J. King Saud Univ., Vol 1, *Eng. Sci.* (1,2), pp. 111-122, Riyadh (1409/1989)

Harmonic Analysis of Three Phase AC Voltage Regulators Using Thyristor – Diode Switches

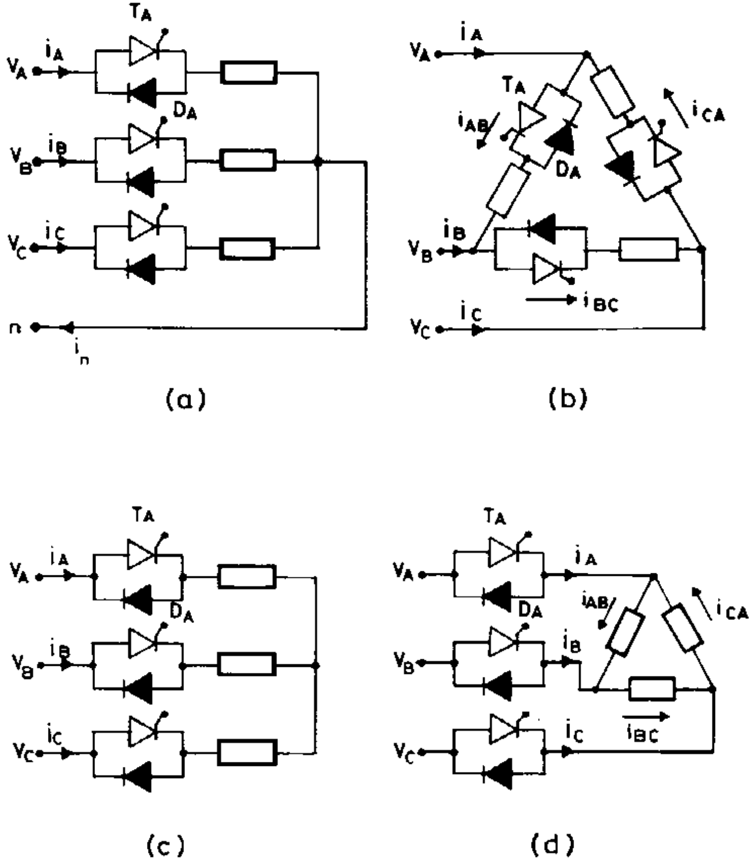
N.H. Malik
Electrical Engineering Department, College of Engineering, King Saud University
P.O. Box 800, Riyadh-11421, Saudi Arabia

Abstract. Based upon numerical evaluation of Fourier coefficients, complete harmonic analysis is carried out for various three phase, AC thyrode voltage regulators. Various quantities of interest are computed and presented in the form of normalized curves for balanced R-L loads of different power factors. It is shown that the line controlled circuits offer the best overall performance and are superior in performance than the conventionally used thyristor voltage controllers.

1. Introduction

Thyristors are now widely used for voltage and power control in both DC and AC circuits [1]. Commonly used methods for voltage control are: (**a**) Phase angle control, (**b**) Integral cycle control, and (c) Pulse width modulation control. In phase angle control [2,3,4] a pair of inverse parallel connected thyristors or a triac are used. By delaying the thyristor firing angle, the rms load voltage can be controlled. If one of the thyristors in each pair is replaced by a diode, voltage control is still possible. An arrangement using such a thyristor - diode combination, is referred to as a thyrode [3].

To supply a three phase load, a number of different circuit configurations (Fig. 1) can be used employing the thyrode switches [3]. In general, the performance characteristics of such three phase voltage controllers are not well understood. This is due to the fact that the harmonic analysis of these configurations have previously been reported for only a few specific conditions [3,5,6]. This paper describes the harmonic analysis of three phase thyrode controllers feeding balanced R-L loads of different power factors. A generalized computer program has been developed to determine the comprehensive harmonic characteristics of these voltage controllers and the results are reported here.



(a) (b)

(c) (d)

Fig. 1 . Various circuit configurations for three phase thyrode voltage controllers

2. System Equations

Various three phase configurations shown in Fig. 1 can broadly be classified as: (i) Branch controlled, Fig. 1(a) and 1(b), and (ii) Line controlled, Fig. 1(c) and 1(d). In branch controlled circuits, each phase operates in a single phase mode, independent of the other two phases whereas in the line controlled circuits the conduction in any one line also depends upon the other two lines. Therefore, for voltage and current relationships, one has to examine each configuration.

Let the three phase supply voltages be balanced and given by:

$$v_A = \sqrt{2}\, V \sin(\omega t) \tag{1}$$

$$v_C = \sqrt{2}\, V \sin\left(\omega t - \frac{2\pi}{3}\right) \tag{2}$$

$$v_B = \sqrt{2}\, V \sin\left(\omega t + \frac{2\pi}{3}\right) \tag{3}$$

Furthermore, let the load be passive and balanced having a per phase impedance of $Z\angle\phi$. To keep balanced three phase operation, the thyristors in phases (or lines) A, B and C are gated at α, $\alpha + 2\pi/3$, and $\alpha - 2\pi/3$ radians respectively.

2.1. Phase Controlled Circuits

In the phase controlled circuit of Fig. 1(a) phase A thyristor (T_A) starts conduction when firing pulse is applied at $\omega t = \alpha$. The conduction continues till current becomes zero. Then the conduction starts in the reverse direction through the diode till D_A current again becomes zero at $\omega t = \varkappa$. Thus the equations for phase A load voltage (V_{LA}) and phase A current (i_A) are given as follows:

$$V_{LA} = v_A \Big]_{\alpha}^{\varkappa} \tag{4}$$

$$i_A = \frac{\sqrt{2}}{Z} V \left[\sin(\omega t - \phi) - \sin(\alpha - \phi)\, e^{(\alpha - \omega t)\cot\phi}\right]_{\alpha}^{\varkappa} \tag{5}$$

The diode extinction angle $\varkappa$ can be obtained by substituting $\omega t = \varkappa$ and $i_A = 0$ in equation (5) and solving it numerically. The resulting value of $\varkappa$ must be in the range $2\pi + \phi \leqslant \varkappa \leqslant 3\pi$. In Fig. 1(a), the other two phases will have similar behaviour with appropriate phase shifts. The neutral current will be the sum of all three phase currents.

For the delta connected circuit of Fig. 1(b), the thyristor and diode conduction patterns will be similar to those described earlier. Therefore, phase load voltages and currents of the two circuits will have similar general behaviour. If, as before, α and $\varkappa$ are measured with respect to v_A, the equations for phase AB load voltage (V_{LAB}) and load current (i_{AB}) for the circuit of Fig. 1(b) are given as follows:

$$V_{LAB} = v_{AB} \Big]_{\alpha+\pi/6}^{\varkappa+\pi/6} \tag{6}$$

$$i_{AB} = \frac{\sqrt{6}}{Z} V \Big[\sin(\omega t + \pi/6 - \phi) - \sin(\alpha+\pi/6-\phi)\, e^{(\omega t-\alpha)\cot\phi}\Big]_{\alpha+\pi/6}^{\varkappa+\pi/6} \tag{7}$$

The other two phases will have similar voltages and currents with appropriate phase shifts. The line current (i_A) will then be equal to

$$i_A = i_{AB} - i_{CA} \tag{8}$$

2.2. Line Controlled Circuits

Both of the circuits of Fig. 1(c) and 1(d) are line controlled and as far as line quantities are concerned, the behaviour of both circuits is similar. For the current to flow in any one line, either one or both of the other two lines must also conduct simultaneously. Thus, these circuits exhibit complicated conduction patterns depending upon values of α and ϕ. For the line controlled circuits, α ranges from ϕ to $7\pi/6$ radians. When all three lines conduct, the phase A load voltage equals v_A. When only two lines conduct simultaneously, the phase A load voltage is one half of either v_{AB} or v_{AC} depending upon whether lines AB or AC are conducting simultaneously. During one complete cycle of supply voltage, there may be periods when all three or only two (AB or BC or CA) or none of the lines conduct at a given time. Details of the conduction patterns and appropriate equations for voltages and currents are given in reference [3] and will not be repeated here.

3. Method of Solution

In order to compute various quantities of interest, N values of load voltage and current (line as well as phase) were calculated and stored using the appropriate equations for one complete cycle of the supply voltage. From these N values, Fourier coefficients were calculated by using a Fast Fourier transform subroutine [2]. It was found that this procedure gives reliable and accurate results. Once, the Fourier coefficients are known, it is a simple matter to determine the various parameters of interest.

For the circuits of Fig. 1, computations were performed using N = 360, Z (star) = 1 pu, Z(delta) = 3 pu, $\phi = 0, 15, 30, \ldots 90°$ and different α values to cover the full control range for each configuration. The above values of Z resulted in the same pu values of line currents and three phase power for full conduction (*i.e.* $\alpha \leqslant \phi$). In the next section, the main results will be presented in the form of normalized curves showing the variation of desired parameters with α as well as ϕ.

4. Results and Discussion

Figure 2 shows variation of the rms load voltage with α and ϕ for branch and line controlled circuits. When $\alpha \leqslant \phi$, the load voltage is 1 pu. As α is increased, the load voltage decreases gradually. The line controlled circuits offer a wider range of voltage control from 1 pu to 0 pu, irrespective of the value of ϕ. However, for the branch controlled circuits, the range of voltage control is limited and highly dependent upon the value of ϕ.

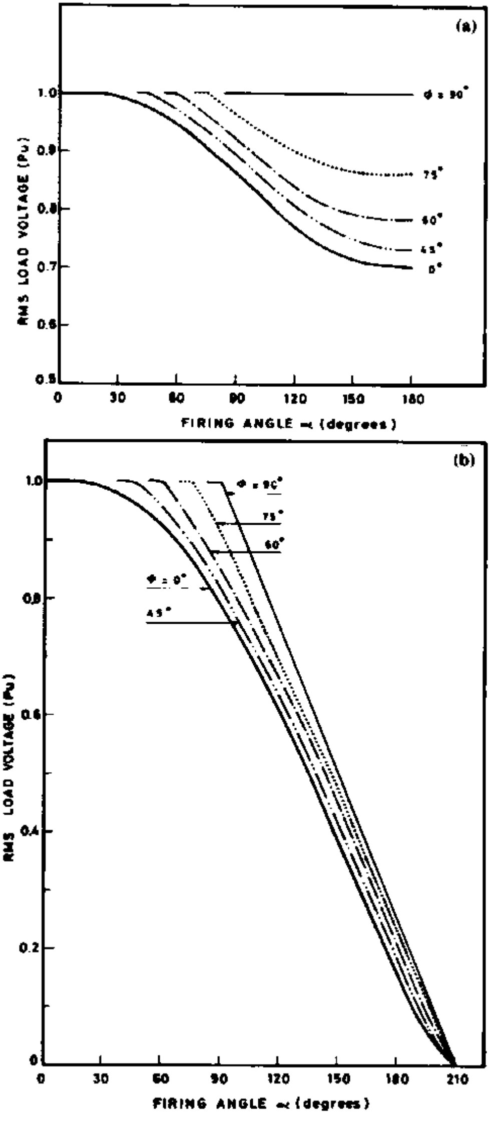


Fig. 2. Variation of rms load voltage with α and ϕ for:
(a) Branch controlled circuits.
(b) Line controlled circuits.

Figure 3 shows the variation of rms line current with α and ϕ for various circuit configurations. For line controlled circuits, full range of current control is available. For branch controlled circuits, the current control is only partial. Moreover, in such cases, for highly inductive loads the current increases with α over a certain control range. This is due to the presence of a large transient component which decays very slowly with time. Therefore, such configurations have a large value of DC component in the phase currents. For the delta connected case, this DC component and other triplen harmonics do not appear in the line currents. Therefore, the variation of rms line current with α and ϕ is different for the two branch controlled circuits as is obvious from Fig. 3.

Figure 4 shows the total rms harmonic content of the line currents for various configurations. It is obvious that the branch controlled delta connected circuit have the lowest overall line current harmonic content. Among the other circuits, the line controlled circuits have lower harmonic content whereas the branch controlled star connected configuration has the worst performance, as far as line current harmonics are concerned. Moreover, this circuit also has very large and undesirable neutral current as is evident from Fig. 5. This neutral current is entirely made up of DC component and triplen harmonics.

Although, the branch controlled delta connected circuit has minimal line current harmonics, the same is not the case as far as phase current harmonics are concerned. The phase currents contain DC as well as triplen harmonics in addition to the usual harmonics present in the line currents. Thus, the phase currents harmonics will be as shown in Fig. 4(a).

Considering all the configurations, the line controlled circuits have the best overall performance because these offer full range of voltage and power control, have wider range of firing angles for the control purposes and offer relatively lower values of line current harmonics. The line current does not contain any DC or triplen harmonics. However, all the other harmonics are present. The amplitude of various harmonics depends upon α, ϕ and the order of the harmonic. Fig. 6 shows variation of *2nd*, *4th* and *5th* harmonic with α and ϕ for the line controlled circuit of Fig. 1(c).

A comparison of the line controlled thyrode controllers with line controlled thyristor controllers discussed in references [2,4] shows that the thyrode controller has a simple control for firing angle (one narrow gate pulse per cycle for the thyrode circuit compared to two sixty degrees wide gate pulses per cycle for the thyristor circuit), the control range of firing angle is wider ($7\pi/6$ radians as compared to $5\pi/6$ radians) and results in higher values of distortion and power factors. Its only drawback is the existance of some even order non triplen harmonics in the line current. Therefore, if such harmonics may be tolerated in the line current, a thyrode controller which will also be less expensive compared to thyristor controller should be preferred.

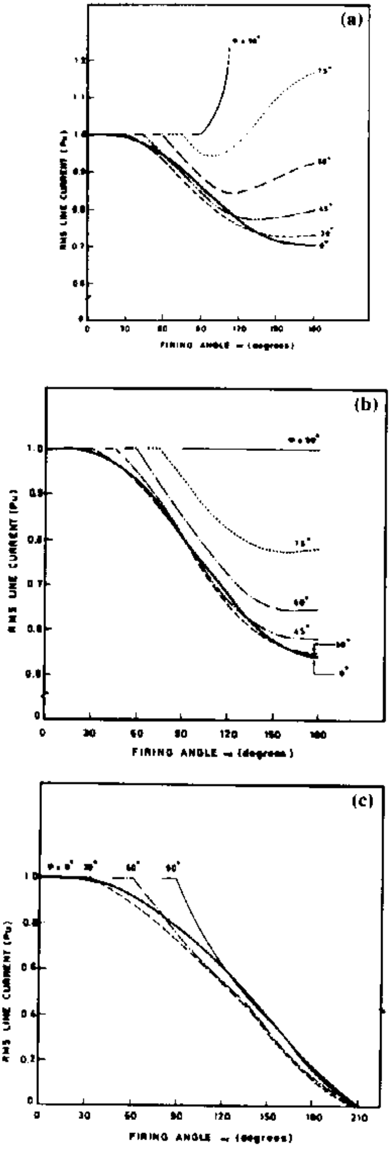


Fig. 3. Variation of rms line current with α and ϕ for:
(a) Branch controlled circuit of Fig 1(a).
(b) Branch controlled circuit of Fig. 1(b).
(c) Line controlled circuits.

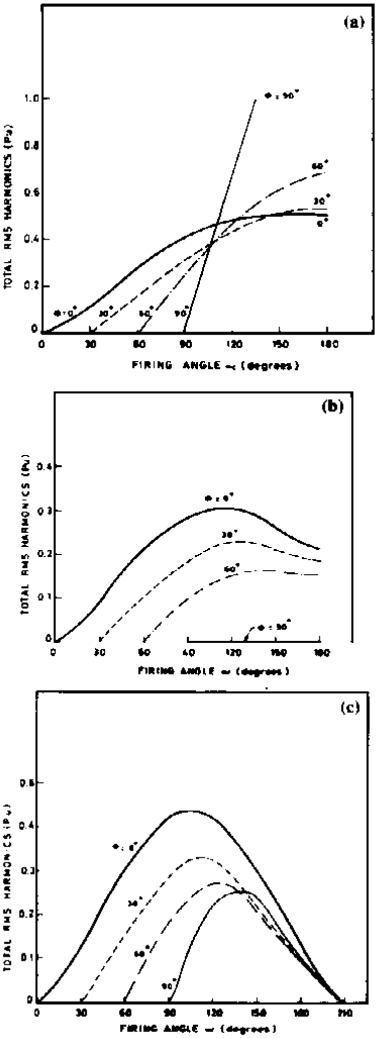


Fig. 4. Total rms harmonic content of line current for:
(a) Branch controlled circuit of Fig. 1(a).
(b) Branch controlled circuit of Fig. 1(b).
(c) Line controlled circuits.

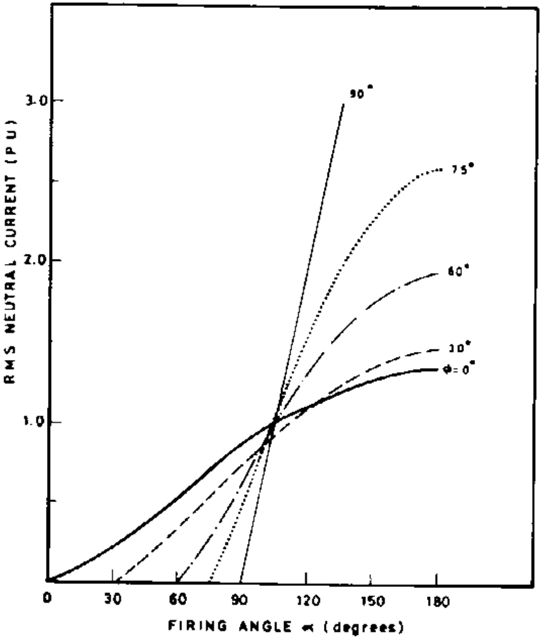


Fig. 5. Rms neutral current for the circuit of Fig. 1(a).

5. Conclusions

Harmonic analysis is presented for three phases AC voltage regulators employing thyristor diode switches. Of the various configurations considered, it is found that the branch controlled circuits are unsuitable since these provide limited voltage variation. These also have DC and other even as well as triplen harmonics in the phase currents. In the case of three phase four wire arrangement, there is a large and undesirable neutral current which becomes very significant for highly inductive loads at large values of α. Among the various configurations considered, the line controlled circuits have the best overall performance. Such circuits are also less expensive, simpler and superior in performance when compared with conventional thyristor controllers of similar configuration.

Acknowledgements: The author would like to acknowledge the support and facilities provided by King Saud University, Riyadh, Saudi Arabia to carry out this work. He would also like to thank his colleagues Drs. A. Mazi and A. Al-Arainy for useful discussions.

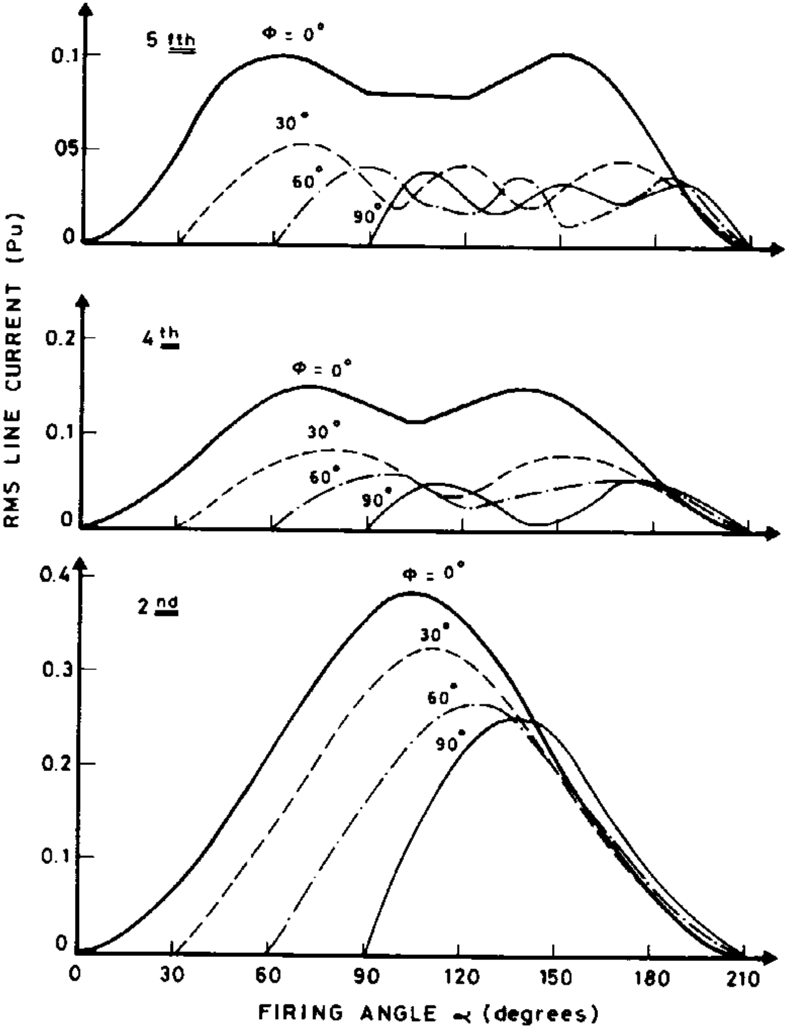


Fig. 6. Variation of major line current harmonics with α and ϕ for the line controlled thyrode controllers.

References

[1] **Dewan, S.B.** and **Straughen, A.** *Power Semiconductor Circuits.* England, London: John Wiley, 1975.

[2] **Malik, Nazar H.** *et al.* "Analysis and performance of three phase, phase controlled thyristor AC voltage controllers." *IEEE Trans.*, **IE-32**, 3 (1985), 192–199.

[3] **Shepherd, W.** *Thyristor Control of AC Circuits.* England, St. Albans: Crosby Lockwood Staples Ltd., 1975.

[4] **Malik, N.H.** and **Haque, S.E.** "Harmonic analysis of three phase AC voltage controllers." *Conf. Record of 1986 IEEE/IAS Annual Meeting,* Denver, Colorado, USA, (1986), 1078–1085.

[5] **Sharma, V.K.** and **Fleming, R.Z.** "Analysis of a three phase delta connected inductive load controlled by an SCR-diode switch." *IEEE Trans.*, **IA-21**, 3 (1985), 555–561.

[6] **Odiete, G.C.E.**; **Nakanishi, S.** and **Shepherd, W.** "Analysis and control characteristics of thyristor and thyrode controllers with purely resistive load." *Conf. Record of 1986 IEEE/IAS Annual Meeting,* Denver, Colorado, USA, (1986), 758–762.

(*Manuscript Received:* 5-9-1987; *Accepted:* 20-2-1988)

التحليل التوافقي لمنظم الجهد ذو الثلاثة أوجه من نوع ثايرد (THYRODE)

نذر حسين مالك

قسم الهندسة الكهربائية، كلية الهندسة، جامعة الملك سعود، ص.ب ٨٠٠،
الرياض ١١٤٢١، المملكة العربية السعودية

ملخص البحث: باستخدام التقويم العددي لعوامل فورير تم تحليل توافقي كامل لعدة أشكال من منظمات الجهد ذو الثلاثة أوجه من نوع ثايرد (THYRODE). وهذا البحث يختص بالأحمال المتزنة من نوع (R-L) وبمعامل قدرة متغيرة حيث حسبت عدة علاقات ذات أهمية ورسمت بشكل نسبي. وتبين نتائج هذا البحث أن الدائرة المحكمة خطيا تعطي أحسن الأداء وتتميز عن الطريقة الاعتيادية والتي تتحكم بالجهد عن طريق حاكم الثايرستور.