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SHORT COMMUNICATION

A Computer Aided Tool for the Analysis of Telephone Traffic on High-Usage Routes

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Abstract. The investigation of telephone traffic on high-usage routes is one important problem in the evaluation of the busy-hour service of telephone exchanges. In this paper, the acceptable level of offered traffic on a high-usage route is considered to be that which results in a specified last trunk occupancy assuming sequential hunting. Using teletraffic analysis, a mathematical formula that represents the case is derived; and a computer program written in Pascal is described. The application of the program is demonstrated by means of two examples; the first is related to the development of traffic tables, and the second is concerned with the investigation of high-usage routes on some telephone exchanges in practical use.

Introduction

In telephone networks, a telephone exchange is usually connected to other exchanges via two types of route: high-usage routes, and final routes, with each of such routes consisting of a group of trunks, and with each trunk being able to serve a single telephone call. The high-usage routes are deliberately offered high-traffic, so that the busy-hour usage of the trunks is high. As a result, high traffic congestion takes place on such routes. The congested traffic on a number of high-usage routes is overflowed to a final route usually called an alternative final route, to distinguish it from a direct final route which does not accept any overflowed traffic. The basic advantage of this arrangement is related to the fact that fewer additional trunks would be required for the alternative final route to carry the overflowed traffic, than would be needed if additional trunks were added to each of the high-usage routes to provide satisfactory grade of service which is usually specified by a given acceptable congestion level [1-3].

The determination of the number of trunks that should be provided to a high-usage route, with a given busy-hour offered traffic, is an important problem in the design of the high-usage arrangement in telephone exchanges. One solution to this problem is based on finding the optimum number of trunks that if used, for the high-usage route concerned, it would result in a specific required last trunk occupancy assuming sequential hunting, which is the least trunk occupancy under the assumed hunting. This paper is concerned with describing the implementation of the solution on the computer, together with the presentation of some examples of the application of the implementation to practical investigations. Such implementation and demonstration would be useful to students and engineers in the field, especially with the present unsatisfactory level of teletraffic training, in many parts of the world, generally noticed by the ITU [4,5]. The paper addresses the following major points:

*The teletraffic analysis required for solving the problem.

*The computer programming needed to implement the analysis.

*Examples on the use of the resulted programs including the development of design tables for high-usage routes, and the practical investigation of the traffic flow on the high-usage routes of some Saudi telephone exchanges.

Teletraffic Analysis

Bear [2], has given the following formula for the probability [P (n,c,a)] of having the last c trunks of a full-availability group of n trunks busy, irrespective of the state of the first (s = n − c) trunks, assuming sequential hunting, and a specific offered traffic, a.

$$P(n,c,a) = B(n,a)\ F(n,c,a) \qquad (1)$$

where

B(n,a) is the Erlang B congestion.
F(n,c,a) is the last c trunks occupancy factor.

B(n,a) is given as follows:

$$B(n,a) = \frac{a^n/n!}{\sum_{i=1}^{i=n} (a^i/i!)} \qquad (2)$$

F(n,c,a) is given as follows:

$$F(n,c,a) = \frac{\sum_{m=0}^{s} \binom{c+m}{m} \frac{a^{s-m}}{(s-m)!}}{\sum_{m=0}^{s} \binom{c-1+m}{m} \frac{a^{s-m}}{(s-m)!}} \tag{3}$$

P (n,c = 1,a), or in short P(n,a), gives the occupancy of the last trunk. For this case, F (n,c = 1,a) or F(n,a) becomes as follows:

$$F(n,a) = \frac{\sum_{i=0}^{i=n-1} [(n-i)\, a^i/i!]}{\sum_{i=0}^{i=n-1} (a^i/i!)} \tag{4}$$

Using equation (1), p(n,a) becomes:

$$P(n,a) = \frac{a^n/n!}{\sum_{i=0}^{i=n} (a^i/i!)} \quad \frac{\sum_{i=0}^{i=n-1} [(n-i)\, a^i/i!]}{\sum_{i=0}^{i=n-1} (a^i/i!)} \tag{5}$$

Computer Programming

Using the teletraffic analysis given above, computer programs are required to implement the analysis, so that the problem of the evaluation of traffic on high-usage routes, can be solved. Fig. 1 presents a top-down refinement view of the problem, showing the three major questions that should be taken into account. The basic question of the three is finding the last trunk occupancy, given the number of trunks and traffic offered. For this question equation (5) should be programmed. The other two major questions, which are concerned with: finding the optimum number of trunks (given the traffic offered and the occupancy required), and finding the optimum offered traffic (given the number of trunks and the occupancy required), can then be programmed using the programs of the last trunk occupancy formula.

Computer programs have been developed for dealing with the three major questions considered above. The computer language Pascal has been chosen for this purpose, and the resulted programs has been run on an IBM-PC. The structured nature

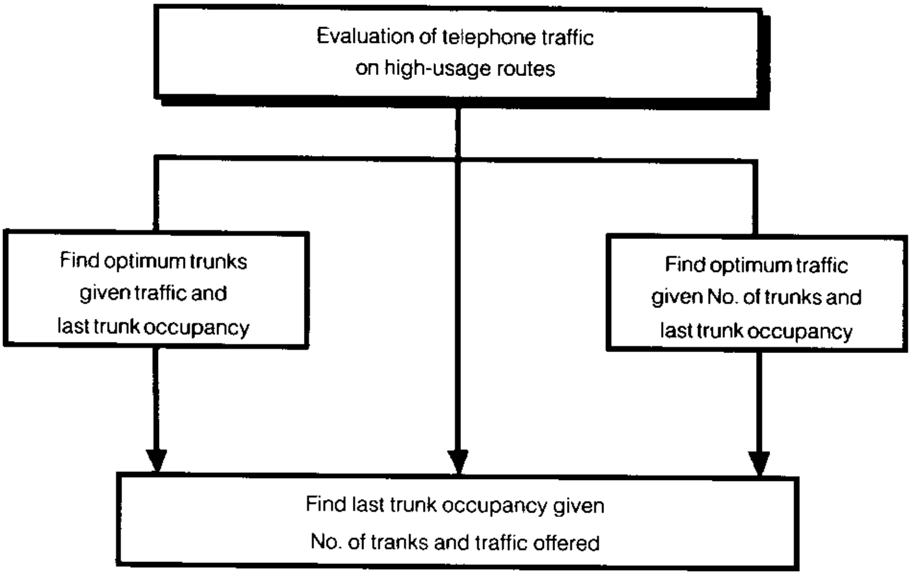

Fig. 1. A top-down refinement view of the problem of the evaluation of telephone traffic on high-usage routes

of the language, its readability, even if the human reader is not familiar with the language, and its availability on most computers, large and small, have influence in choice [6]. The resulted programs are of modular construction, with each module concerned with a specific task. This is illustrated in Figs 2 through 4.

Figure 2 presents a Pascal function for computing the last trunk occupancy according to equation (5), given the number of trunks of the high-usage route and the traffic offered. The algorithm used in this function is divided into two main blocks, one for computing B(n,a), and the other for computing F(n,a). Fig. 3 presents a Pascal function for computing the optimum number of trunks, while Fig. 4 shows a Pascal function for computing the optimum traffic offered, with both functions using the function of Fig. 2.

Application Examples

Two examples on the use of the computer programs presented above are given here. The first is concerned with the development of design tables for relating the number of trunks on high-usage routes to the traffic offered, and the required last trunk occupancy. For this purpose, a Pascal procedure has been developed using the Pascal functions described above. Table 1 shows an example of the results produced by the procedure. The Table provides for a specified range of trunks, against a specified range of last trunk occupancy, the optimum traffic expected to be offered.

COMPUTE LAST TRUNK OCCUPANCY

```
FUNCTION LtrOccp        (n : integer      { Number of Tranks                 } :
                          a : real        { Computation Variables            }) : real;

VAR
     i                  : integer    { Counter                              } :
     el, sum, nom       : real       { Computation Variables                } :
     b, f               : real       { Congestion and Occupancy Factor      } :

{ ---------------------------      Compute ErlangB Formula : B(n ,a)      ------------------------- }

BEGIN
     el        := 1; sum := el ;
     FOR i     := 1  TO  n  DO
        BEGIN
              el  := el * a / i;
              sum  := sum + el;
              END;
     b  := el / sum;

{ ---------------------------      Compute Occupancy Factor : F(n ,a)      ------------------------- }
     el        := 1; sum := el; nom := n;
     FOR i     := 1  TO  n -1)  DO
        BEGIN
              el  := el * a / i;
              sum  := sum + el;
              nom  := nom + (n + 1)* el
              END;
     f  := nom / sum;

{ ------------------------------------------      Result: P(n, a)      ---------------------------------------- }
     LTrOccp  := b * f
END ;
```

Fig. 2. A Pascal function for computing the occupancy of the last trunk of a high-usage telephone route.

```
COMPUTE OPTIMUM TRUNKS FOR HUR

FUNCTION HUOpr      (a  { Traffic                   },
                     q  { Last Trunk Occupancy      } : real) : integer ;

VAR
    trunks  : integer     { Number of Trunks     } ;
    occup   : real        { Occupancy            } ;

BEGIN
    trunks := round (a);
    occup := LTrOccp(trunks,a);
    WHILE  (occup > q)  DO
       BEGIN
            trunks := trunks + 1 ;
            occup := LTrOccup(trunks,a)
       END;
    HUOpTr := trunks
END;
{ ---------------------------------------------------------------------------------------------------- }
```

Fig. 3. A Pascal function for computing the optimum number of trunks on a high-usage telephone route

The second example considers the traffic flow on the high-usage routes of some Saudi telephone exchanges [7], and provides, for each route, an investigation of the optimum number of trunks against the available number of trunks, for a given busy-hour traffic, and a specified last trunk occupancy. Table 2 presents the results of the investigation.

COMPUTE OPTIMUM TRAFFIC FOR HUR

```
FUNCTION HUOpTf      (a  { Number of trunks         },
                      q  { Last Trunk Occupancy     } : real) : real ;

VAR

    traffic, occup, decrement : real { computation Variables } ;

    BEGIN

      decrement    := n/100 ;

      traffic      := n;

      occup        := LTrOccup(n,traffic);

      WHILE  (occup > q)  DO

         BEGIN

            traffic := traffic - decrement;

            occup := LtrOccup(n,traffic)

         END ;

    IF(n = 1)    THEN HUOpTf := q

                 ELSE HUOpTf := traffic

END;

{ ................................................................................................ }
```

Fig. 4. A Pascal function for computing the optimum traffic expected to be offered on a high-usage telephone route.

Conclusions

The teletraffic analysis, computer implementation, and applications presented here, for the investigation of telephone traffic on high-usage routes, using the last trunk occupancy rules, would be useful for students and professionals in the field, in their investigation of various related practical problems.

Table 1. Design table for high-usage telephone routes (with full-availability groups of trunks)

Number of trunks	Expected offered traffic (for different last trunk occupancy)				
	Last trunk occupancy				
	0.300	0.350	0.400	0.450	0.500
10	6.900	7.400	7.800	8.300	8.900
11	7.700	8.250	8.690	9.240	9.790
12	8.520	9.000	9.600	10.080	10.680
13	9.360	9.880	10.400	11.050	11.570
14	10.220	10.780	11.340	11.900	12.600
15	11.100	11.550	12.150	12.750	13.500
16	11.840	12.480	13.120	13.760	14.400
17	12.750	13.260	13.940	14.620	15.300
18	13.500	14.220	14.760	15.480	16.200
19	14.440	15.010	15.770	16.530	17.100
20	15.200	16.000	16.600	17.400	18.200
21	16.170	16.800	17.430	18.270	19.110
22	16.940	17.600	18.480	19.140	20.020
23	17.940	18.630	19.320	20.010	20.930
24	18.720	19.440	20.160	21.120	21.840
25	19.500	20.250	21.250	22.200	22.750
26	20.540	21.320	22.100	22.880	23.660
27	21.330	22.140	22.950	23.760	24.570
28	22.120	22.960	23.800	24.640	25.760
29	23.200	24.070	24.650	25.520	26.680
30	24.000	24.900	25.800	26.700	27.600

Table 2. Investigation of the high-usage routes of some Saudi telephone exchanges in practical use for a given busy-hour traffic and (0.40) last trunk occupancy

Exchange identity	HU Route index	Number of available trunks	Busy-hour traffic offered (Erlang)	Optimum number of trunks required	Trunks to be added (+) or saved (−)
JEDASR02CG2 (Rouwais) local	1	18	7.4	10	−8
	2	20	6.0	8	−12
	3	19	8.8	12	−7
	4	66	34.4	40	−26
	5	33	29.3	34	1
	6	29	9.3	12	−17
	7	17	9.4	12	−5
	8	28	16.1	20	−8
	9	12	9.0	12	0
	10	24	21.1	25	1
	11	25	17.8	22	−3
DMAMSR02CG0 (Lasilki) local	1	76	71.6	79	3
	2	44	19.5	24	−20
	3	18	18.1	22	4
	4	31	11.3	14	−17
	5	16	6.2	9	−7
	6	22	5.4	8	−14
	7	33	12.3	16	−17
	8	27	22.2	27	0
	9	12	10.3	13	1
RYADSR0120T/C (Murabba) local	1	60	4.7	7	−53
	2	34	15.8	19	−15
	3	30	15.9	20	−10
	4	30	15.5	19	−11
	5	31	22.2	27	−4
	6	90	59.7	67	−23
	7	24	23.0	27	3
	8	15	0.9	2	−13

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وسيلة حاسوبية لتحليل الحركة الهاتفية على الطرق كثيرة الاستعمال

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ملخص البحث. تعتبر دراسة الحركة الهاتفية على الطرق كثيرة الاستعمال مسألة مهمة في تقويم خدمات ساعة الزحمة في المقاسم الهاتفية. ويؤخذ مستوى الحركة المقبول، على طريق كثير الاستعمال، على أنه المستوى الذي يؤدي إلى تحقيق معدّل تشغيل محدّد لدارة الربط الأخيرة في هذا الطريق، وذلك بفرض أن الحصول عن دارة ربط حرة يتم تسلسليًا. ويقدم البحث علاقة رياضية لهذا الغرض مستنتجة من نظرية حركة الاتصالات، ويعطي أيضًا برنامجًا بلغة الباسكال لتسهيل تطبيقها. كما يقدم البحث تطبيقين توضيحيين: أحدهما يختص بحساب جداول الحركة للطرق كثيرة الاستعمال، والآخر يهتم بدراسة حالة عملية.