.1304	0.1486	0.1876
46	0.1283	0.1466	0.1820
47	0.1251	0.1425	0.1795
48	0.1232	0.1407	0.1760
49	0.1209	0.1380	0.1732
50	0.1178	0.1341	0.1685

A numerical example

Consider the data in Abouammoh *et al* [11]. These data represent 40 patients suffering from blood cancer from one of Ministry of Health Hospitals in Saudi Arabia and the ordered life times (in days) are:

115, 181, 255, 418, 441, 461, 516, 739, 743, 789, 807, 865, 924, 983, 1024, 1062, 1063, 1165, 1191, 1222, 1222, 1251, 1277, 1290, 1357, 1369, 1408, 1455, 1478, 1549, 1578, 1578, 1599, 1603, 1605, 1696, 1735, 1799, 1815, 1852.

It was found that the test statistic for the set of data, by using equation (2.7) is $\hat{\Delta}_{F_n} = 1.29$.

It is clear from Table 2.3 the computed value of the test statistic ($\hat{\Delta}_{F_n}$) that we accept H_1 which states that the set of data have NBUA property under significant level at 95% upper percentile.

On Testing Exponentiality against NBUA Alternatives in the Censored Case

Test for NBUA in case of right censored data

In this section, a test statistic is proposed to test H_0 versus H_1 with randomly right censored data. Such a censored data is usually the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modeled as follows. suppose n objects are put on test, and $X_1, \ldots, X_n$ denote their true lifetime. We assume that $X_1, \ldots, X_n$ be independent, identically distributed (i.i.d) according to a continuous life distribution F. Let $Y_1, \ldots, Y_n$, be i.i.d. according to a continuous life distribution F. Also we assume that X's are independent Y's. In the randomly right-censored model, we observe the pairs $(Z_j, \delta_j), j=1,\ldots,n$ where $Z_j = \min(X_j, Y_j)$ and,

$$\delta_j = \begin{cases} 1 & \text{if } Z_j = X_j \text{ (} j^{th} \text{ observn is ucensored)} \\ 0 & \text{if } Z_j = Y_j \text{ (} j^{th} \text{ observation is censored).} \end{cases}$$

Let $Z(0) = 0 < Z_{(1)} < \ldots < Z_{(n)}$ denote the ordered Z's and $\delta_{(j)}$ is the δ_j corresponding to $Z_{(j)}$, respectively.

Using the censored data $(Z_j, \delta_j), j = 1,\dots,n$, Kaplan and Meier [12] proposed the product limit estimator.

$$\begin{aligned} \overline{F}_n(X) &= 1 - F_n(X) \\ &= \prod_{[j:Z_{(j)} \le 1]} \{(n-j)/(n-j+1)\}^{\delta(j)}, \; x \in [0, Z_{(n)}] \end{aligned} \tag{3.1}$$

Now for testing $H_0 : \Delta_F = 0$, against $H_1 : \Delta_F > 0$, using the randomly right censored data, we propose the following test statistic.

$$\hat{\Delta}^c_{F_n} = 3\int_0^\infty v_n(x) dF_n(x) - 2\int_0^\infty \int_0^\infty v_n(x+t) d F_n(x) d F_n(t) \tag{3.2}$$

where $v_n(x) = \int_0^x \overline{F}_n(u) du$ and $\overline{F}_n$ is the product limit estimator, given in (3.1).
For computational purpose, $\hat{\Delta}^c_{F_n}$ in (3.2) may be rewritten as

$$\begin{aligned} \hat{\Delta}^c_{F_n} = {} & 3\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j-1} C_k^{\delta(k)} \left(Z_{(j)} - Z_{(j-1)}\right) \left\{\prod_{m=1}^{i-2} C_m^{\delta(m)} - \prod_{m=1}^{i-1} C_m^{\delta(m)}\right\} \\ & -2\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{l} \prod_{m=1}^{k-1} C_m^{\delta(m)} \left(Z_{(k)} - Z_{(k-1)}\right) \\ & \left\{\prod_{p=1}^{i-2} C_p^{\delta(p)} - \prod_{p=1}^{i-1} C_p^{\delta(p)}\right\} \cdot \left\{\prod_{q=1}^{j-2} C_q^{\delta(q)} - \prod_{q=1}^{j-1} C_q^{\delta(q)}\right\} \end{aligned} \tag{3.3}$$

where $dF_n(z_j) = \overline{F}_n(z_{j-1}) - \overline{F}_n(z_j)$, $C_k = [n-k][n-k+1]^{-1}$. And

$$l = \begin{cases} \# Z\text{'s} \le Z_{(i)} + Z_{(j)} & \text{if } Z_{(i)} + Z_{(j)} < Z_{(n)} \\ n & \text{if } Z_{(i)} + Z_{(j)} \ge Z_{(n)} \end{cases}$$

Table 3.1 gives the critical values percentiles of $\hat{\Delta}^{c}_{F_n}$ test for sample size 5(1)50.

Table 3.1. Critical values for percentiles of $\hat{\Delta}^{c}_{F_n}$

N	% 90	% 95	% 99
5	0.370	0.506	0.833
6	0.313	0.443	0.721
7	0.290	0.397	0.682
8	0.269	0.358	0.635
9	0.247	0.335	0.616
10	0.221	0.296	0.560
11	0.205	0.282	0.484
12	0.189	0.263	0.449
13	0.164	0.236	0.412
14	0.155	0.213	0.395
15	0.147	0.204	0.377
16	0.140	0.191	0.374
17	0.133	0.192	0.331
18	0.117	0.165	0.306
19	0.110	0.163	0.295
20	0.103	0.147	0.269
21	0.094	0.138	0.271
22	0.084	0.124	0.251
23	0.083	0.122	0.226
24	0.074	0.111	0.211
25	0.071	0.104	0.212
26	0.067	0.104	0.184
27	0.059	0.092	0.173
28	0.059	0.095	0.201
29	0.052	0.082	0.160
30	0.050	0.086	0.172
31	0.045	0.075	0.166
32	0.040	0.071	0.153
33	0.039	0.068	0.154
34	0.035	0.064	0.152
35	0.034	0.060	0.134
36	0.029	0.055	0.133
37	0.029	0.054	0.124
38	0.023	0.046	0.116
39	0.021	0.048	0.119
40	0.016	0.040	0.109
41	0.015	0.040	0.105
42	0.014	0.038	0.097
43	0.010	0.031	0.092
44	0.010	0.032	0.088
45	0.007	0.030	0.090
46	0.005	0.027	0.091
47	0.002	0.022	0.081
48	-0.002	0.020	0.076
49	-0.001	0.020	0.072
50	-0.003	0.016	0.074
60	-0.017	0.000	0.032
70	-0.028	-0.015	0.007
81	-0.037	-0.023	0.022

Example 3:

Consider the data in Susarla and Vanryzin [6], which represent 81 survival times (in weeks) of patients of melanoma. Out of these 46 represents non-censored data, and the ordered values are: 13, 14, 19, 20, 21, 23, 23, 25, 26, 26, 27, 27, 31, 32, 34, 37, 38, 40, 46, 50, 53, 54, 57, 58, 59, 60, 65, 66, 70, 85, 90, 98, 102, 103, 110, 118, 124, 130, 136, 138, 141, 234.

The ordered consored observations are: 16, 21, 44, 50, 55, 67, 73, 76, 80, 81, 86, 93, 100, 108, 114, 120, 124, 125, 129, 130, 132, 134, 140, 147, 148, 151, 152, 158, 181, 190, 193, 194, 213, 215.

Now, ignoring the censored data, one can apply the methodology of section 2 to test the hypothesis H_0: the survival times are exponential versus H_1: the survival times follow NBUA and not exponential.

Computing $\hat{\Delta}_{F_n}$ from (2.7) non-censored data, we get $\hat{\Delta}_{F_n}$ =1.062, exceeds the critical value in Table 2.3 at %95 upper percentile. Then we accept H_1 which states that the set data have NBUA property.

But, taking into account the whole set of survival data (both censored and uncensored), and computing the statistic ?? from (3.3) censored data, we get $\hat{\Delta}^c_{F_r}$ = - 2.525, less than the critical value in Table (3.1) at %95 upper percentile. Then we reject, H_1 which states that the set data have NBUA property.

Concluding Remarks

In this paper we considered the NBUA class of life distributions. Our main objective was to derive an U-test for testing whether any sample from a life distribution follows an exponential distribution against the alternative hypothesis that it follows the NBUA class. Tabulated critical values for the test are calculated and presented in section 2 for sample sizes 5(1)50. The test statistics has been discussed also in (2.1). An application of the test statistics was applied on the survival data of patients of blood cancer disease and melanoma. We found that these data follow the NBUA class of life distributions and not the exponential distribution. We have also extended the test to handle the case of right censored data.

Acknowledgment. The authors are greatly indebted to Professor Ibrahim A. Ahmad for his constructive comments and stimulating discussion leading to this work.

References

[1] Coa, J. and Wang, Y. "The NBUC and NWUC Classes of Life Distributions". *J. Appl. Prob.*, 28 (1991), 473-479.

[2] Stoyan, D. (1983). *Comparison Methods for Queues and Other Stochastic Models.* New York: John Wiley, 1983.

[3] Deshpande, J.V., Kochar, S.C. and Singh, H. "Aspects of Positive Aging". *J. Appl. Prob.*, 23 (1986), 748-758.

[4] Abouammoh, A. M. and Ahmed, A. N. "Two Notations of Positive Aging Based on Stochastic Dominance". *Statische Hefte*, 30 (1989), 213-229.

[5] Hendi, M. I and Rady, E.A. "On NBUA Class of Life Distributions". *J. King Saud Univ. Science*, 6, No. 1 (1994), 109-122.

[6] Hollander, M. and Proschan, F. "Test for Mean Residual Life". *Biometrika*, 62 (1975), 585-593.

[7] Susarla, V. and Vanryzin, J. "Empirical Bayes Estimations of a Survival Function Right Censored Observation". *Ann. Statist.*, 6 (1978), 710-755.

[8] Serfling, R. J. *Approximation Theorem of Mathematical Statistics.* New York: John Wiley, 1980.

[9] Lee, A.J. U-statistics. New York: Marcel Dekker, 1989.

[10] Ahmad I. A. "Nonparametric Testing of Classes of Life Distributions Derived from a Variability Ordering". *Parisankhyan Samikkha*, 2 (1995), 13-18.

[11] Abouammoh, A.M., Abdulghani, S. A. and Qamber, I.S. "On Partial Orderings and Testing of New Better than Renewal Used Classes". *Reliability Eng. Syst. Safety*, 43 (1994), 37-41.

[12] Kaplan, E.L and Meier, P. "Nonparameteric Estimation from Incomplete Observations". *J. Amer. Statist. Assoc.*, 53 (1958), 457-481.

دراسة اختبار الفرضية الآسية ضد الفرضية الخاصة بفصل توزيعات الحياة الجديد أفضل من المستخدم في المتوسط (ج ف ت م)

محمود ا. هندي، هشام النشواتي وعبدالرحمن س. الرزيزاء
قسم الإحصاء وبحوث العمليات، كلية العلوم، جامعة الملك سعود، ص.ب ٢٤٥٥ ، الرياض ١١٤٥١ ، المملكة العربية السعودية

(استلم للنشر في ١٤١٩/٢/٧هـ ؛ وقبل للنشر في ١٤١٩/٨/١٣هـ)

ملخص البحث. تم اشتقاق إحصاءة يو لفصل توزيعات الجديد أفضل من المستخدم في المتوسط (ج ف ت م)، لاستخدامها في اختبار الفرضية الآسية ضد الفرضية الخاصة بفصل توزيعات الحياة الجديد أفضل من المستخدم في المتوسط (ج ف ت م). ويعتبر فصل توزيعات الحياة الجديد أفضل من المستخدم في المتوسط (ج ف ت م)، حالة وسط بين فصلي توزيعات الحياة الجديد أفضل من المستخدم (ج ف ت) والجديد أفضل من المستخدم في التوقع (ج ف ت ت) وأكثر واقعية في التطبيق العملي من كليهما.

تم حساب كفاءة اختبار الإحصاءة يو لفصل التوزيعات (ج ف ت م) ثم مقارنتها بكفاءة الاختبارات للإحصاءات الواردة في بحث هولندر ويروشان عام (١٩٧٥م)، وعمل جداول للقيم الحرجة للمئينات لفصل التوزيعات (ج ف ت م) تحت تحقق الفرضية الآسية للعينات الصغيرة ذات الحجم ن = ١ (١) ٥٠ لكل من العينات الكاملة والعينات غير الكاملة (مبتورة من جهة اليمين). تم استخدام مثال لبيانات حقيقية لعينة من مرض السرطان لإبراز مدى تطبيق هذا البحث في الحياة العملية.