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MECHANICAL ENGINEERING

An Automated Procedure for Dimensional Synthesis of Planar Mechanisms

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Abstract. This paper describes the basic principles for the development of an automated planar mechanism dimensional synthesis procedure. The procedure generates the synthesis equations by imposing length constraint along with the constraints of the motion program. The synthesis equations are formulated in a quadratic form. A numerical algorithm designed to solve the systems of quadratic algebraic equations is developed. The algorithm is based on a combination of steepest descent and the second order Newton-Raphson methods. A six-bar and an eight-bar mechanisms were synthesized using the development presented in this paper.

Introduction

Kinematics synthesis, in general, implies the development of methods of computation or graphical construction, that implicitly determine the proper dimensions for the synthesized mechanism. In recent years, a major research effort by many eminent kinematicians has been directed toward kinematics synthesis [1-7]. The design of a mechanism on the other hand is characterized by "human iteration". Typically, a design process is a sequence of decisions, each of which must be evaluated and altered as necessary. The design engineer brings to this process his experience from past successful designs. The ultimate goal of the design process is the discovery of the optimum solution for a given design situation.

Until quite recently, the classical techniques for kinematics synthesis have been graphical, where accurate results depend on the skill of the designer in applying drafting techniques and descriptive geometry. With the development of powerful numerical analysis tools, it is becoming increasingly clear that the traditional graphical techniques can be supplemented and sometimes completely replaced by computational methods.

Dimensional synthesis using analytical techniques has been found to be very useful in mechanism design. Kota and Chiou [2] used orthogonal arrays and newly developed design procedures that account for tolerances and clearances in the links and joints in addition to the vibration due to link elasticity. A good implementation of this method is in obtaining an initial guess or starting point for optimal synthesis procedures especially in path generation problems. Krishnamurty and Turcic [3] presented the application of multiple objective optimization techniques to perform optimal synthesis of general planar mechanisms. Their application was based on the method of non-linear goal programming. The objectives are prioritized and satisfied separately and not combined in a single objective. An automated computer aided synthesis program (AUCASYN) for mechanisms and machines was developed by Tatu and Asko [4]. Their program uses the DADS-3D methods of analysis. it provides complete classification and kinematics synthesis of 4-bar mechanisms. Salem [5] presented a computer-aided design system for mechanisms and machines based on AutoCAD. They utilized the AutoLisp language within AutoCAD for design operations, definitions and representation of mechanisms. It includes analysis and optimization routines and runs on micro-computer stations. Breteler [6] described a general approach employing a finite element method like technique to formulate the synthesis equations of planar mechanisms. He treated the problem of feasibility of the kinematics parameter space and used the singularity of the continuity matrix in the finite element method to yield an exact expression for the parameter space boundary.

Among the popular approaches in analytical techniques for linkage design are the complex number approach developed by Sandor [8] and the displacement matrix approach proposed by Suh [9] and extensively used by Soni [10]. Before such a development can be undertaken, it is however necessary to develop a generalized algorithm to generate automatically the synthesis equations in algebraic form. Azees and Soni [11] used the displacement matrix approach to synthesize a linkage mechanism for point path generation. This approach utilizes Edge-Edge, Vertex-Vertex, and Vertex-Edge matrices. The solution of such synthesis equations by digital computer was not possible at the time of their development. With recent development of algorithms using homotopic mapping and continuation methods by Morgan

[12] and Garcia and Zangwelt [13], it is now possible to solve such synthesis equations to obtain all possible solutions. However, experience in using these algorithms showed that with large number of equations they fail to predict a solution.

In this work, we present a new technique for developing and generating synthesis equations. The technique is partially based on some previous theory that has been worked up by Soni, Dado and Wang [1]. They presented a systematic way for deriving the synthesis equations. The method introduced in this paper is based on considering the mechanism as an assembly of *binary links* related to each other by constraints equations. The fact that each link must preserve constant length throughout all the linkage's positions is used to obtain the synthesis equations. An important part of this work is the testing of a new numerical technique for solving the synthesis equations. The most common technique for practical mechanism design is manual iterations using a CAD system to predict the motion of the links. This technique is a combination of the second order Newton-Raphson method with the steepest descent method using optimum step moves. This technique has managed to solve complicated problems such as six and eight bar mechanisms.

The Generation of Synthesis Equations

The mechanism is considered to be a collection of connected binary links. These binary links are divided into two types; links with known relative orientation and links with unknown relative orientation.

For links with known relative orientation, the coordinates at the n-th position of one of the joints of a binary link is expressed in terms of the first and the n-th positions of the other joint. To find such an expression, refer to Fig. 1. The notation is as follows:

K	is the number of the joint with known n-th position.
L	is the number of the joint with unknown n-th position.
m	is the number of the link under consideration.
O_{mn}	is the relative orientation between the initial position and the n-th position of the m-th link.
(x_{in}, y_{in})	are the coordinates of the i-th joint at the n-th position.
r_m	is the distance between the two joints K and L, (i.e. the length of the link).

Considering Fig. 1, and for known relative orientation θ_{mn} one can write:

$$x_{Ln} = x_{Kn} + r_m \cos(\alpha + \theta_{mn}) \tag{2.1}$$

where

$$\cos\alpha = \frac{x_{L_1} - x_{K_1}}{r_m}$$

$$\sin\alpha = \frac{y_{L_1} - y_{K_1}}{r_m}$$

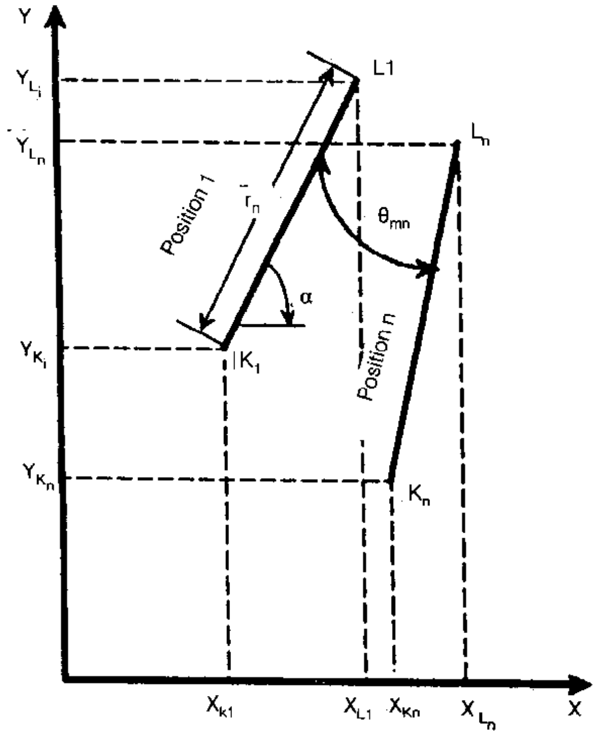

Fig. 1. m-th link at first and n-th position

After simple manipulation, equation (2.1) can be written as:

$$x_{Ln} = x_{Kn} + (x_{L1} - x_{K1})\cos\theta_{mn} - (y_{L1} - y_{K1})\sin\theta_{mn} \qquad (2.2)$$

Similarly

$$y_{Ln} = y_{Kn} + (y_{L1} - y_{K1})\cos\theta_{mn} + (x_{L1} - x_{K1})\sin\theta_{mn} \qquad (2.3)$$

The binary links with unknown relative orientation are used to impose the length constraint between the two joints of the binary link as,

$$(x_{K1} - x_{L1})^2 + (y_{K1} - y_{L1})^2 = (x_{Kn} - x_{Ln})^2 + (y_{Kn} - y_{Ln})^2 \tag{2.4}$$

For a maximum number of positions (N) for a mechanism, the length constraint produces (N-1) equations for each binary link with unknown relative orientation. As equation (2.4) implies, the constraint equations are only quadratic eqautions.

Examples

Several examples were solved using the synthesis equations generation procedure. Two of these examples are solved in the following section where other examples solutions are presented in the results section.

A. Six-bar input-point path coordination mechanism

Problem statement: The orientation of link 2 (θ_2) is coordinated with the specified path of joint 7 for the six bar mechanism shown in Fig. 2. The motion program is given in the results section.

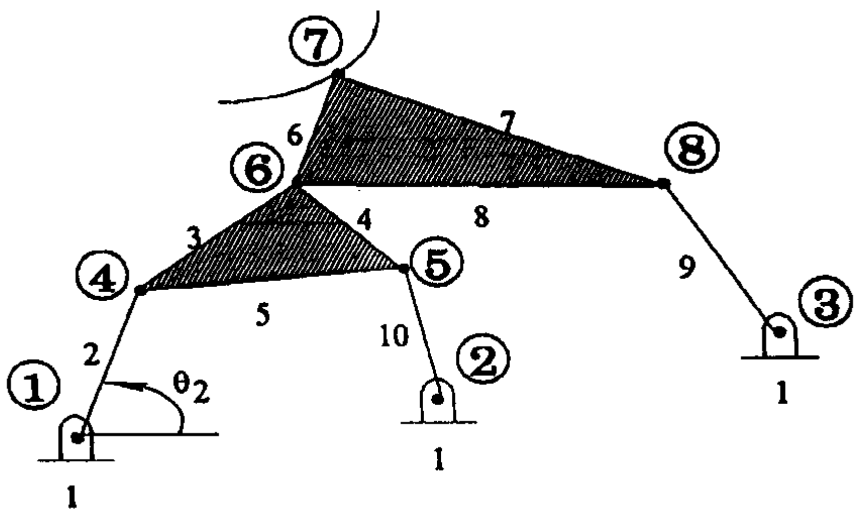

Fig. 2. Six-bar input-point path coordination mechanism .

Unknowns: Initial positions for joints 1,2,3,4,5,6,& 8, give 14 unknowns.

The i-th position for joints 6,8 & 5, which gives 6 (N-1) unknowns.

$$\Longrightarrow \text{No. of unknowns} = 14 + 6\,(N-1)$$

Constraints: links 3,4,5,6,7,8,9 & 10 are binary links with unknown relative orientation, this gives 8 (N-1) equations. Note that the orientation of link 2 is defined by θ_2.

Maximum number of positions: N_{max} is derived as follows:

$$\# \text{ of unknowns} = \# \text{ of equations}$$
$$14 + 6\,(N_{max} - 1) = (N_{max} - 1)$$
$$\Longrightarrow N_{max} = 8 \text{ positions.}$$
which in turn gives 56 equations.

The coordinates of joint 4 are expressed in terms of its initial position, the coordinates of joint 1, and the relative orientation of link 2. Its coordinates are determined using equations (2.2) and (2.3) as follows:

$$x_{4i} = x_1 + (x_{41} - x_1)\cos(\theta_{2i}) - (y_{41} - y_1)\sin(\theta_{2i})$$

$$y_{4i} = y_1 + (x_{41} - x_1)\sin(\theta_{2i}) + (y_{41} - y_1)\cos(\theta_{2i})$$

$$i = 2,\ldots 8$$

Where θ_{2_i} is the relative orientation of link 2 between position 1 and position i.

Synthesis equations

Length constraint on link 3:

$$(x_{41} - x_{61})^2 + (y_{41} - y_{61})^2 = (x_{4i} - x_{6i})^2 + (y_{4i} - y_{6i})^2$$

Length constraint on link 4:

$$(x_{61} - x_{51})^2 + (y_{61} - y_{51})^2 = (x_{6i} - x_{5i})^2 + (y_{6i} - y_{5i})^2$$

Similar equations are written for the length constraint in links 5-10 for each position i = 2,...,8 resulting in 56 synthesis equations. To reduce the number of equations, we can assume some of the unknonws. For example, specifying the coordinates of joint 1, the number of positions is reduced by one, and the number of equations is reduced by eight as follows:

$$12 + 6\,(N_{max} - 1) = 8\,(N_{max} - 1)$$

$$\Rightarrow N_{max} = 7 \text{ positions.}$$

$$\#\text{ of equations} = 8\,(N_{max} - 1) = 48 \text{ equations.}$$

The synthesis equations will remain the same except that they will be written for 7 positions instead of 8 positions, (i.e. i = 2,...). The solution of this mechanism is handled by the second order Newton-Raphson method combined with the steepest descent. Figure 3 shows the synthesized mechanism simulated for the various positions.

It could be observed from Fig. 3 that inversion switches took place between positions 1 & 2 and between 4 & 5. This inversion switching could not be controlled by any analytical precision position synthesis procedure [9,11]. However, optimum synthesis could help in this regard since a smooth continuous path could be traced while tolerating a minimum amount of error.

B. Eight-bar two-body guidance mechanism

Problem Statement: It is required to guide two bodies using the eight-bar mechanism shown in Fig. 4. The paths of joints 8 and 7 are given with the corresponding relative orientation of the two bodies A ($\Delta\theta$) and B ($\Delta\phi$). Synthesize a suitable eight-bar linkage to perform the job.

Unknowns

Initial positions of all the joints except joints 7 and 8. This gives 20 unknowns.

The coordinates of joints 2,3,10 and 11 for the rest of the positions. This gives 8 (N − 1) unknowns.

$$\#\text{ of unknowns} = 20 + 8\,(N - 1)$$

Constraints: all the links are binary links with unknown relative orientation except links 1 (ground link), 7,8,9,10,11,12 and 13. This gives 12 (N − 1) equations.

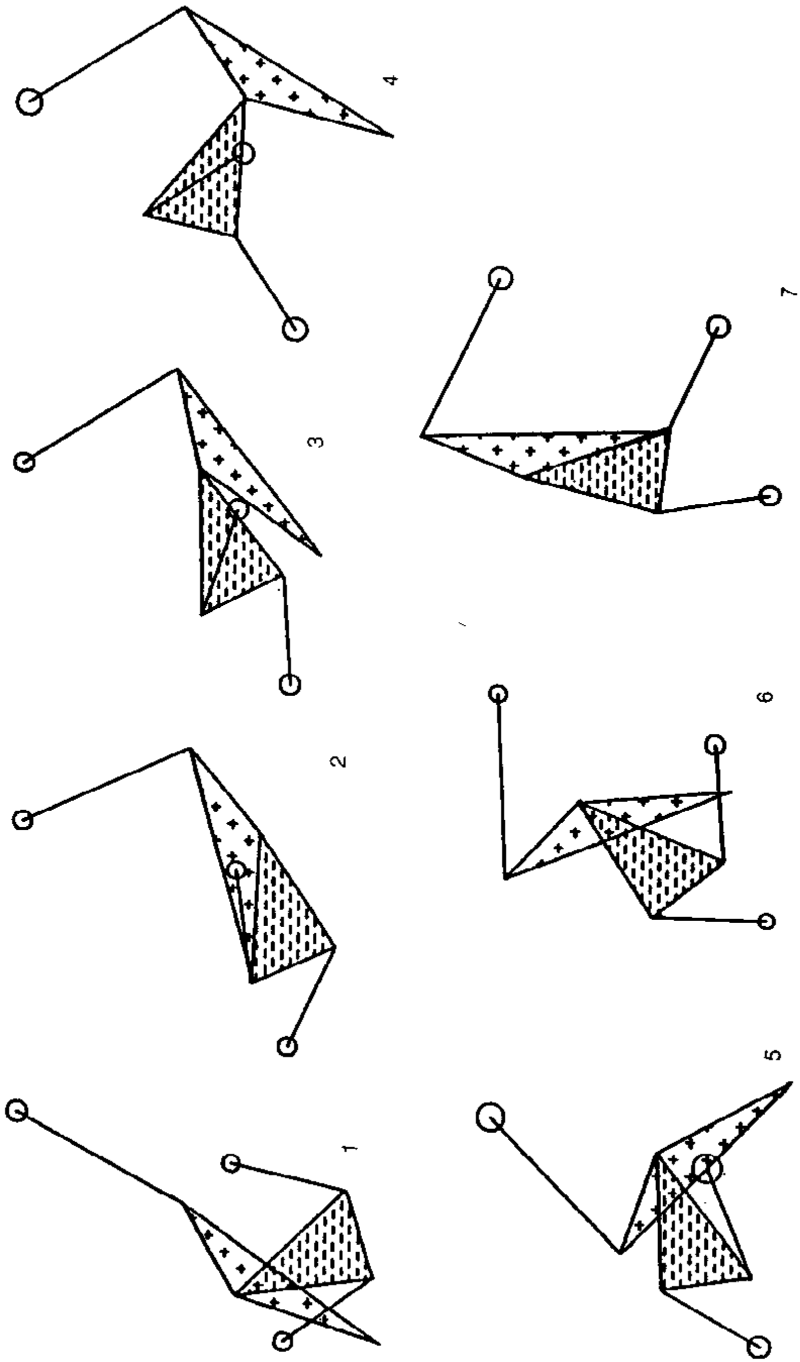

Fig. 3. Six-bar input-point path tracing .

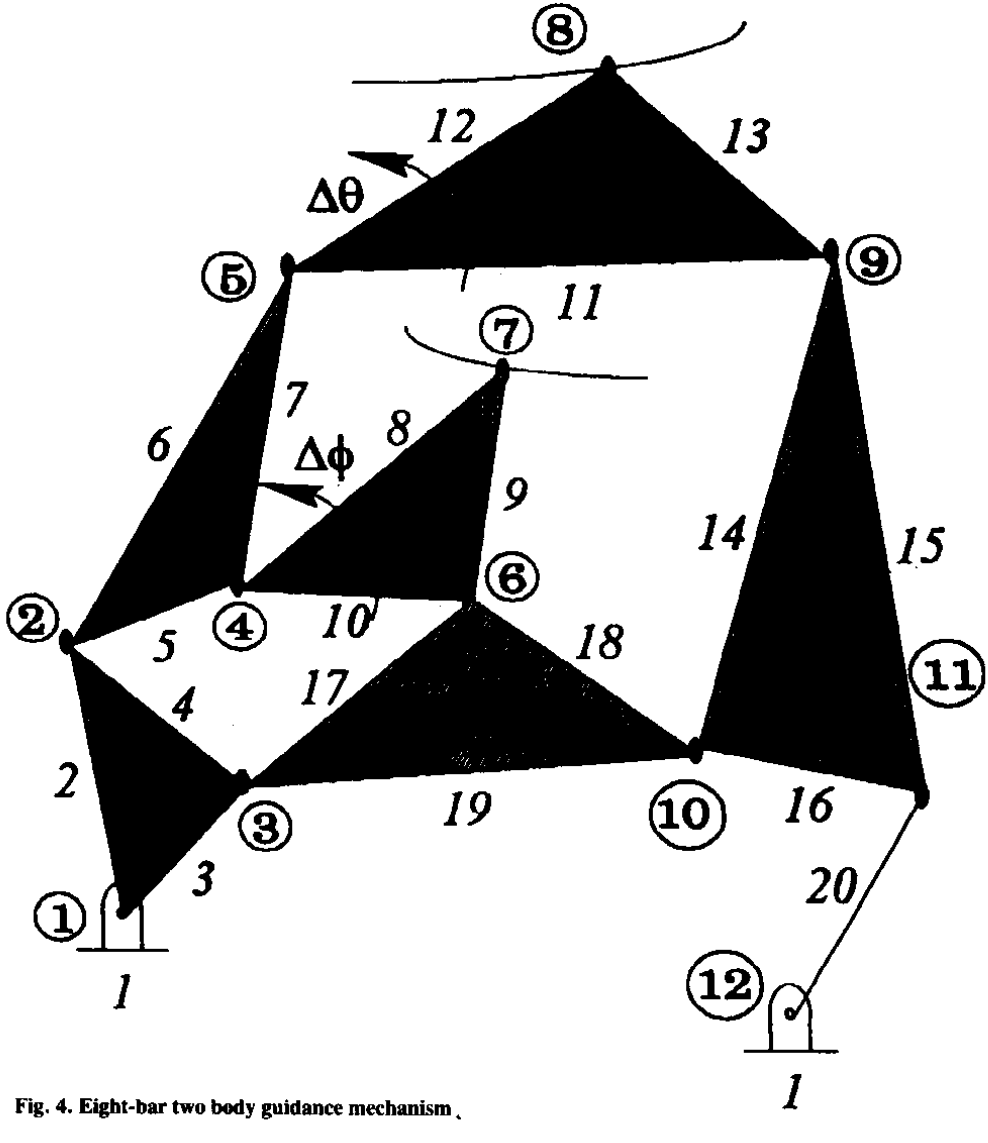

Fig. 4. Eight-bar two body guidance mechanism.

Maximum number of positions (N_{max}):

$$\# \text{ of unknowns} = \# \text{ of equations}$$

$$\Rightarrow 20 + 8\,(N_{max} - 1) = 12\,(N_{max} - 1)$$

$$\Rightarrow N_{max} = 6 \text{ positions.}$$

which in turn gives 60 equations.

The coordinates of joints 5 and 9 can be written in terms of the known path of joint 8 and the relative orientation ($\Delta\theta$) given by the problem statement. The expressions for their coordinates are given by equations (2.2) and (2.3) as follows:

$$x_{5i} = x_{8i} + (x_{51} - x_{81}) \cos(\Delta\theta_i) - (y_{51} - y_{81}) \sin(\Delta\theta_i)$$

$$y_{5i} = y_{8i} + (x_{51} - x_{81}) \sin(\Delta\theta_i) + (y_{51} - y_{81}) \cos(\Delta\theta_i)$$

$$x_{9i} = x_{8i} + (x_{91} - x_{81}) \cos(\Delta\theta_i) - (y_{91} - y_{81}) \sin(\Delta\theta_i)$$

$$y_{9i} = y_{8i} + (x_{91} - x_{81}) \sin(\Delta\theta_i) - (y_{91} - y_{81}) \cos(\Delta\theta_i)$$

$$i = 2,\ldots 6$$

Similarly, the coordinates of joints 4 and 6 can be written in terms of the known path of joint 7 and the relative orientation ($\Delta\phi$) given by the motion program. The expression for their coordinates is given by:

$$x_{4i} = x_{7i} + (x_{41} - x_{71}) \cos(\Delta\phi_i) - (y_{41} - y_{71}) \sin(\Delta\phi_i)$$

$$y_{4i} = y_{7i} + (x_{41} - x_{71}) \sin(\Delta\phi_i) + (y_{41} - y_{71}) \cos(\Delta\phi_i)$$

$$x_{6i} = x_{7i} + (x_{61} - x_{71}) \cos(\Delta\phi_i) - (y_{61} - y_{71}) \sin(\Delta\phi_i)$$

$$y_{6i} = y_{7i} + (x_{61} - x_{71}) \sin(\Delta\phi_i) + (y_{61} - y_{71}) \cos(\Delta\phi_i)$$

$$i = 2,\ldots 6$$

Where ($\Delta\theta_i$) and ($\Delta\phi_i$) are the differences between the orientation of the body at the i-th positions and its orientation at the first position for bodies A and B, respectively.

Synthesis equations

Length constraint on link 2:

$$(x_1 - x_{21})^2 + (y_1 - y_{21})^2 = (x_1 - x_{2i})^2 + (y_1 - y_{2i})^2$$

Length constraint on link 3:

$$(x_1 - x_{31})^2 + (y_1 - y_{31})^2 = (x_1 - x_{3i})^2 + (y_1 - y_{3i})^2$$

Similar equations are written for length constraint on links 4-6 and 14-20 for each i = 2,...,6 positions resulting in 60 synthesis equations. To reduce the number of equations we can assume some unknowns. For example, specifying the coordinates of joints 1 and 12 will reduce the number of positions by one, and as a result, the number of equations will be reduced to 48 equations.

Numerical Methods for Solving the Synthesis Equations

Three different numerical methods were used to solve the synthesis equations. The first is the second order Newton-Raphson method, the second is the steepest descent method with optimum step moves. The third is a combination of the first two methods.

Second order Newton-Raphson method

This method is a modification of the first order Newton-Raphson method, which linearizes the equations at the trial point, and then uses the intersection of the slope at the trial point with the zero line as a feedback to the solution process. The first order method uses the first two terms of Taylor series to proceed towards the next solution point. By considering the third term, more accuracy can be achieved. Moreover, it was found that the convergence speed is increased more than before even with large scale problems that involved relatively high number of second order equations. For the second order approximation, Taylor series for a single variable function can be written as follows:

$$f(x) \cong f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2 \tag{3.1}$$

where x_0 is the point around which approximation takes place.

Setting $f(x) = 0$, we get:

$$f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2 = 0 \tag{3.2}$$

or

$$f(x_0) + f'(x_0)(\Delta x) + \frac{1}{2} f''(x_0)(\Delta x)^2 = 0 \tag{3.3}$$

where: $\Delta x = x - x_0$.

By analogy, for a system of equations given by $\{F(x)\} = \{0\}$, the previous equation can be written as follows:

$$\{F\}_0 + [J]\,\{\Delta X\} + \frac{1}{2}\,[DJ]\,\{\Delta X\} = \{0\} \tag{3.4}$$

Where:

- $\{\Delta X\}$ is the correction vector.
- $\{F\}_0$ is the residual vector evaluated at $\{X\}_0$.
- [J] is the Jacobian matrix of the functions evaluated at $\{X\}_0$.
- [DJ] is given by:

$$[DJ] = [\{JP\}_1\,\{JP\}_2 \ldots\ldots \{JP\}_n] \tag{3.5}$$

where:

$$\{JP\}_i = \frac{\partial}{\partial x_i}\,[J]\,\{\Delta X\} \tag{3.6}$$

The correction vector $\{\Delta X\}$ in equation (3.6) is obtained using the first order Newton-Raphson method. Using this substitution one gets:

$$\{JP\}_i = -\,\frac{\partial}{\partial x_i}\,[J]\,.\,\{[J]^{-1}\,\{F\}_0\} \tag{3.7}$$

Equation (3.3) could be solved for the correction vector as:

$$\{\Delta X\} = -\,[\,[J] + \frac{1}{2}\,[DJ]\,]^{-1}\,\{F\}_0 \tag{3.8}$$

Equation (3.8) gives an approximation to $\{\Delta X\}$ since the first order Newton-Raphson method was used to substitute for $\{\Delta X\}$ used in equation (3.6).

This method gives faster convergence over relatively large number of second order equations. It managed to solve sixteen equations generated by the synthesis of the input-point path four bar mechanism, with an excellent accuracy, see the results section.

Steepest descent method with optimum step strategy

First, we will introduce two concepts that are extensively used throughout the development of this method. Namely, the error function and the gradient vectors. The error function of the synthesis equations is simply the sum of the squares of the residuals. For the system of equations $\{F(x)\}=\{0\}$, the error function at $\{X\}_0$ is defined as:

$$U(x_1, x_2, \ldots, x_n) = F_1^2 + F_2^2 + \ldots + F_n^2 \tag{3.9}$$

Where:

$$F_1 = F_1(x_1, x_2, \ldots x_n)$$

$$F_2 = F_2(x_1, x_2, \ldots x_n)$$

$$F_n = F_n(x_1, x_2, \ldots x_n)$$

The elements of the gradient vector of the error function are the derivatives of U with respect to each unknown and they are given by:

$$G_1 = \frac{\partial U}{\partial x_1} = 2\left(F_1 \frac{\partial F_1}{\partial x_1} + F_2 \frac{\partial F_2}{\partial x_1} + \ldots + F_n \frac{\partial F_n}{\partial x_1}\right)$$

$$G_2 = \frac{\partial U}{\partial x_2} = 2\left(F_1 \frac{\partial F_1}{\partial x_2} + F_2 \frac{\partial F_2}{\partial x_2} + \ldots + F_n \frac{\partial F_n}{\partial x_2}\right)$$

$$G_n = \frac{\partial U}{\partial x_n} = 2\left(F_1 \frac{\partial F_1}{\partial x_n} + F_2 \frac{\partial F_2}{\partial x_n} + \ldots + F_n \frac{\partial F_n}{\partial x_n}\right)$$

(3.10)

Applying the initial guess to the new system of equations would produce a direction $\{G\}$ of maximum rise for the n-dimensional surface of the error function U. Traveling in the opposite direction of this vector will give the steepest descent along the surface, which will lead us to the minimum point on the surface.

It is required to move a step with a length that yields the minimum value of the function along the direction of motion. Such a step is called the *optimum step*. Employing the method of finding the optimum step will give a step (α) along $\{G\}$ but in the reverse direction (*i.e.* steepest descent). The new iterations would look like this:

$$\{X\} = \{X\}_0 + \alpha\{G\} \tag{3.11}$$

Where:

- $\{X\}$ is the new solution vector.
- $\{X\}_0$ is the old solution vector.
- $\{G\}$ is the gradient vector evaluated at $\{X\}_0$.
- α is the optimum step length.

The vector $\{X\}$ is substituted into the error function U, with α being unknown, $\{X\}_0$ and $\{G\}$ being known.

A fourth order equation in α is obtained and minimized with respect to α. This value of α is the optimum step which is then used to evaluate the new solution vector using equation (3.11).

The fourth order equation in α is obtained by substituting equation (3.11) into equation (3.9). This fourth order equation could be written as:

$$U(\alpha) = A\,\alpha^4 + B\alpha^3 + C\alpha^2 + D\alpha + E \qquad (3.12)$$

The values of A,B,C,D, and E are determined by evaluating the error function at five different values of α using equation (3.9). The resulting five simultaneous equations are solved for A,B,C,D and E.

For $U_{(\alpha)}$ to be minimum,

$$\frac{\partial U}{\partial \alpha} = 0 \qquad (3.13)$$

Differentiatioin of equation (3.9) gives:

$$\frac{\partial U}{\partial \alpha} = A'\,\alpha^3 + B'\,\alpha^2 + C'a + D' \qquad (3.14)$$

Where:

$$A' = 4A, \quad B' = 3B, \quad C' = 2C, \quad D' = D.$$

Solving equation (3.13) gives the *optimum step* at which the error function is minimum along the direction of move. The solution of equation (3.13) is found exactly using the exact solution of the cubic equation. Substituting α_{opt} in equation (3.11) will give the new solution vector $\{X\}$, which is then used in the next iteration. The algorithm used in finding the optimum step is illustrated in Fig. 5.

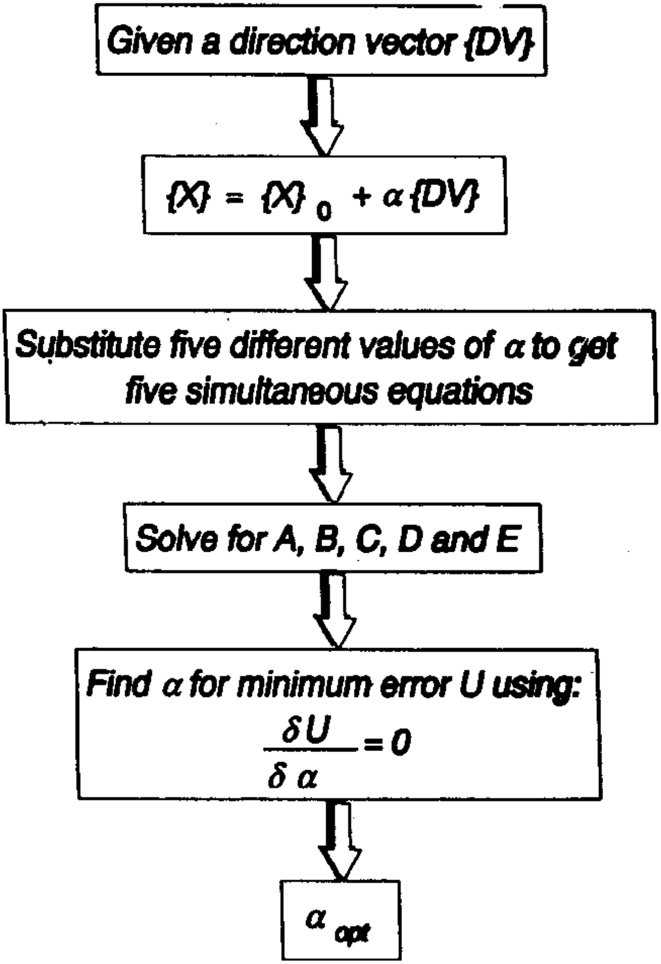

Fig. 5. Finding the optimum step.

One problem will arise when the step on the steepest descent direction approaches a local minimum. In this case, the step size becomes very small. What decides whether this is a local minimum or a global minimum is the value of the error function itself at this particular point. An error higher than the specified maximum would cause the solution to be rejected, and will introduce a random step far from the local minimum region, from which the solution process is repeated again. On the other hand, if the desired accuracy is achieved, the point is considered as the solution.

The disadvantage of this method falls in the slow action around the minimum point in moving from one optimum step to another. This problem can be overcome by switching to the second order Newton-Raphson method whenever the step starts to get very small. Introducing a switch would be on the basis of a certain minimum value for the optimum step (α_{min}). This is further discussed in the next method.

Combining steepest descent method with 2-nd order Newton Raphson

This combination can be employed to increase the speed of convergence by switching to the second order Newton-Raphson method whenever the step reaches a certain minimum value. The newton-Raphson method here is used also with the optimum step moves. The equation for the new direction in this case is:

$$\{X\}^{new} = \{X\}^{old} + \alpha \{\Delta X\} \quad (3.15)$$

Where $\{\Delta X\}$ is given by the second order Newton-Raphson method described earlier, and α is the step to be moved on the direction $\{\Delta X\}$. Comparing this equation with equation (3.11), it can be seen that the steepest descent direction $\{G\}$ has been replaced by $\{\Delta X\}$. Figure 6 describes the flow of computation of this method.

Results

Several examples were solved using the synthesis equations generation and solution procedures presented in this paper. Following are the motion programs for each example with the results of the synthesis process. It could be seen that a wide range of mechanism synthesis problems are solved using this procedure accompanied with the numerical solution technique.

1) Four-bar/input-output, 5 positions (Fig.7):

Motion program

$\Delta\theta_2$ [rad]	1.1	1.5	1.8	2.0
$\Delta\theta_4$ [rad]	3.3	4.4	5.4	6.0

Fixed points:
O_2 (0,0), O_4 (1,0).

Links lengths:
O_2A: 1.59, O_4B: 0.52, AB: 1.34

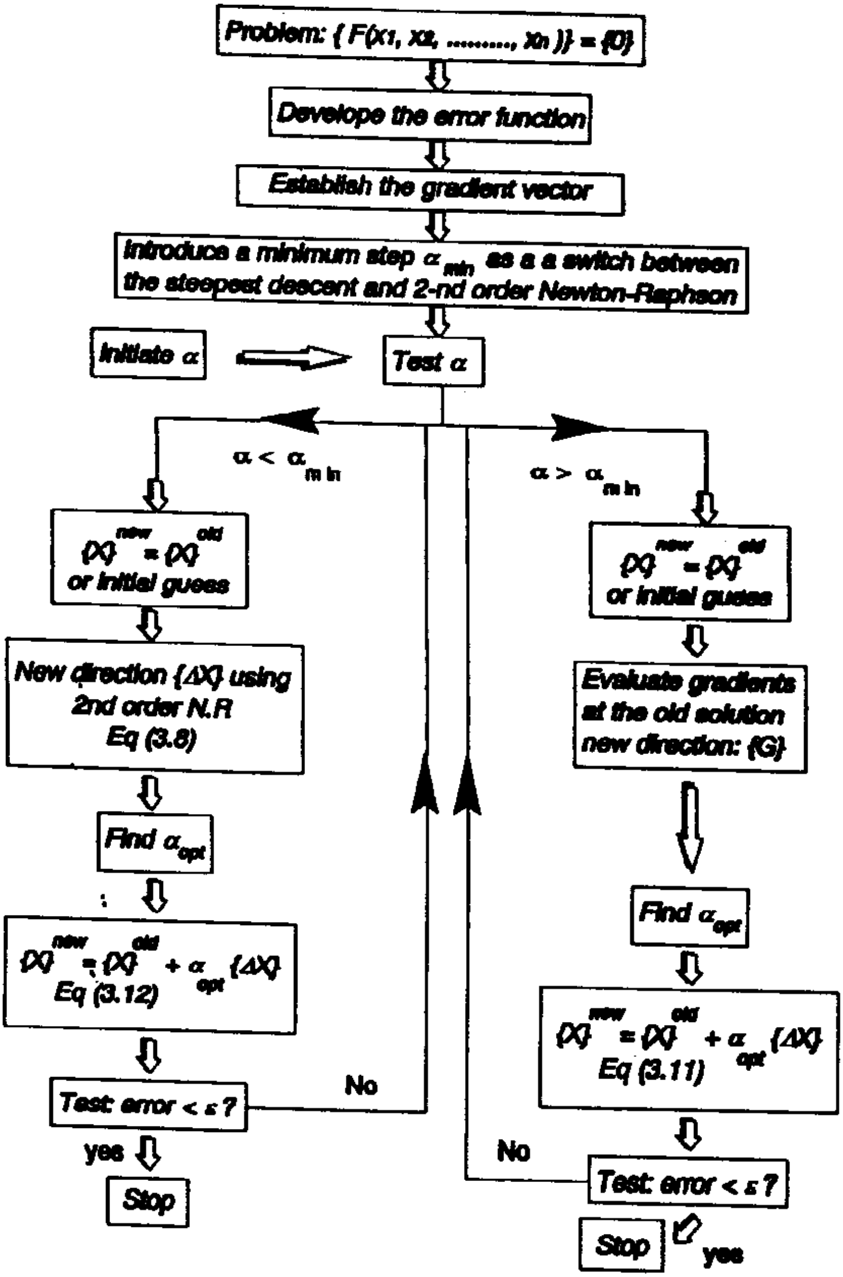

Fig. 6. The steepest descent combined with 2nd order Newton Raphson.

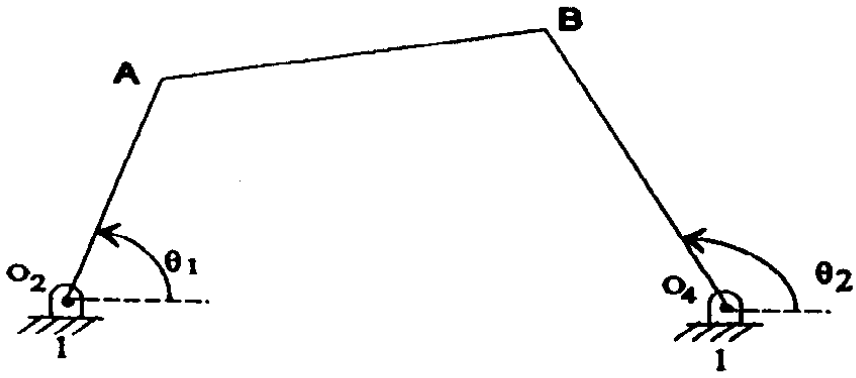

Fig. 7. Four-bar input-output mechanism .

2) Four-bar/input-point path tracing, 5 positions (Fig. 8).

Motion program

$\Delta\theta_2$ [rad]	0.0	1.0	2.0	3.0	4.0
X_c	1.0	2.0	3.0	5.0	6.0
Y_c	2.0	4.0	5.0	8.0	9.0

Fixed points:

O_2 (6.2, 3.5), O_4 (7.2, 1.3).

Links lengths:

O_2A: 0.99, O_4B: 1.66, BC: 6.68, AB: 2.24, AC: 4.48.

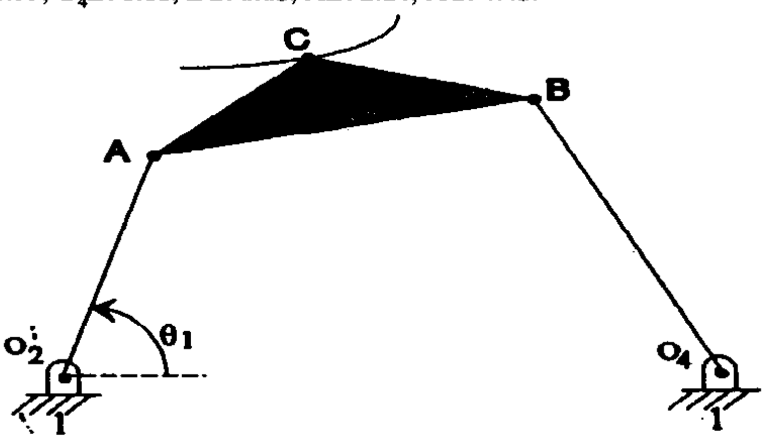

Fig. 8. Four-bar input-point path mechanism .

3) Four-bar/point path trracing, 9 positions (Fig. 9).

Motion program

X_c	8.1	10.5	12.2	11.4	9.5	7.6	6.2	5.2	6.6
Y_c	6.0	5.1	4.1	5.35	6.6	7.3	7.5	7.5	6.8

Fixed points:
O_2 (4.7, 0.26), O_4 (4.8, 0.27).

Links lengths:
O_2A: 0.92, O_4B: 0.94, BC: 7.38, AB: 0.13, AC: 7.5.

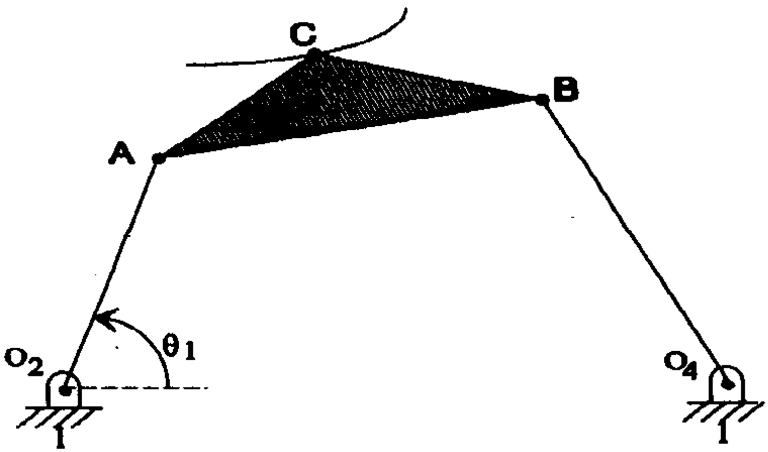

Fig. 9. Four-bar point path generation mechanism .

4) Slider/input-output, 7 positions (Fig. 10)

Motion program

S_i	1	2	3	4	5	6	7
S_o	7	6	5	4	3	2	1

Fixed points:
O_2 (−0.2, 0.6), O_4(0.2,0.4).

Link length and sliders path orientations:
AB: 7.84, α_i : 80.1°, α_0 : 260°.

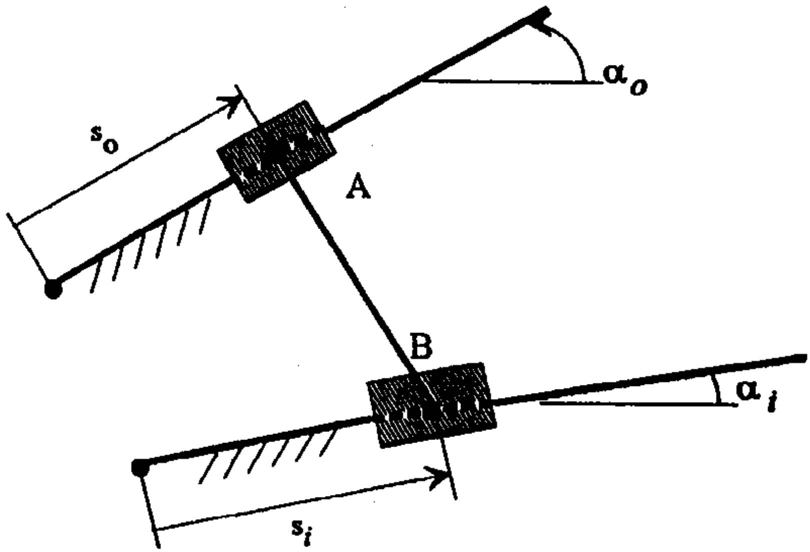

Fig. 10. Slider mechanism for input-output.

5) Six-bar/input-point path tracing, 7 positions (Fig. 11):

Motion program

$\Delta\theta_2$ [rad]	0.5	1.0	1.5	2.0	2.5	3.0	3.2
X_c	0.0	1.0	2.0	3.0	4.0	2.0	1.0
X_c	1.0	3.0	2.0	1.0	2.0	3.0	4.0

Fixed points:

O_2 (0.0, 2.5), O_4 (2.7, 3.2), O_6 (3.4, 6.5).

Links lengths:

O_4E: 1.77, O_6D: 2.79, DC: 3.65, BC: 2.28, BD: 1.63, BE: 2.34, AE: 1.40, AB: 2.09.

6) Eight-bar/two-body guidance, 5 positions (Fig. 12).

Motion program

First body:

$1X_s$	− 5.0	− 1.7	1.74	5.0	7.7
Y_s	8.66	9.85	9.85	8.7	6.4
$\Delta\theta$ [rad]	0.0	0.35	0.7	1.1	1.4

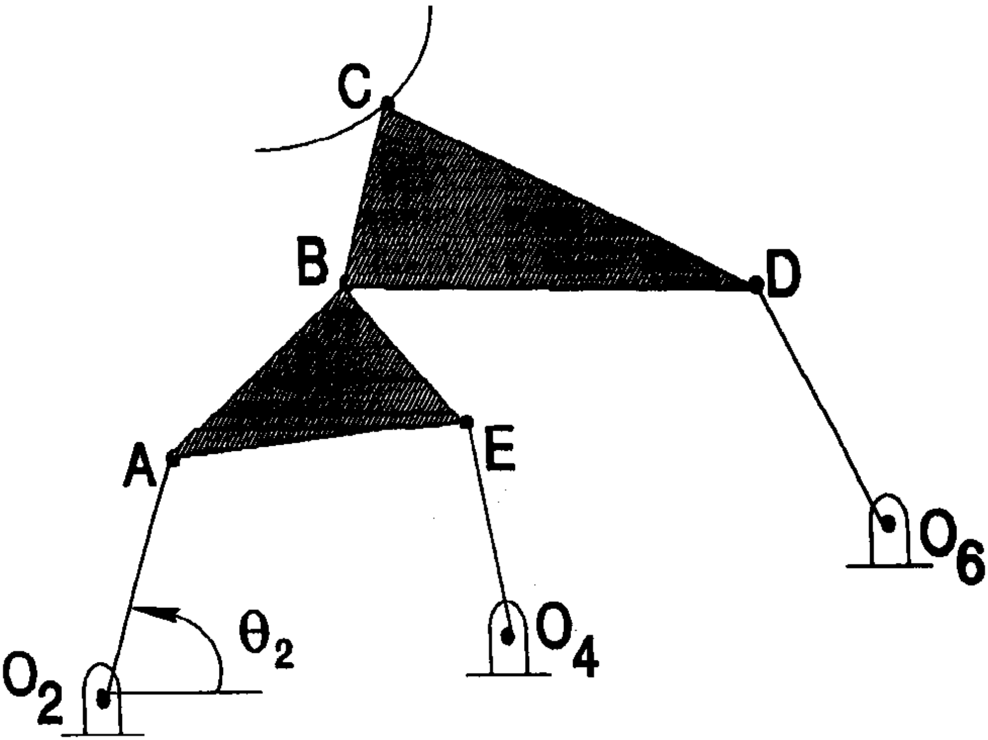

Fig. 11. Six-bar input-point path mechanism .

Second body:

X_p	−18.9	−16.4	−12.9	−8.4	−3.47
Y_p	6.84	11.47	15.32	18.13	19.7
$\Delta\varphi$ [rad]	0.0	0.26	0.52	0.79	1.05

Fixed points:
O_2 (0.0, 0.0), O_4 (0.0, 10.0).

Links lengths:
O_2A: 5.20, O_2B: 4.33, O_4G: 9.69, AB: 1.20, BF: 9.07, FG: 13.17, CA: 18.33, DA: 13.53, BE: 19.50, FE: 10.44, HF: 8.06, HG: 10.61.

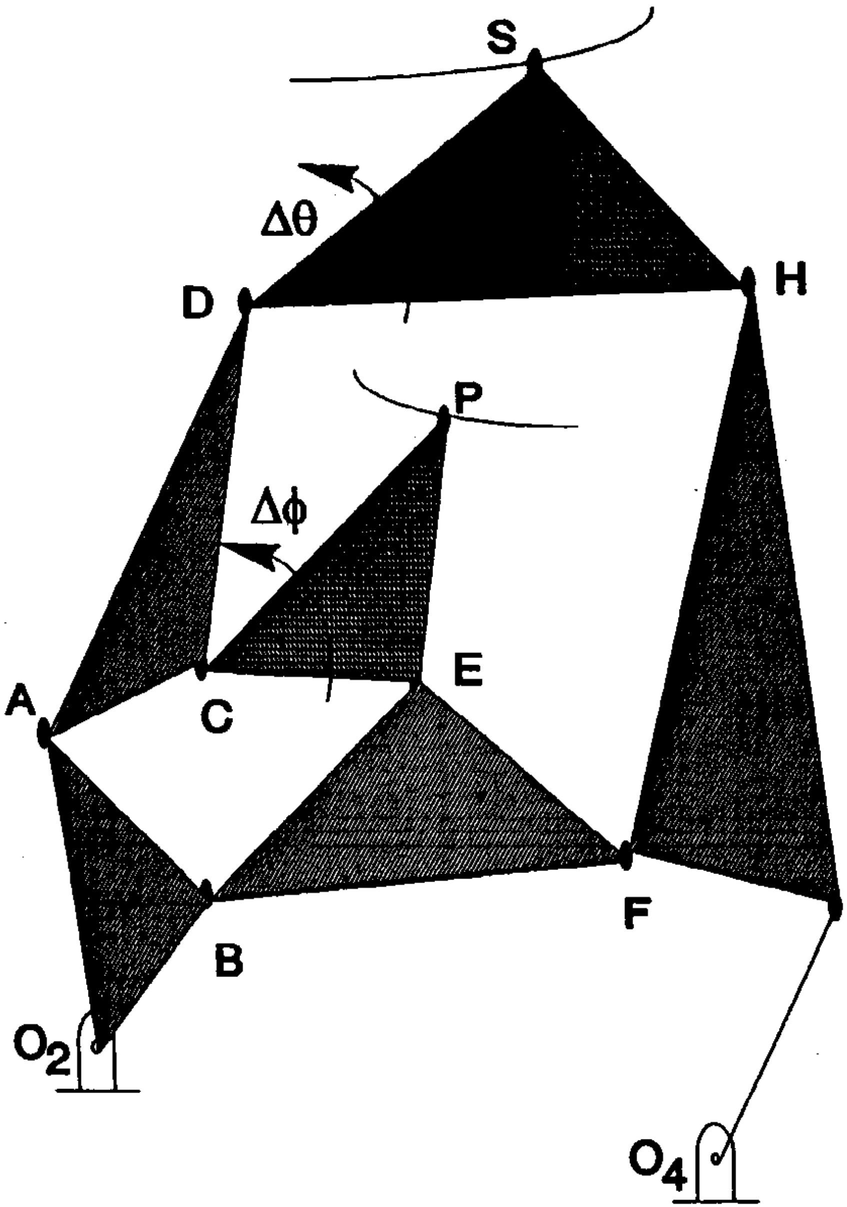

Fig. 12. Eight-bar two-body guidance mechanism .

Conclusions and Recommendations

The analytical synthesis of mechanisms is a topic that is still not explored completely. This work is considered an advanced step ahead towards the complete solution of the analytical mechanism synthesis problem. It has been shown that complex mechanisms can be formulated easily by imposing length constraints on the mechanism's links. This synthesis procedure is developed in such a way that only second order equations are generated. The second order synthesis equations generated are better suited for the numerical solution procedure developed for such systems. The numerical procedure is a combination of second order Newton-Raphson method and steepest descent method with optimum step moves. It managed to solve six bar mechanisms to an excellent accuracy. An eight-bar two-body guidance mechanism was also solved by this method with a reasonable accuracy.

Finally, it is recommended that additional constraints are imposed on the synthesized mechanism to ensure non-trivial solutions sometimes generated by the numerical procedure. Moreover, simulation must be done in order to check for the various mechanism defects usually expected after the synthesis process. A full dynamic and kinematic analysis must be done as a final step before the final adoption of the mechanism. This work forms the core for a complete package that can be developed in this field.

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طريقة آلية لاحتساب أبعاد الآلات المستوية

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ملخص البحث. تصف هذه الورقة المبادىء الأساسية المستخدمة في تطوير طريقة آلية لاحتساب أبعاد الآلات المستوية. هذه الطريقة تولد معادلات التصميم عن طريق فرض مقيدات الطول بالإضافة إلى مقيدات برنامج الحركة للآلة. معادلات التصميم المولدة تكون بشكل معادلات جبرية من الدرجة الثانية فقط.

وتم تطوير طريقة عددية خاصة بحل مثل هذه المعادلات الجبرية من الدرجة الثانية. وترتكز هذه الطريقة العددية على الجمع بين طريقة الانحدار الأقصى وطريقة نيوتن ـ رافسون ذات الدرجة الثانية.

صممت آلة سداسية الأذرع وآلة ثمانية الأذرع باستخدام الطريقة المقدمة في هذه الورقة بالإضافة إلى آلات مستوية أخرى.