Influence of Superconducting Generators on Steady-State Stability of Multi-Machine Power Systems

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Abstract. This paper deals mainly with how steady state-stability of interconnected power systems is influenced by introducing a superconducting alternator unit. A linearized model for a multi-machine system containing four generating units, one of whih is superconducting, is obtained. Results are presented which bring out the significant influences of a superconducting alternator on the steady-state stability of power systems. These indicate the directions in which improved performance may be achieved. A quantitative comparison between the influence of the superconducing generator and its conventional counterpart on the stability of multi-machine systems is also provided.

1. Introduction

Steady-state stability of interconnected power systems is of considerable importance. Different techniques have been suggested for modelling of multi-machine systems for small disturbance stability studies. Using matrix elimination techniques, Laughton [1] has proposed a method for extracting the system matrix A, from algebraic and differential equations of the whole system. Undrill [2] described a method for building up the A matrix from the submatrices describing the individual machine units of the power system. This method requires the inversion of a matrix of the same order as the system model itself. Anderson [3] has suggested the PQR method for a single machine infinite busbar system. This method facilitates computer manipulation of the system equations into a state space form. Abdalla, *et al.* [4] have described a direct method for constructing the matrix A. This method saves computer time by eliminating the need for the inversion of matrices of high order.

Superconducting alternators are expected to be introduced into large power networks because of their potential advantages and capability to supply greater base loads if compared with conventional synchronous machines. However, these alternators are air-cored with a cryogenic field winding surrounded by two concentric rotor screens. The outer screen acts as a damper having a time constant suitable for damping. The inner screen acts as an electromagnetic shield having a longer time constant to protect the superconducting elements from time-varying magnetic fields. The air-cored construction of superconducting alternator reduces its synchronous reactance as well as inertia constant. The former upgrades stability whereas the latter may cause an increase in frequency of oscillations which degrades stability. Furthermore, the mutual interaction between the screens reduces inherent damping, which also degrades stability [5]. The extent to which each of these factors affects the steady-state stability of a multi-machine system, is not clear yet. This paper is mainly concerned with the influence of superconducting alternators on the steady state-stability of a multi-machine power system.

2. Steady-State Stability Analysis

A linearized model of the multi-machine power system may be obtained in the standard state-space form,

$$\dot{X} = AX + BU \qquad \ldots\ldots\ldots \qquad (1)$$

where X is the state vector, U is the input vector, A is the state coefficient matrix and B is the input coefficient matrix. The state coefficient matrix A of the system under consideration is obtained using the method described in [4]. However, this method requires some modifications to facilitate the inclusion of a superconducting alternator with two rotor screens. A brief description of the method and the necessary modifications are given in the Appendix.

The eigenvalues of the matrix A contains the necessary informatin on the steady-state stability of the multi-machine system. The system is stable if all the eigenvalues have negative real parts. These eigenvalues may be classified into two types. First, real roots which are related to exponential components of the time response. Secondly, complex conjugate pairs of roots which are associated with oscillatory modes of the time response. These have the form $-\alpha \pm j\omega$, where ω gives the frequency of oscillations and α defines the damping factor *i.e.* the reciprocal of the time constant by which the oscillations are decayed. This type of roots may further be divided into two groups, high and low frequency modes. The former are highly damped modes corresponding to stator circuits, whereas the latter are lightly damped modes associated with rotor oscillations. The most critical eigenvalues are those related to rotor oscillations.

3. Multi-Machine System Description

The multi-machine power system considered in this paper, shown in Fig. 1, comprises four generating units of different types and ratings. These are two steam, one superconducting and one hydro units. The superconducting alternator is replaced by two steam units of the same size. This replacement is necessary to provide a quantitative comparison between the influence of the superconducting alternator and its substitute on steady-state stability of power systems. It should be emphasized that the synchronous reactance of the chosen conventional counterpart ($X_d = 2$ p.u.) is on the conservative side. The value of X_d for a conventional turbo-alternator unit of 2000 MVA is expected to have a value of about 3 p.u. [6]. The parameters for the conventional units, Table 1, are calculated from manufacturers data [7]. Those for the superconducting alternator, Table 2, are computed using three dimensional field analysis [8].

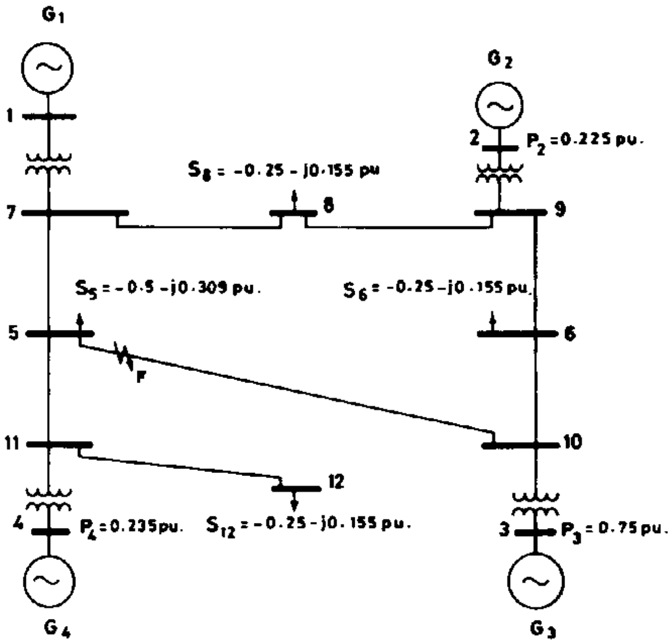

Fig. 1. Four-machine Power System

Table 1. Conventional machines parameters [9]. (reactions in p.u., time constant in seconds)

	Steam unit G_1	Steam unit G_2	Conventional counterpart steam unit G_3	Hydro unit G_4
MVA	192	590	2×1000	615
X_d	1.651	2.11	2.0	0.9
X_q	1.59	2.02	1.92	0.65
X_{ad}	1.55	1.95	1.86	0.66
X_{aq}	1.49	1.86	1.77	0.41
X_F	1.69	2.1	1.97	0.72
X_D	1.58	2.07	1.94	0.67
X_Q	1.57	1.93	1.96	0.46
r_a	0.0026	0.0046	0.005	0.0014
r_F	0.00076	0.00013	0.0015	0.00026
r_D	0.2	0.02	0.0078	0.012
r_Q	0.031	0.024	0.0084	0.02
H,sec	3.3	2.32	3.25	5
K_A	25	200	200	200
T_A	0.2	0.3575	0.05	0.02
K_F	0.091	0.0529	0.0	0.01
T_F	0.35	1.0	1.0	1.0

Table 2. Superconducting alternator parameters [9]. (reactions in p.u.)

MVA	2000
$X_d = X_q$	0.545
$X_{D1} = X_{Q1}$	0.257
$X_{D2} = X_{Q2}$	0.423
X_F	0.541
$X_{dF} = X_{dD1} = X_{dD2} = X_{FD1} = X_{D1D2}$	0.237
X_{FD2}	0.39
$X_{qQ1} = Y_{qQ2} = X_{Q1Q2}$	0.237
R_a	0.003
Outer Screen time constant, sec	0.081
Inner Screen time constant, sec	1.0
Field winding time constant, sec	750
H (KW.Sec/KVA)	3
R_g	0.04
T_g	0.1
T_b	0.35

The system network is represented using the π method. Each load is represented by a constant admittance, which is added to the admittance of the node at which the load is connected. Each conventional generator is represented by a seventh-order model. These models are based on Park's equations. The steam and hydro units are each equipped with a static exciter. Fig. 2a shows the block diagram of the exciter model. The mechanical input power to each of the steam and hydro units is assumed constant. Since exciter control has no appreciable effect in the case of superconducting alternators [9], thus a governor control is considered. Fig. 2b shows the model of the turbine and governor used for the superconducting alternator.

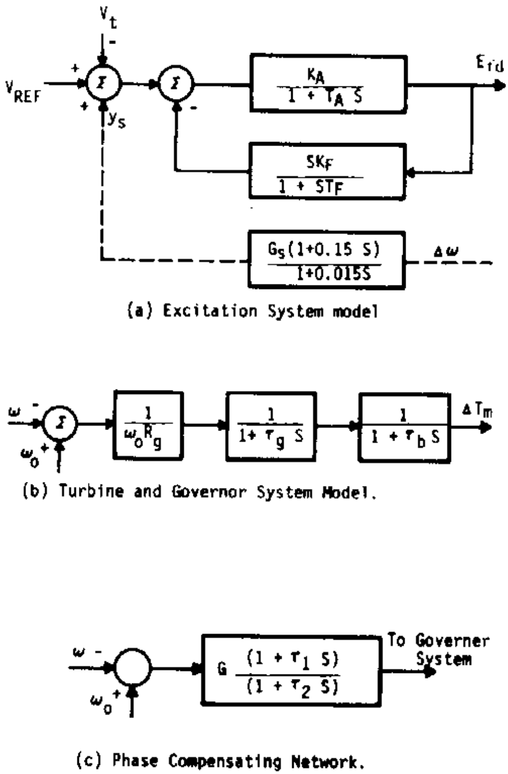

Fig. 2. Power system controllers

4. Results and Discussion

The results of the load flow solution are included in Table 3. Since the system has no infinite bushar, any of the machines may be chosen as reference [2]. The eigenvalues are computed by a standard algorithm using a standard computer subroutine. In all cases considered it has been found that system eigenvalues contain; four complex conjugate pairs of eigenvalues having a frequency of about 60 Hz corresponding to the stator circuit; three pairs of complex conjugate with lower frequency modes in the neighborhood of 1.5 – 2.5 Hz which corresponds to different rotor oscillaitons. The rest of eigenvalues are purely real. The incorporation of power system controllers is also associated with additional eigenmodes, some of which are complex with a frequency lower than 1 Hz.

4.1. Uncontrolled Systems

The eigenvalues of the uncontrolled systems (*i.e.* systems with inoperative controllers) are listed in columns (1) and (2) of Table 3. The modes in the first column are those of the system in which the superconducting unit is replaced by its conventional counterpart. The second column contains the eigenvalues of the system which includes the superconducting unit. It is clear from these results that the introduction of the superconducting alternator leads to an inherently stable multi-machine system. This is indicated by the immigration of the unstable root $\lambda(27)$ of column (1) to the stability region as shown in column (2). However, the eigenvalues associated with rotor oscillaitons, $\lambda(9)$ to $\lambda(14)$, show degradation in damping. This is indicated by the reduction in the absolute value of all the real parts of those eigenvalues. These results have been repeated at different operating conditions. It has been found that the introduction of superconductor alternator renders the system stable out of unstable condition. It may be concluded that the introduction of a superconducting alternator into power systems increases steady state stability limits by increasing the stability margins, making the whole system eigenvalues stable over a wider range of operating conditions. Similar conclusions are drawn in reference [9] concerning transient stability.

4.2. Effect of Power System Controllers

An automatic voltage regulator (AVR) is attached to each of the conventional units, and a conventional speed governor (CSG) is attached to the superconducting alternator. The results obtained in this case are shown in column (3). A comparison of these results with those of column (2) shows that the damping of two oscillatory modes namely $\lambda(11,12)$ and $\lambda(13,14)$ is slightly reduced. This is eventually due to the AVR and the CSG. To enhance both stability and damping of oscillatory modes, power system, stablizers (PSS) are introduced into the excitation system of each conventional machine. The block diagram of the stablizer used is included in Fig. 2a. This is a speed stablizer, in which a signal from the shaft speed is applied to the refer-

Table 3. System Eigenvalues

	Four conventional generator without controllers	The superconducting alternator introduced at busbar 3	As column 2 – AVR on conventional units and CSG on superconducting generator	As column 3 – PSS on conventional units	As column 4 + compensating network on superconducting generator	As column 3 + phase compensating network on superconducting generator
λ(1,2)	2185 ± j918.25	−1619.3±j782.35	−1619.3±j782.4	−1619.3±j782.4	−1619.3±j782.4	−1619.29±j782.4
λ(3,4)	−24.34 ± j565.1	−13.7 ± j519.67	−13.69± j519.67	−13.69 ±j519.67	−13.69 ±j519.67	−13.69 ±j519.67
λ(5,6)	−18.87 ±j515.72	−18.45 ±j516.31	−18.44 ±j516.33	−18.44 ±j516.33	−18.44 ±j516.33	−18.44 ±j516.33
λ(7,8)	−13.92 ±j434.04	−13.83 ±j433.8	−13.83 ±j433.77	−13.83 ±j433.77	−13.83 ±j433.77	−13.83 ±j433.77
λ(9,10)	−1.89 ±j13.365	−1.743 ±j12.89	−1.799 ±j13.05	−2.44 ±j14.5	−2.53 ±j14.66	−2.0 ±j13.24
λ(11,12)	−2.145 ±j9.92	−2.04 ±j9.76	−2.0 ±j9.78	−2.09 ±j9.88	−2.23 ±j9.93	−2.35 ±j9.84
λ(13,14)	−0.736 ±j9.05	−0.55 ±j8.68	−0.52 ±j8.83	−1.27 ±j9.05	−1.95 ±j9.6	−1.39 ±j9.13
λ(15)	−75.07	−75.06	−31 ±j0.68	−68.95 ±j2.998	−68.95 ±j2.998	−31.1 ±j0.68
λ(16)	−69.94	−69.24	−1.35 ±j2.36	−1.98 ±j3.911	−5.09 ±j6.64	−4.43 ±j5.83
λ(17)	53.97	−53.67	−2.86 ±j0.165	−0.545 ±j0.55	−0.55 ±j0.56	−2.98 ±j0.196
λ(18)	−37.36	−37.24	−0.5 ±j0.603	−0.565 ±j0.2	−0.56 ±j0.2	−0.5 ±j0.595
λ(19)	−30.92	−31.17	−0.56 ±j0.239	−151.27	−2.55 ±j0.82	−0.56 ±j0.24
λ(20)	−23.64	−30.34	−152.5	−84.04	−151.27	−1.3 ±j0.27
λ(21)	−19.43	−27.74	−73.36	−66.77	−100.76	−152.54
λ(22)	−3.45	−20.98	−66.113	−54.59	−84.04	−100.76
λ(23)	−0.73	−2.96	−53.68	−50.16	−66.77	−73.36
λ(24)	−0.48	−1.00	−39.94	−43.99	−54.59	−66.11
λ(25)	−0.37	−0.77	−39.04	−39.12	−50.16	−53.69
λ(26)	−0.297	−0.38	−30.55	−31.19	−43.99	−39.94
λ(27)	+0.02	−0.291	−26.92	−30.475	−39.116	−39.03
λ(28)		−0.037	−20.26	−27.225	−31.19	−30.58
λ(29)		−0.00114	−11.47	−23.88	−30.51	−26.92
			−1.43	−20.0	−27.23	−20.47
			−1.0	−11.53	−23.88	−1.0
			−0.00146	−2.93	−20.27	−0.00146
				−2.59	−2.93	
				−1.22	−1.13	
				−1.0	−1.0	
				−0.00146	−0.00146	

Operating point	G_1	G_2	G_3 Conven.	G_3 Super	G_4
P	0.045	0.225	0.75	0.75	0.235
Q	0.06	0.04	0.11	0.11	0.145
δ°	20.6	49.9	49.3	23.9	20.1
V	1.01 | 0.0	1.01 | 2.2	1.01 | 7.8	1.01 | 7.8	1.01 | 1.11

ence of the voltage regulator through a lead/lag network. The parameters of the PSS should be carefully selected for each machine. The selection method used is described in [4,9]. The time constants of the PSS are chosen first, then the gain G_s is varied. The gain G_s is taken as one-third of the value that makes the system unstable for any of the eigenvalues. The eigenvalues obtained in this case are listed in column (4). These show an improvement in damping of the critical osillatory mode λ(9-14).

This improvement is limited to that shown in the Table. No further improvement could be obtained by readjusting the PSS gains. Thus, further upgrading of damping of the multimachine system should be achieved via upgrading the damping of the superconducting alternator. For this purpose the phase compensating network (PCN) of Fig. 2c is incorporated in the governor feedback loop of the superconducting alternator. The design and choice of parameters of such network are described elsewhere [10]. The eigenvalues obtained after the inclusion of this network are given in column (5). Comparing these eigenvalues with those listed in column (4) indicate a further improvement in damping of all the oscillatory modes λ(9-14).

To demonstrate the influence of the superconducting alternator unit on the steady-state stability of the multi-machine system and also to estimate the contribution of this unit to damping of the oscillatory modes, the PSS on all conventional units are disconnected, while keep the PCN on the superconducting unit. The eigenvalues obtained in this last case are those of column (6). The most critical interaction eigenvalues are those related to the rotor oscillations, namely λ(9) through λ(14). The eigenvalues of the superconducting rotor oscillations are λ(13,14). It is clear from comparison of columns (4) and (6) that the introduction of the phase advance network increases the damping of some critical oscillatory modes, namely λ(11,12) and λ(13,14). The other interaction modes, namely λ(9,10), show a slight deterioration in damping. However, this does not affect the overall system damping since these roots are already well damped. The comparison between columns (4) and (6) has been repeated at different operating points and similar conclusions have been drawn. This has also been confirmed by results reported in Ref. [9] concerning transient stability. This indicates that the influence of the superconducting unit when equipped with PCN is greater than the combined influence of the remaining conventional units, even when each of them is equipped with PSS. Therefore, the superconducting alternator unit is dominating the steady-state stability of the multimachine system.

5. Conclusions

Extensive investigation into steady-state stability of a multi-machine power system containing different types of generating units have shown that the introduction of superconducting alternators into large power networks results in inherently stable systems. The use of PSS on the conventional units without upgrading the damping of the superconducting alternator would not significantly improve the steady-state stability. As substantial improvement in both the damping of critical oscillatory modes and stability of the multi-machine system could be achieved through the use of a carefully designed control scheme for the superconducting alternator.

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Appendix

The linearization technique of reference [4] will be described here after necessary modifications required to accommodate a superconducting alternator. This description is given for a multi-machine system consisting of two generating units, one of which is superconducting alternator.

Network Equations

A load flow solution of the power system is obtained first, then the steady-state values of the system variables are calculated. The non-synchronous loads are representred by a constant admittance, the later is added to the self admittance of the node at which the load in connected. Then all the load nodes are eliminated using the well known Kron reduction method. Thus mode equations of the reduced network, which contains only machine nodes, may by written as:

$$I = Y\,V \tag{2}$$

Reference Axes

The node equations are referred to D and Q axes which rotate at the angular frequency of the network current. Equation (2) can therefore be expressed in real form as:

$$\begin{bmatrix} I_{D1} \\ I_{Q1} \\ I_{D2} \\ I_{Q2} \end{bmatrix} = \begin{bmatrix} G_{11} & -B_{11} & G_{12} & -B_{12} \\ B_{11} & G_{11} & B_{12} & G_{12} \\ G_{21} & -B_{21} & G_{22} & -B_{22} \\ B_{21} & G_{21} & B_{22} & G_{22} \end{bmatrix} \begin{bmatrix} V_{D1} \\ V_{Q1} \\ V_{D2} \\ V_{Q2} \end{bmatrix}$$

or in symbolic form

$$I_N = Y_N V_N \tag{3}$$

The common reference axes may be chosen arbitrary. The system variables are referred to the common reference frame (D and Q) of the network, assuming the frequency of the network is identical to that of any selected machine [2].

Axes Transformation

The equations of each machine are usually referred to its rotor d and q axes. Under steady state conditions the axes of the machines rotate at a constant speed with angular differences δ_1 and δ_2 with respect to the common D-Q-axes. Therefore the system equations referred to the common reference axes are:

$$\begin{bmatrix} V_{D1} \\ V_{Q1} \\ V_{D2} \\ V_{Q2} \end{bmatrix} = \begin{bmatrix} \cos\delta_1 & -\sin\delta_1 & 0 & 0 \\ \sin\delta_1 & \cos\delta_1 & 0 & 0 \\ 0 & 0 & \cos\delta_2 & -\sin\delta_2 \\ 0 & 0 & \sin\delta_2 & \cos\delta_2 \end{bmatrix} \begin{bmatrix} V_{d1} \\ V_{q1} \\ V_{d2} \\ V_{q2} \end{bmatrix}$$

In symbolic form the above equation can be written as

$$V_N = T\, v_m \tag{4}$$

Linearization of equation (4) gives

$$\begin{bmatrix} \Delta V_{D1} \\ \Delta V_{Q1} \\ \Delta V_{D2} \\ \Delta V_{Q2} \end{bmatrix} = [T] \begin{bmatrix} \Delta v_{d1} \\ \Delta v_{q1} \\ \Delta v_{d2} \\ \Delta v_{q2} \end{bmatrix} + \begin{bmatrix} -v_{d1}\sin\delta_1 & -v_{q1}\cos\delta_1 & 0 & 0 \\ v_{d1}\cos\delta_1 & -v_{q1}\sin\delta_1 & 0 & 0 \\ 0 & 0 & -v_{d2}\sin\delta_2 & -v_{q2}\cos\delta_2 \\ 0 & 0 & v_{d2}\cos\delta_2 & -v_{q2}\sin\delta_2 \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \end{bmatrix}$$

which may be written in symbolic form as:

$$\Delta V_N = T\,\Delta v_m + D\,\Delta\delta \tag{5}$$

The linearized form of the network node equation (3) is:

$$\Delta I_N = Y_N\,\Delta V_N \tag{6}$$

From the power invariance theorem of Kron

$$i_m = T^t\,I_N \tag{7}$$

where i_m is the vector of the machines currents and T^t is the transpose of the transformation matrix.

For a small perturbation, we have

$$\begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} = [T]^t \begin{bmatrix} \Delta I_{D1} \\ \Delta I_{Q1} \\ \Delta I_{D2} \\ \Delta I_{Q2} \end{bmatrix} + \begin{bmatrix} i_{q1} & 0 \\ -i_{d1} & 0 \\ 0 & i_{q2} \\ 0 & -i_{d2} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \end{bmatrix}$$

or in compact form

$$\Delta i_m = T^t\,\Delta I_N + E\,\Delta\delta \tag{8}$$

Eliminating ΔI_N and ΔV_N from equations (5, 6 and 8) and solving for Δv_m, we have

$$\Delta v_m = G\Delta\, i_m + Q\, \underline{\Delta\delta} \tag{9}$$

where

$$G = T^t\, Y_N^{-1}\, T \quad \text{and} \quad Q = -GE - T^t D$$

Generating Units

The linearized equations of the stator d-,q-axes of each machine are arranged in the matrix form:

$$\begin{bmatrix} \Delta\dot{\psi}_{d1} \\ \Delta\dot{\psi}_{q1} \\ \Delta\dot{\psi}_{d2} \\ \Delta\dot{\psi}_{q2} \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & 0 & 0 \\ -\omega_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_0 \\ 0 & 0 & -\omega_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\psi_{d1} \\ \Delta\psi_{q1} \\ \Delta\psi_{d2} \\ \Delta\psi_{q2} \end{bmatrix} + \begin{bmatrix} \omega_0 R_1 & 0 & 0 & 0 \\ 0 & \omega_0 R_1 & 0 & 0 \\ 0 & 0 & \omega_0 R_2 & 0 \\ 0 & 0 & 0 & \omega_0 R_2 \end{bmatrix} \begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} +$$

$$\begin{bmatrix} \omega_0 & 0 & 0 & 0 \\ 0 & \omega_0 & 0 & 0 \\ 0 & 0 & \omega_0 & 0 \\ 0 & 0 & 0 & \omega_0 \end{bmatrix} \begin{bmatrix} \Delta v_{d1} \\ \Delta v_{q1} \\ \Delta v_{d2} \\ \Delta v_{q2} \end{bmatrix} + \begin{bmatrix} \psi_{q1} & \\ -\psi_{d1} & \\ & \psi_{q2} \\ & -\psi_{q2} \end{bmatrix} \begin{bmatrix} \Delta\omega_1 \\ \Delta\omega_2 \end{bmatrix}$$

compactly

$$\Delta\dot{\psi} = J\Delta\psi + K\,\Delta i_m + W\Delta v_m + P\,\underline{\Delta\omega} \tag{10}$$

Eliminating Δv_m from Equation (10) by using Equation (9), we obtain

$$\Delta\dot{\psi} = J\Delta\psi + F\,\Delta i_m + C\Delta\delta + P\,\underline{\Delta\omega} \tag{11}$$

where $\qquad F = WG + K, \qquad C = WQ$

The angular velocity equations of the machines can be combined to give

$$\begin{bmatrix} \Delta\dot{\delta}_1 \\ \Delta\dot{\delta}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta\omega_1 \\ \Delta\omega_2 \end{bmatrix} \quad \ldots\ldots (12)$$

Also, the linearized swing equations of the machines are

$$\begin{bmatrix} \Delta\dot{\omega}_1 \\ \Delta\dot{\omega}_2 \end{bmatrix} = \begin{bmatrix} \frac{\omega_0 i_{q1}}{2H_1} & \frac{-\omega_0 i_{d1}}{2H_1} & 0 & 0 \\ 0 & 0 & \frac{\omega_0 i_{q2}}{2H_2} & \frac{-\omega_0 i_{d2}}{2H_2} \end{bmatrix} \begin{bmatrix} \Delta\psi_{d1} \\ \Delta\psi_{q1} \\ \Delta\psi_{d2} \\ \Delta\psi_{q2} \end{bmatrix} +$$

$$\begin{bmatrix} -\frac{\omega_0 \psi_{q1}}{2H_1} & \frac{\omega_0 \psi_{d1}}{2H_1} & 0 & 0 \\ 0 & 0 & -\frac{\omega_0 \psi_{q2}}{2H_2} & \frac{\omega_0 \psi_{d2}}{2H_2} \end{bmatrix} \begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} + \begin{bmatrix} \frac{\omega_0}{2H_1} & 0 \\ 0 & \frac{\omega_0}{2H_2} \end{bmatrix} \begin{bmatrix} \Delta T_{m1} \\ \Delta T_{m2} \end{bmatrix} \quad (13)$$

Equations (11, 12 and 13) may be combined with the field and damper equations of the conventional machine [2] together with the field and screens equations of the superconducting alternator [5]. The result of this combination is given at the end of the appendix, which has the compact form:

$$\dot{X} = LX + M\Delta I + BU \quad (14)$$

The linearized model of the multi-machine system in state space form may be obtained as follows: multiply the matrix M by the inverse matrix of the machine reactances. The latter is composed diagonally of the machine reactances, which will be a

12×12 matrix. The result of the multiplication may be added to the matrix L to obtain the state space form of equation (1).

The Reference Axes

In the previous analysis, the network reference axes were assumed rotating at constant speed ω_o. Since this assumption is unvalid [2], the network frequency is assumed to be equal to that of one arbitrary selected machine unit. Thus, the network (D-Q) axes, rotates in synchronism with the reference machine (d-q) axes. Therefore $\Delta\delta$ of the reference machine is zero. The row and column of $\Delta\delta$ of the reference machine are eliminated [2,4].

The Expanded Form of Equation (14)

$$
\begin{bmatrix}
\Delta\dot{\psi}_{F1} \\ \Delta\dot{\psi}_{d1} \\ \Delta\dot{\psi}_{D} \\ \Delta\dot{\psi}_{q1} \\ \Delta\dot{\psi}_{Q} \\ \Delta\dot{\psi}_{F2} \\ \Delta\dot{\psi}_{d2} \\ \Delta\dot{\psi}_{D1} \\ \Delta\dot{\psi}_{D2} \\ \Delta\dot{\psi}_{q2} \\ \Delta\dot{\psi}_{Q1} \\ \Delta\dot{\psi}_{Q2} \\ \Delta\dot{\delta}_{1} \\ \Delta\dot{\delta}_{2} \\ \Delta\dot{\omega}_{1} \\ \Delta\dot{\psi}_{2}
\end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \omega_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{11} & c_{12} & \psi_{q1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \omega_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{21} & c_{22} & -\psi_{d1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_0 & 0 & 0 & c_{31} & c_{32} & 0 & \psi_{q2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\omega_0 & 0 & 0 & 0 & 0 & 0 & c_{41} & c_{42} & 0 & -\psi_{d2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-\omega_0 i_{q1}}{2H_1} & 0 & \frac{\omega_0 i_{d1}}{2H_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-\omega_0 i_{q2}}{2H_2} & 0 & 0 & \frac{\omega_0 i_{d2}}{2H_2} & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta\psi_{F1} \\ \Delta\psi_{d1} \\ \Delta\psi_{D} \\ \Delta\psi_{q1} \\ \Delta\psi_{Q} \\ \Delta\psi_{F2} \\ \Delta\psi_{d2} \\ \Delta\psi_{D1} \\ \Delta\psi_{D2} \\ \Delta\psi_{q2} \\ \Delta\psi_{Q1} \\ \Delta\psi_{Q2} \\ \Delta\delta_{1} \\ \Delta\delta_{2} \\ \Delta\omega_{1} \\ \Delta\omega_{2}
\end{bmatrix}
+
$$

$$
\begin{bmatrix}
-\omega_0 r_F & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & F_{11} & 0 & F_{12} & 0 & 0 & F_{13} & 0 & 0 & F_{14} & 0 & 0 \\
0 & 0 & -\omega_0 r_D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & F_{21} & 0 & F_{22} & 0 & 0 & F_{23} & 0 & 0 & F_{24} & 0 & 0 \\
0 & 0 & 0 & 0 & -\omega_0 r_Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\omega_0 R_F & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & F_{31} & 0 & F_{32} & 0 & 0 & F_{33} & 0 & 0 & F_{34} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_0 R_{D1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_0 R_{D2} & 0 & 0 & 0 \\
0 & F_{41} & 0 & F_{42} & 0 & 0 & F_{43} & 0 & 0 & F_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_0 R_{Q1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_0 R_{Q2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\omega_0 \psi_{q1}}{2H_1} & 0 & \frac{-\omega_0 \psi_d}{2H_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\omega_0 \psi_{q2}}{2H_2} & 0 & 0 & \frac{-\omega_0 \psi_{d2}}{2H_2} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta i_{F1} \\ \Delta i_{d1} \\ \Delta i_D \\ \Delta i_{q1} \\ \Delta i_Q \\ \Delta i_{F2} \\ \Delta i_{d2} \\ \Delta i_{D1} \\ \Delta i_{D2} \\ \Delta i_{q2} \\ \Delta i_{Q1} \\ \Delta i_{Q2}
\end{bmatrix}
+
\begin{bmatrix}
\frac{\omega_0 r_F}{X_{ad}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & \frac{\omega_0 R_F}{X_{aF}} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \frac{\omega_0}{2H_1} & 0 \\
0 & 0 & 0 & \frac{\omega_0}{2H_2}
\end{bmatrix}
\begin{bmatrix}
\Delta E_1 \\ \Delta E_2 \\ \Delta T_{m1} \\ \Delta T_{m2}
\end{bmatrix}
$$

تأثير المولدات ذات التوصيل الفائق على استمرار الشبكات الكهربائية

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ملخص البحث. تبحث هذه الورقة في كيفية تأثير المولدات الكهربائية ذات التوصيل الفائق على استقرار شبكات القوى الكهربائية التي تشتمل على عدة مولدات مختلفة النوع والحجم. تم إعداد النموذج الخطي لشبكة كهربائية تحتوي على أربع مولدات إحداها ذو توصيل فائق والذي بواسطته يتم دراسة استقرار الشبكات الكهربائية. كما تم إعداد وشرح مقارنة كمية بين تأثير تلك المولدات وتأثير المولدات العادية على استقرار الشبكات.