J. King Saud Univ., Vol. 5, Eng. Sci. (2), pp. 171–190, Riyadh (A.H. 1413/1993)

Strain-Curvature Characteristics of R.C. Sections Externally Reinforced in a Loaded State

Rajeh Zaid Al Zaid and Ghulam Husain Siddiqi*

*Assistant Professor and *Associate Professor, Department of Civil Engineering, King Saud University, P.O. Box 800, Riyadh 11421, Saudi Arabia*

(Received 20/5/1991; Accepted 17/3/1992)

Abstract. The study presents a numerical model for the development, in usable form, of a strain-step history relating concrete strain, strains in tension and compression steels, curvature, moment, neutral axis position and crack height of reinforced concrete sections which are externally reinforced at a chosen state of preload. The determination of the balanced area of external steel poses some difficulties. This is because it is a function of the available area of internal steel (both tension and compression) and the strains induced in them by the existing service loads. The model affords an evaluation of the pseudo-balanced value of external steel for a reinforced concrete section in a state of strain under existing load, the nominal capacity that the section may develop after application of a desired ratio of the balanced value of external steel and the ductility level attained. The history developed furnishes strain and depth of neutral axis values in the service range of the loading.

The results of the analysis by the model are compared with published test results. A parametric study of the effect of preload-moment on the pseudo-balanced ratio of external reinforcement and the corresponding moment capacity is made. An example of design for external reinforcing of an existing section is also presented.

Introduction

The increasing application of external reinforcement in the form of plates to the tension side of under-reinforced beams to improve their strength capacity or serviceability performance has resulted in several experimental investigations, for example [1,2,3 and 4]. Most of these studies employ epoxy resins for bonding external reinforcement while some investigate mechanical bonding of the reinforcement to the beam.

The authors are aware of several unreported applications of plating made through epoxy glues or mechanical bonding. In most cases the external reinforcement was applied using engineering judgement which may result in over-reinforcing of the given sections leading to an undesirable brittle failure.

A reinforced concrete (RC) beam section in which ultimate concrete strain and yielding of the tensile steel occur simultaneously is referred to as a balanced section and the reinforcement as the balanced steel area. The RC beams are reinforced to a maximum of seventy five percent of the balanced amount to maintain ductility in the beam.

For the design of externally reinforced beams, therefore, it is necessary to determine the balanced area of the external steel at the very outset. However, this parameter is a function of the available area of the internal steel (tension and compression) and the (difficult-to-calculate) strains induced in it by the existing service loads. Development of a sound theoretical basis for the design of external reinforcing of a given section is, therefore, very much desirable and is accomplished here.

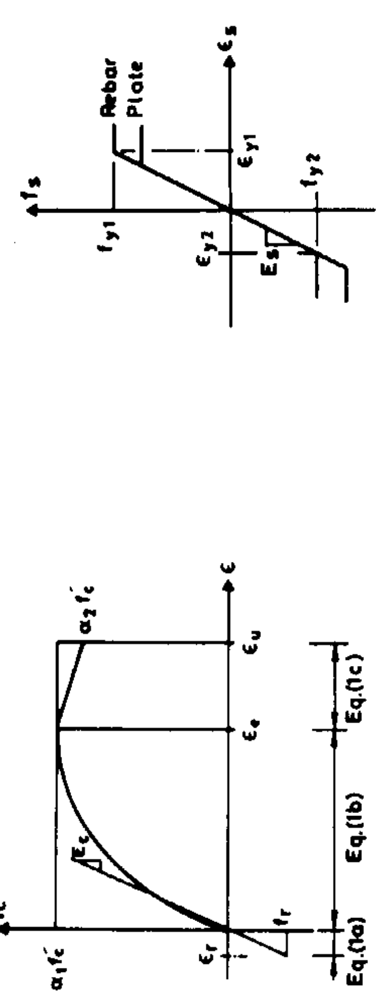
An analysis-cum-design model enabling computation of the strain-status at an existing state of loading is developed in this study. The model employs this preload strain-status to find the maximum plating requirement which would yield at the usable concrete strain level. It determines the section capacity at a desired percentage of that maximum plating requirement in order to maintain ductility of the retrofitted member. It also develops a history of its behavior under service load.

The model employs a tri-curvilinear stress-strain relationship for concrete and a bi-linear relationship without strain hardening for steel reinforcement, with rebar and plate steels having different yield levels. Strain-curvature status is employed to evaluate the compressive and tensile forces in concrete and steels, and equilibrium conditions are used to find the moment of resistance at the status.

A complete history of section behavior is developed by computer implementation of the model. The history includes strains in tension and compression steels, yield sequence in the steels, the depth of the neutral axis, the crack height and the moment capacity at chosen strain steps.

Research Significance

The numerical model presented affords design-investigation of a beam section externally reinforced in a state of loading prior to plating, provides a rational basis for the evaluation of ductility of such a treatment, allows calculation of strains at service load levels and permits total planning of a load test of a beam specimen.



(a) Concrete,

(b) Rebar and plate steels

Fig. 1. Stress-Strain relationships.

Material Behavior

The stress-strain curve of concrete is assumed to be tri-curvilinear as shown in Fig. 1(a). The stress-strain relations in the three ranges [5,6] shown in the figure are,

$$f_c = E_c \epsilon \qquad \epsilon_r \leq \epsilon \leq 0 \tag{1a}$$

$$f_c = \alpha_1 f_c' \{ 2 (\frac{\epsilon}{\epsilon_e}) - (\frac{\epsilon}{\epsilon_c})^2 \}, \qquad 0 \leq \epsilon \leq \epsilon_e \tag{1b}$$

$$f_c = f_1 - f_2 \epsilon, \qquad \epsilon_e \leq \epsilon \leq \epsilon_u \tag{1c}$$

where,

$$f_1 = \{ \alpha_1 + \frac{\alpha_1 - \alpha_2}{\epsilon_u - \epsilon_e} \epsilon_e \} f_c' \tag{1d}$$

$$\overline{f_2 =} \ \frac{\alpha_1 - \alpha_2}{\epsilon_u - \epsilon_e} f_c' \tag{1e}$$

$$\epsilon_r = \frac{f_r}{E_c} \tag{1f}$$

α_1 , α_2 are constants and f_r , f_c' and E_c are modulus of rupture, cylinder compressive strength and modulus of elasticity of concrete as defined in Ref. [7], while ϵ_u is the maximum usable value of strain in concrete.

In Refs. [5,6] α_1 is assumed equal to 0.85, α_2 as square of α_1 and,

$$\epsilon_e = 2 \ (\frac{\alpha_1 \ f_c'}{E_c}) \tag{2}$$

The stress-strain curve of steel is assumed to be bi-linear as shown in Fig. 1b. The rebar yield stress f_{y1} is different and generally larger than the plate yield stress f_{y2}.

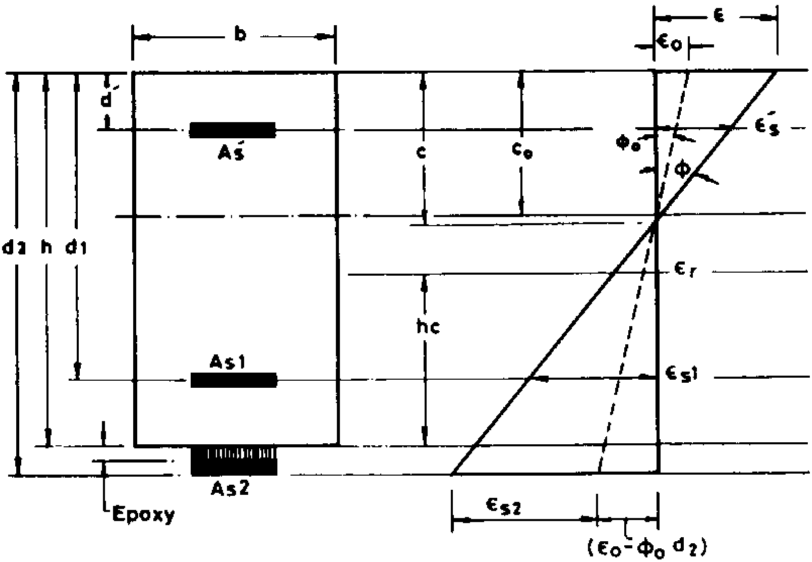
Behavioristic Assumptions

The following behavioristic assumptions are made in developing the model:

1. Plane sections remain plane after bending.
2. Concrete cracks at a stress level of f_r resulting in an inward propagation of a crack from the tension face. Only the concrete from the crack tip to the neutral axis contributes to the tensile forces on the section.
3. There is a perfect bond between concrete and rebar steel and concrete and plate steel.

4. The effect of strain hardening in the two tensile steels, internal and external, is neglected.
5. The section attains its theoretical flexural capacity before any other failure takes place.

It may be pointed out that the external plate steel can be (and usually is) applied at non-zero strain levels in concrete and internal steels.

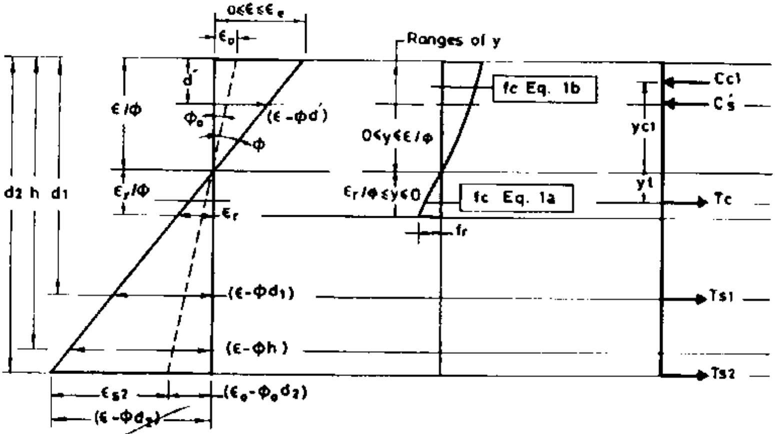


(2a) Beam section (2b) Strains at various levels

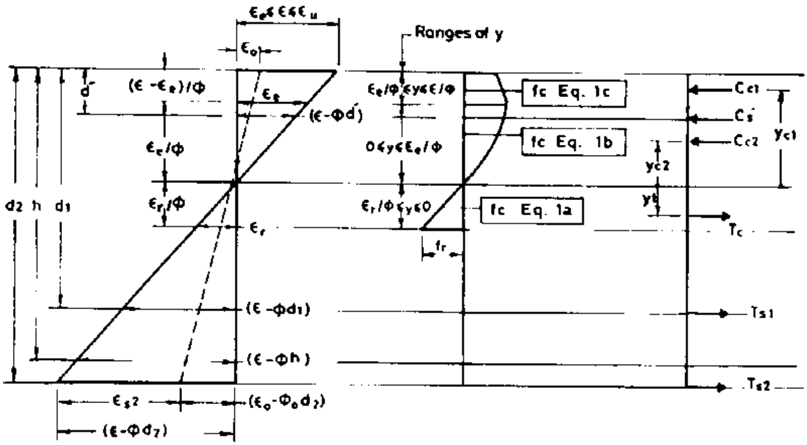
Development of the Model

Section kinematics

Figure 2(a) defines the section dimensions, and steel areas and their positions. Figure 2(b) shows the strain diagram at a top fiber concrete strain, ϵ, and corresponding curvature, ϕ. These two variables completely define the kinematics of a section. The depth of the neutral axis, c, the height of crack (determined by tensile strain in concrete), h_c, the steel strains ϵ_s', ϵ_{s1}, and ϵ_{s2} are expressed in terms of ϵ and ϕ and shown on strain diagrams of Figs. 2(c) and 2(d). Figure 2(c) shows strain and stress



(2c) Concrete extreme fiber strain in the range $0 \leq \epsilon \leq \epsilon_e$ and corresponding stresses



(2d) Concrete extreme fiber strain in the range $\epsilon_e \leq \epsilon \leq \epsilon_u$ and corresponding stresses

Fig. 2. Beam section, Strain-Stress distribution over the section and stress block resultants

diagrams in the range of $0 \leq \epsilon \leq \epsilon_e$ while Fig. 2(d) does the same in the range of $\epsilon_e \leq \epsilon \leq \epsilon_u$. All the limiting distances of the various stress-levels over the depth are also shown there. It may be noted that the strain in the top concrete fiber induced by the preplating moment, M_o, is denoted as ϵ_o and the corresponding curvature as ϕ_o. The compressive strains in concrete or steel are considered positive. However, the yield strains in tension steels, ϵ_{y1} and ϵ_{y2} are considered to be absolute quantities by the formulation.

Stress resultants and their locations

By employing $\int b f_c\, dy$ and $\int b f_c\, y dy$, in the concrete strain ranges of Figs. 2c and 2d, the compression stress block resultants and their positions from the neutral axis may be expressed in terms of the basic variables ϵ and ϕ.

The first crack in concrete in tension appears at a curvature of $(\epsilon - \epsilon_r)/h$. The tension stress block resultants and their positions from the netural axis, in this case, may also be expressed in terms of ϵ and ϕ in the two ranges of pre- and post-cracking curvatures.

The strains in the various steels of Figs. 2(c) and 2(d) may be employed to develop expressions for the forces induced in them. It is understandable that these expressions take separate forms over different ranges of strains and curvatures.

Equations of Equilibrium

The equation of equilibrium of the stress resultants in the horizontal direction yields the curvature in terms of concrete strain as,

$$\phi = \frac{-B - \sqrt{B^2 - 4AC}}{2A}, \qquad A \neq 0 \tag{3a}$$

$$\phi = -\frac{C}{B}, \qquad A = 0 \tag{3b}$$

where

$$A = a_s' + a_{s1} + a_{s2} + a_{tc} \tag{4}$$

$$B = b_s' + b_{s1} + b_{s2} + b_{tc} \tag{5}$$

$$C = C_{cc} + C_{tc} \tag{6}$$

The terms in the right hand components of Eqs. (4) through (6) are listed in Table 1 along with their respective limits of validity.

The total moment of resistance of the section is obtained by summing the moments of the various stress resultants about the internal tension steel.

Table 1. Right hand side components of Equs. 4 thru 6

Component name	Expression	Range
	$-(E_s - E_c)\, A'_s\, d'$	$\varepsilon \leq \varepsilon_c$ and $\phi < \phi'_y$
$a'_s =$	$(E_s + f_2)\, A'_s\, d'$	$\varepsilon > \varepsilon_c$ and $\phi < \phi'_y$
	0	$\phi \geq \phi'_y$
	$(E_s - E_c)\, A_{s1}\, d_1$	$\phi < \phi_r$
$a_{s1} =$	$-E_s\, A_{s1}\, d_1$	$\phi_r \leq \phi < \phi_{y1}$
	0	$\phi \geq \phi_{y1}$
$a_{s2} =$	$-E_s\, A_{s2}\, d_2$	$\phi < \phi_{y2}$
	0	$\phi \geq \phi_{y2}$
$a_{tc} =$	$-\dfrac{1}{2} E_c\, b\, h^2$	$\phi < \phi_r$
	0	$\phi \geq \phi_r$
	$(E_s - E_c)\, \varepsilon\, A'_s$	$\varepsilon \leq \varepsilon_c$ and $\phi < \phi'_y$
$b'_s =$	$\{(E_s + f_2)\, \varepsilon - f_1\}\, A'_s$	$\varepsilon > \varepsilon_c$ and $\phi < \phi'_y$
	$f_{y1}\, A_{s1}$	$\phi \geq \phi'_y$

Table 1. Continued

Component name	Expression	Range
	$(E_s - E_c)\,\varepsilon\, A_{s1}$	$\phi < \phi_r$
b_{s1} =	$E_s\,\varepsilon\, A_{s1}$	$\phi_r < \phi < \phi_{y1}$
	$-f_{y1}\, A_{s1}$	$\phi \geqslant \phi_{y1}$
b_{s2} =	$E_s\,\{(\varepsilon - \varepsilon_0) + \phi_0 d_2\}\, A_{s2}$	$\phi < \phi_{y2}$
	$f_{y2}\, A_{s2}$	$\phi \geqslant \phi_{y2}$
b_{tc} =	$E_c\, b\, h\, \varepsilon$	$\phi < \phi_r$
	0	$\phi \geqslant \phi_r$
C_{cc} =	$\alpha_1 f'_c\, b\,\{3\varepsilon_c - \varepsilon\}\,\dfrac{\varepsilon^2}{3\varepsilon_c^2}$	$\varepsilon \leqslant \varepsilon_c$
	$\dfrac{b}{6}\,\{(\alpha_0 \pm'_c - 3 f_c)\,\varepsilon_c + 3\,(\alpha_1 f'_c + f_c)\,\varepsilon\}$	$\varepsilon > \varepsilon_c$
C_{tc} =	$-\dfrac{1}{2}\, E_c\, b\, \varepsilon^2$	$\phi < \phi_r$
	$-\dfrac{1}{2}\, E_c\, b\, \varepsilon_r^2$	$\phi \geqslant \phi_r$

Pseudo-balanced Quantity of External Steel

A given section with A_{s1} and A'_s may be loaded to a certain level so that M_o is the resulting preplating moment carried by the section. It is desirable to find the external steel quantity (additional to the internal steel which is already there and is strained proportional to M_o) such that at the ultimate concrete strain, either of the tension steels has already yielded while the other is about to do so. Such an amount of external steel may be defined as a pseudo-balanced amount, $A_{s2\text{-bal}}$, and evaluated as shown below.

At the balanced condition the concrete strain is equal to ϵ_u and the corresponding curvatures ϕ^*_{y1} and ϕ^*_{y2} at which the internal and external steels yield are,

$$\phi^*_{y1} = \frac{\epsilon_u + \epsilon_{y1}}{d_1} \tag{7}$$

$$\phi^*_{y2} = \frac{\epsilon_u - \epsilon_0 + \epsilon_{y2} + \phi_0 d_2}{d_2} \tag{8}$$

Letting ϕ^* be the larger of the above two curvatures, so as to ensure yielding of both the steels, the balanced amount of external steel may be obtained from Eq. (3) by using values of A, B and C evaluated at ϵ_u and ϕ^*. Table 2 presents the right hand side components of the variables A, B and C evaluated at ϵ_u and ϕ^*.

Table 2. Right hand side components of Equs. 4 thru 6 at ϵ_u and ϕ^* values

Component name	Expression	Range
$a'_s =$	$-(E_s + f_2) A'_s d'$	$\phi^* < \phi'_y$
	0	$\phi^* \geq \phi'_y$
$b'_s =$	$\{(E_s + f_2)\epsilon_u - f_1\} A'_s$	$\phi^* < \phi'_y$
	$f_{y1} A'_s$	$\phi^* \geq \phi'_y$
$b_{s1} =$	$-f_{y1} A_{s1}$	
$b_{s2} =$	$-f_{y2} A_{s2}$	
C_{cc}	$\frac{b}{6}\{(\alpha_1 f'_c - 3f_c)\varepsilon_e + 3(\alpha_1 f'_c + f_c)\varepsilon_u\}$	
$C_{tc} =$	$-\frac{1}{2} E_c b \varepsilon_t^2$	

Notes:

1) The components, a_{s1}, a_{s2}, a_{tc}, and b_{tc}, defined in Table 1 are identically equal to zero in this case.

2) Components are valid irrespective of ranges where they are not indicated.

Employing the definitions of variables of Table 2, the balanced amount of external steel from Eq. (3) is,

$$A_{s2\text{-}bal} = \left[b_s' + b_{s1} + \left(\frac{C_{cc} + C_{tc}}{\phi^*} + a_s' \phi^* \right) \right] \frac{1}{f_{y2}} \tag{9}$$

Ductility Ratio

The ratio of curvature at the prescribed value of the ultimate compressive strain in concrete to the curvature at the yielding of the either tensile steel, internal or external, is considered as a measure of ductility and called ductility ratio. Simultaneous yielding of the two steels occurs only in the balanced state of reinforcing. In the other situations the subsequent yielding furnishes a lesser value of the ductility ratio.

Results of Analysis

The results which are now presented are obtained by computer implementation of the model described above.

Comparison with test results

The test results used for comparison are reported by Swamy *et al.* [1]. They employed identical test beams bearing following size (in mm) and reinforcing (in mm^2),

$$h = 255,\ d_1 = 220,\ d_2 = 257.25,\ d' = 0.0,\ b = 155,\ \text{and}\ A_{s1} = 942,$$

and the average mechanical properties (in MPa) of the reinforcement used were,

$$f_{y1} = 430,\ f_{y2} = 245,\ \text{and}\ E_s = 200{,}000.$$

The test beams were without compression steel and were plated at different load levels. Table 3 presents the test information furnished by the reference.

Table 3. Test parameters of test beams of ref. [1]

Test beam	f_c', MPa (a)*	A_{s2}, mm^2	M_0, kN-m
F01	43.49	000.0	00.00
F12	41.75	187.5	00.00
F22	43.25	187.5	24.93
F23	43.50	187.5	42.19
F24	43.25	187.5	57.53

* Taken as 0.83 of the cube strength reported in Ref. [1].

Comparison of the ultimate moment capacity of the test beams with the model results is presented in Table 4.

Table 4. Comparison with moment capacity of the test beams of ref. [1]

Test beam	Moment (kN-m) Ref. [1] Test (a)	Moment (kN-m) Ref. [1] Theory (b)	Model Moment (kN-m) (c)	Test [1] / model
F01	80.54	74.6	73.8	1.09
F12	88.97	83.1	81.8	1.09
F22	89.05	87.3	81.9	1.09
F23	89.36	87.6	82.0	1.09
F24	89.05	84.8	81.9	1.09

Notes:

(a) Ultimate concrete strains reported in Ref. [1] are upto 0.006; the measured moments consequently include strain-hardening effect.

(b) Theoretical values employed by Ref. [1] are based on Hognestad block.

(c) Moment predicted by the model is at 0.003 strain and without strain hardening effect.

It is noteworthy that the ultimate concrete strains induced in the test beams are of the order of 0.006, which possibly resulted in strain hardening of the tension steel. Since strain hardening is not accounted for in the formulation, the moments obtained by the model are lower than those attained by the test beams. However, a constant ratio of test beam to model moments of 1.09 is indicative of consistency of the results predicted by the model.

The reference reports rigidities (EI values) of the test beams at 100 kN load; these are used to calculate the corresponding curvatures which are compared with those from the model in Table 5.

Table 5. Comparison with curvature, at 100 kN load, of test beams of ref. [1]

Test beam	Curvature Ref. [1] EI (Test) $N \cdot mm^2\ 10^{12}$	Curvature Ref. [1] M/EI $\frac{1}{mm}\ 10^{-6}$	Curvature Ref. [1] (M/EI)/Theory	Model curvature* $\frac{1}{mm}\ 10^{-6}$	Test [1] / model
F01	3.97	9.660	0.92	9.452	1.02
F12	5.04	7.609	0.94	7.871	0.97
F22	4.77	8.040	0.89	8.988	0.89
F23	3.98	9.636	0.92	9.460	1.02
F24	4.08	9.400	0.94	9.445	0.99

* Curvatures from the model are interpolated values between strain-steps

Finally in Table 6 the depths of the neutral axis at various load levels of the test beams are compared with those from the model.

Table 6. Comparison with neutral axis depth of test beams of ref. [1]

Test beam	Test ref. [1] neutral axis depth (mm)			Model neutral axis depth (mm)		
	@ 20 kN	@ 100kN	@ 200kN	@ 20kN	@ 100 kN	@ 200 kN
F01 : [1]	119.0	107.0	100.5	134.5	104.2	–
F12 : [1]	139.0	122.2	115.0	139.4	112.0	115.9
F22 : [1]	120.2	124.3	116.1	134.5	106.2	115.7
F23 : [1]	121.1	106.9	115.0	134.5	104.1	115.2
F24 : [1]	116.4	103.3	113.5	134.5	104.2	114.3
	Test / Theory [1]			Test [1] / Model		
F01 : [1]	0.86	1.10	0.96	0.83	1.03	–
F02 : [1]	0.98	1.15	0.97	1.00	1.09	0.99
F22 : [1]	0.87	1.16	0.98	0.89	1.17	1.00
F23 : [1]	0.88	1.10	0.97	0.90	1.03	1.00
F24 : [1]	0.84	1.06	0.96	0.86	1.99	0.99

Notes:

(a) Netural axis depth is reported at loads of 20 kN, 100 kN and 200 kN.

(b) The netural axis depths from the model are interpolated values between strain-steps.

(c) In case of test beam F01 load level of 200 kN was not reached by the model.

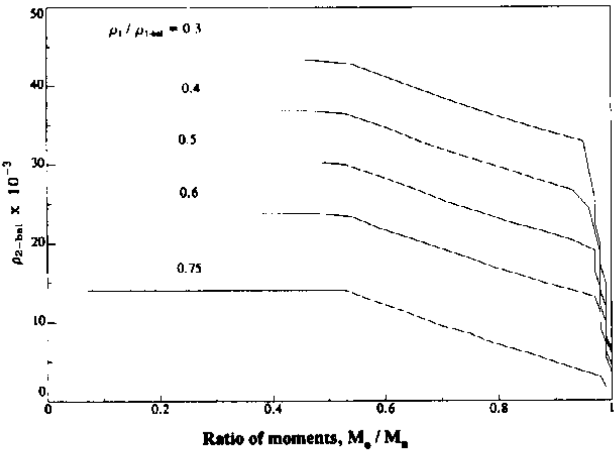


Fig. 3. Variation of balanced external steel with preplating moment for a given section

Study of pseudo-balanced amount of external steel

A parametric study of $\rho_{2\text{-bal}}$ with the ratio of preplating moment, M_0, to moment capacity, M_n, of a given unplated section at different ratios, ρ_1, of internal steel is presented in Fig. 3. The symbol $\rho_{2\text{-bal}}$ is defined as the ratio of $A_{s2\text{-bal}}$ to effective area of concrete, b d_t and ρ_1 as the ratio of A_{s1} to b d_1. The curves in Fig. 3 are evaluated at $\rho_1 / \rho_{1\text{-bal}}$ of 0.3, 0.4, 0.5, 0.6 and 0.75. The symbol $\rho_{1\text{-bal}}$ denotes balanced value of internal steel.

It may be noted that the point of intersection of the horizontal with the sloping leg of a curve in Fig. 3 determines the M_0-value at which both tension steels yield simultaneously -- the definition of the true balanced condition according to Ref. 7. This value may be calculated from the model by employing a finer strain step but is of no practical use; hence the definition of the pseudo-balanced condition.

The intersection of the first sloping leg with the second occurs at the moment ratio at which the internal steel yields. The second sloping leg, therefore, represents the range over which the internal steel has already yielded before the application of the external steel.

The value of $\rho_{2\text{-bal}}$ along the horizontal line is controlled by the internal steel and beyond that by the external steel.

Design-investigation of a given section for external reinforcing

The model is capable of design and/or investigation of a given cross-section which requires to be externally reinforced to a desired percentage of $A_{s2\text{-bal}}$ and is implemented by a computer code.

The section analysis is performed at prescribed strain steps until a moment of resistance equal to or greater than M_0 is reached. The balanced amount of external steel is calculated at this step using the equations of Table 2. The desired percentage of this steel is added to the cross-section as external reinforcement and the analysis continued until the ultimate concrete strain is reached.

Table 7. Design-investigation of beam section described in input-data part of this Table

DESIGN-INVESTGN OF A GIVEN SECTION-(AS1 + AS') UNDER M0.
HISTORY UPTO M0;
AS2-BAL AT M0;
HISTORY BEYOND M0 OF THE SECTION – (AS1 + AS' + RATIO*AS2-BAL)

INPUT-DATA

SECTION DIMENSIONS, mm: H = 500 D1 = 470 D2 = 502.25 D' = 30 B = 250

REINFORCEMENT AREAS, sq.mm: As1 = 1747 As' = 200

CONCRETE STRESS-STRAIN PARAMETERS, MPa: Fc' = 43.5
ALPHA1 = .85 ALPHA2 = .7225 ECE = .00239 ECU = .003 EC–INC = .0001

STEEL YIELD STRESSES & MODULI, MPa: Fy1 = 430 Fy2 = 245
Es1 = 200000 Es2 = 200000

As2: PRESTRAIN MOMENT (kN – m) & RATIO: Mo = 100 RATIO = .75

RESULTS OF ANALYSIS

Mo WAS REACHED AT STEP # = 4 As2 EFFECTIVE FROM STEP # = 5

AS2–BAL = 4682.011 AS2 = RATIO * AS2–BAL = 3511.508

AS1 YIELDED AT STEP # = 25 DUCTILITY RATIO = 1.284

AS2 YIELDED AT STEP # = 20 DUCTILITY RATIO = 1.659

AS' YIELDED AT STEP # = 22

BETA1	ROBBAL	ROMAX	RO	RO'	Fr	Er
0.74200	0.03717	0.02788	0.01487	0.00170	–4.617E+00	–1.489E–04

Notes:
(a) ECE: ϵ_e, ECU: ϵ_u, EC-INC: strain step
(b) Fr: modulus of rupture, Er: strain at rupture

Table 7. Continued

Strain	PHI	C	Crack HT	Moment	Remarks
	1/mm	mm	mm	kN-m	
1.000E-04	3.778E-07	264.7	0.0	3.59D + 01	
2.000E-04	8.981E-07	222.7	111.5	5.57D + 01	1st crack
3.000E-04	1.620E-06	185.1	223.0	7.58D + 01	
4.000E-04	2.261E-06	176.9	257.3	1.01D + 02	M0 rchd
5.000E-04	2.640E-06	189.4	254.2	1.39D + 02	As2 added
6.000E-04	3.017E-06	198.9	251.7	1.75D + 02	
7.000E-04	3.390E-06	206.5	249.6	2.10D + 02	
8.000E-04	3.759E-06	212.8	247.6	2.45D + 02	
9.000E-04	4.126E-06	218.2	245.7	2.79D + 02	
1.000E-03	4.488E-06	222.8	244.0	3.12D + 02	
1.100E-03	4.847E-06	227.0	242.3	3.43D + 02	
1.200E-03	5.202E-06	230.7	240.7	3.74D + 02	
1.300E-03	5.553E-06	234.1	239.1	4.04D + 02	
1.400E-03	5.900E-06	237.3	237.5	4.33D + 02	
1.500E-03	6.243E-06	240.3	235.9	4.61D + 02	
1.600E-03	6.582E-06	243.1	234.3	4.88D + 02	
1.700E-03	6.917E-06	245.8	232.7	5.14D + 02	
1.800E-03	7.247E-06	248.4	231.1	5.39D + 02	
1.900E-03	7.574E-06	250.9	229.5	5.62D + 02	
2.000E-03	7.906E-06	253.0	228.2	5.84D + 02	As2 ylds
2.100E-03	8.37E-06	251.6	230.6	5.99D + 02	
2.200E-03	8.842E-06	248.8	234.4	6.18D + 02	As' ylds
2.300E-03	9.260E-06	248.4	235.5	6.30D + 02	
2.400E-03	9.670E-06	248.2	236.4	6.41D + 02	
2.500E-03	1.022E-05	244.7	240.7	6.42D + 02	As1 ylds
2.600E-03	1.082E-05	240.3	245.9	6.44D + 02	
2.700E-03	1.141E-05	236.6	250.3	6.45D + 02	
2.800E-03	1.199E-05	233.5	254.1	6.45D + 02	
2.900E-03	1.256E-05	230.9	257.2	6.45D + 02	
3.000E-03	1.311E-05	228.8	259.9	6.45D + 02	

Notes:

(a) rchd: reached ylds: yields

The sectional dimensions, internal reinforcing areas, the material properties and strain parameters of a beam section are defined in the "input-data" section of Table 7. This beam, which is internally reinforced to 40 percent of $\rho_{1\text{-bal}}$, is designed for three preplating moments viz. 100, 200 and 300 kN-m. $M-\phi$ curves of these design calculations, along with that of the section without plating, are presented in Fig. 4. The amount of external reinforcement applied to the section is 75 percent of the corresponding design value of $\rho_{2\text{-bal}}$. The moment capacities and ductilities attained in these beams along with the required design values of balanced external steel areas are presented in Table 8. The Table also shows the sequence of yielding of the tension steels. Fig. 4 clearly shows the effect of plating on the section. The monument capacity and stiffness of the beam are significantly augmented at the cost of ductility.

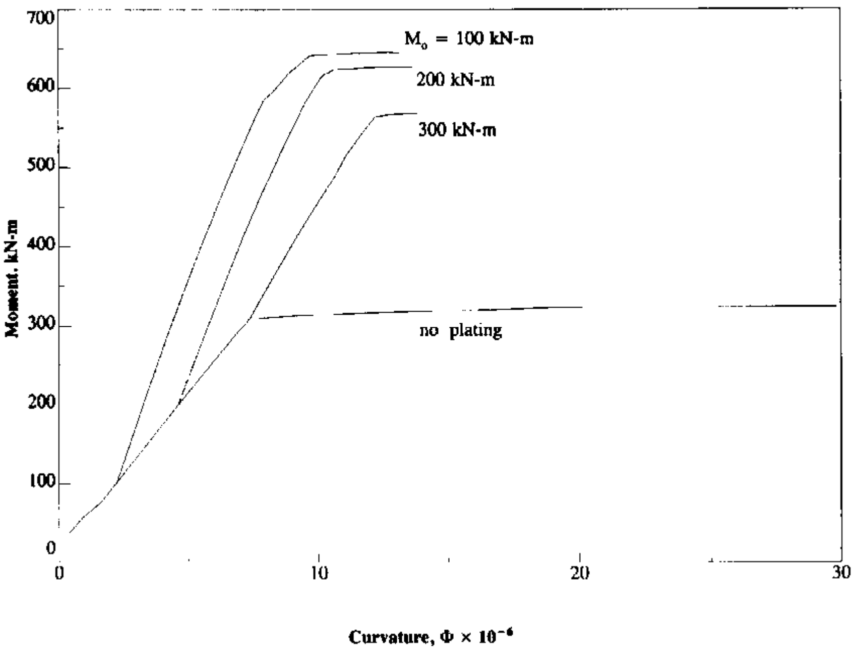


Fig. 4. Moment-curvature relations at different preplating moments of the section defined in Table 7

Table 8. Design-investigation of beam described in input-section of Table 7 at different preplating moments

M_0 kN-m	As2-bal mm^2	As2 mm^2	Moment capacity kN-m	Ductility Ratio	Ductility Control (a)
100.0	4682.0	3512.0	645.0	1.28	I
200.0	4369.0	3277.0	627.0	1.28	E
300.0	3430.0	2572.0	568.0	1.21	E
400.0 (b)	0.0	0.0	324.0	4.05	I

Notes:

(a) I and E refer to internal and external steel control of ductility, respectively.

(b) At $M_0 = 400$ no external steel is permissible; moment capacity is that of the section with internal steel only.

The results of design calculations produced in Table 7 present strain-step history of the section at preplating moment, $M_0 = 100$ kN-m. This moment was reached at the fourth strain-step, the value of $A_{s2\text{-bal}}$ calculated was 4682 mm^2, the desired percentage (75%) of this steel area was applied in the fifth strain-step and analysis continued until the ultimate concrete strain was reached. The nominal moment capacity predicted by the model is 645 kN-m. The curve at $M_0 = 100$ kN−m, in Fig. 4, is developed from the history.

In this beam, yielding of A_{s2} (at twentieth step) before that of A_{s1} (at twenty fifth step) indicates that ϕ^{*}_{y1} controls evaluation of $A_{s2\text{-bal}}$ and the corresponding ductility ratios are 1.66 and 1.28 respectively.

The neutral axis depth decreases until step four; from step five (when external steel is added) it takes an increasing trend until the yield of the external steel at step twenty; beyond this step it decreases again.

The crack height follows a trend opposed to that of the neutral axis depth between these strain-steps.

Conclusions

The difficulty in direct evaluation of steel strains at preload, which are required for the determination of the balanced area of external steel, are circumvented. The model presented offers a rational basis for design and/or investigation of reinforced concrete sections which may be externally reinforced at an existing state of loading.

A pseudo-balanced external steel area for a given section at an existing state of load can be determined so as to ensure ductility of the retrofit.

Crack height, ductility and moment capacity information in pre-and post-retrofit states provided by the model are useful in judging the effectiveness of such a treatment.

The computer implementation of the model may easily be carried out with the help of the equations presented in Tables 1 and 2.

A strain-step history of the section behavior may be developed by the program. Such information, besides offering a design-investigation capability, also provides strain information at service load levels and allows for the total planning of load test of a beam specimen.

References

[1] Swamy, R.N.; Jones, R. and Charif, A. "The Effect of External Plate Reinforcement on the Strengthening of Structrurally Damaged RC Beams." *The Structural Engineering* 67, No. 3 (February 1989), 45-56.

[2] Swamy, R.N.; Jones, R., and Bloxham, J.W. "Structural Behavior of Reinforced Concrete Beams Strengthened by Epoxy-bonded Steel Plates." *The Structural Engineer* 65A, No. 2, (February 1987), 59-68

[3] Shaheen, H.H.; Agag, Y.; Mahmoud, N.S. and Yousef, A. "The Flexural Behavior of Cracked Reinforced Concrete Beams Strengthened with Bonded External Steel Plates." *Proceedings of the Third Arab Structural Engineering Conference*, Vol. 2, (U.A.E. Univ. Al-Ain, UAE; March, 1989, 571-592.

[4] Siddiqi, G.H. and Al-Qahtany, J.N. "Flexural Behavior of Beams Externally Reinforced Through Shot-pin Shear Connectors." *Journal of King Saud University*, Vol. 2, *Engineering Sciences*, No. 1 (1990).

[5] Park, R. and Paulay, T. *Reinforced Concrete Structures*, John Wiley & Sons, Inc. 1975.

[6] *Notes on ACI 318-83 Building Code Requirements for Reinforced Concrete*. Skokie, Illinois: Portland Cement Association, 1986.

[7] ACI Committee 318. *Building Code Requirements for Reinforced Concrete* (ACI 318-89). Detroit: American Concrete Institute, 1989.

خصائص الانفعال والتقوس للمقاطع الخرسانية المسلحة سابقة التحميل بعد تسليحها من الخارج

راجح زيد الزيد و غلام حسين صديقي

قسم الهندسة المدنية، كلية الهندسة، جامعة الملك سعود، ص.ب ٨٠٠،
الرياض ١١٤٢١، المملكة العربية السعودية

(استُلم في ٢٠/٥/١٩٩١م؛ قُبل للنشر في ١٧/٣/١٩٩٢م)

ملخص البحث. تقدم الدراسة أنموذجًا عدديًا لدراسة أداء المقاطع الخرسانية المسلحة سابقة التحميل بعد تقويتها من الخارج بصفائح من الحديد وذلك من خلال ربط الانفعال في الخرسانة والانفعالات الناتجة في الطبقات المختلفة لحديد التسليح وكذلك التقوس مع عزم المقاومة وموضع محور الحياد وعمق التشققات.

إن إيجاد مساحة حديد التسليح الخارجي اللازمة لحصول فشل متوازن لمثل هذه المقاطع تعترضه بعض الصعوبات وذلك نظرًا لاعتمادها على كمية الحديد الداخلي والانفعالات المستحثة به نتيجة للأحمال السابقة، وهذا ما أمكن التغلب عليه من خلال الأنموذج المقترح كما أن الأنموذج يعطي تقويمًا لعزم المقاومة ومعامل اللدونة لمقاطع مقواه بنسبة معينة من كمية الحديد اللازمة لفشل متوازن هذا إلى جانب إعطاء وصف شامل لسلوك المقاطع المقواة تحت تأثير أحمال التشغيل. ولقد تمت مقارنة النتائج المحسوبة باستخدام الأنموذج مع نتائج تجريبية لأبحاث منشورة كما عملت دراسة مقارنة لأثر مستوى الحمل السابق على كمية حديد التسليح الخارجي اللازمة لحصول فشل متوازن وكذلك على عزم المقاومة الناتج. واشتملت الدراسة أيضا على مثال لتوضيح عملية تصميم الحديد الخارجي اللازم لتقوية مقاطع تحت الاستخدام لضمان وجود مستوى مقبول من اللدونة.