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Mobility Degradation Models for Electrons in Inversion Layers of Silicon

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Abstract. Carrier mobility in silicon inversion layers is an important parameter that reflects carrier transport mechanisms. It is very important to accurately model the effective mobility of carriers for the accurate calculation of the drain current using device simulation programs. This becomes more important in submicron devices using higher substrate doping, thinner gate dielectrics, and much higher electric fields. Several models for the effective mobility of carriers , as a function of the effective transverse electric field in MOS inversion layers have been reported . Extraction from experimental data of parameters best fitting to an empirical formula has been used. To find out the physically appropriate model for mobility of inversion layer electrons at room temperature, we compare calculated effective mobility versus transverse electric field using recently reported empirical model for mobility degradation. The drift velocity versus electric field characteristics were calculated, using different models for mobility degradation, and compared to previously reported experimental results. The MOSFET saturation drain current, which is of most importance because of its effect on circuit speed, was calculated using these models and compared to recent theoretical and experimental results that take the effects of mobility degradation, velocity saturation and series resistance into consideration.

Introduction

Carrier mobility is an important parameter that reflects the physical carrier transport mechanisms. In MOSFET inversion layers the carriers are confined to a very thin layer due to the strong gate electric field, and therefore, the low-field mobility in the channel is much smaller than in the bulk. It is very important to accurately model the effective mobility of carriers in silicon MOSFET inversion layers. Mobility is the parameter that mostly affects the accuracy of the results of the drain current calculation using two- and three-dimensional simulation programs. The importance of an accurate inversion layer mobility model becomes more pronounced in modern submicron devices using higher substrate doping, thinner gate dielectrics, and much higher electric fields, and in particular as scaling down of MOSFET devices continues further and further.

A "universal" inversion layer mobility model should accommodate a wide range of process parameters, such as substrate doping, oxide thickness, as well as of the externally applied voltage. The accuracy of the mobility model enables the accurate prediction of the drain current and estimation of the device performance.

Several models for the effective mobility of carriers in inversion layers, as a function of the effective transverse electric field in MOS inversion layers have been reported. Efforts to obtain an accurate MOSFET inversion layer mobility model have started since the early field-effect transistor was developed (Mansour *et al.* [1, 2], Ezawa [3], and Nakamura [4]). The interest in obtaining an accurate model for the inversion layer mobility has recently increased because of its utmost importance in device analysis and design. Extraction of the best fitting parameters from experimental data has usually been the approach used, as well as using empirical formulae

The transverse electric field dependence of the MOSFET channel mobility under strong inversion is usually modeled by a universal μ E_{eff}) model, where the effective electric field, E_{eff} is usually defined as the inversion, Q_i, and the bulk, Q_b, charges per unit area; $E_{eff} = (\eta Q_i + Q_b)/\varepsilon_s$, where ε_s is the dielectric permittivity of silicon, and η is between 0 and 1, Sabins and Clemens [5], which is actually equal to the average electric field experienced by carriers in the inversion layer, Ko [6]. The ability to model the inversion layer mobility as a function of a single variable, E_{eff} is a unique feature of room temperature operation, Gildenblat and Huang [7]. In the general case μ is a complicated function of Q_b and Q_i.

In this paper a detailed investigation of the different inversion layer mobility models for electrons in silicon at room temperature is carried out in order to find out the simplest, most suitable, physically appropriate, and most accurate model to be used. An evaluation of the effective mobility of electrons comparing the various models is performed. The impact of the various mobility models on the drift velocity of electrons and on the calculated saturation drain current for a submicron MOSFET is studied.

Mobility Models for Inversion Layer Electrons

The effective mobility of electrons in silicon inversion layers is calculated using Mathiessen's rule, summing the effects of the three main scattering mechanisms: Coulomb ionized impurity scattering, lattice phonon scattering, and surface roughness scattering, as follows:

$$1/\mu = 1/\mu_c + 1/\mu_{ph} + 1/\mu_{sr}$$

Several models for the effective mobility of electrons in silicon inversion layers, as a function of the effective transverse electric field at room temperature have been reported. A widely used expression for the effective electron mobility due to carrier heating is, Mansour *et al.* [1;2] and Selberherr [8]:

$$\mu_{eff} = \mu \; / \; [\, 1 + (E_{eff} \, / \, E_{crit})^{\alpha} \,]^{1/\alpha} \quad ,$$

Where E_{crit} is a critical transverse electric field, $E_{crit} = 6.98 \times 10^{3} . (T(K)/300)^{1.55}$ V/cm. $\alpha = 1.11 \; (T(K)/300)^{0.66}$, for silicon.

In Huang and Arora [9] the effective inversion layer electron mobility at room temperature was modeled by :

$$\mu_{eff} = \mu_0 / [\, 1 + a\, E_{eff}^{\;1/3} + b\, E_{eff}^{\;2} + c\, (1 + Q_i \, / \, Q_b)^{2} \,] .$$

where the 1/3 power dependence models the acoustic phonon scattering and the 2 power dependence is for surface roughness scattering. , where μ_0, a, b and c are fitting parameters.

In Yue *et al.* [10] the effective inversion layer electron mobility at room temperature is modeled by fitting to experimental data :

$$\mu_{eff} = 1481 / [\, 1 + 0.0738\, E_{eff}^{\;0.25} + 2.63 \times 10^{-12}\, E_{eff}^{\;2} \,]$$

Chen *et al.* [11] modeled the measured effective inversion layer electron mobility at room temperature by an empirical model :

$$\mu_{eff} = 540 \; / [\, 1 + (10^{6}\, E_{eff} / 0.9)^{1.85}]$$

In Gildenblat and Huang [7] the effective inversion layer electron mobility at room temperature is modeled by Mathiessen rule:

$$1/\mu = 1/\mu_c + 1/\mu_{ph} + 1/\mu_{sr} \text{ , where.}$$

$$1/\mu_c = \alpha (1 + Q_i \, / \, Q_0)^{-2} \; , \quad 1/\mu_{ph} = \gamma \; E_{ff}^{\;1/3} + \delta \; ,$$

and

$1 / \mu_{sr} - \beta E_{eff}^{2.1}$, where α, β, γ, δ, and Q_0 are fitting parameters.

Near room temperature the mobility reduces to :

$$1 / \mu \approx 1 / \mu_{ph}(E_{eff}) + 1 / \mu_{sr}(E_{eff})$$

Lombardi *et al.* [12] modeled the effective inversion layer electron mobility at room temperature by a semi-empirical Mathiessen's rule:

$$1/\mu = 1/\mu_c + 1/\mu_{ph} + 1/\mu_{sr} ,$$

where,

$$\mu_c = \mu_0 + \frac{\mu_{max}(T) - \mu_0}{1 + (N_A/C_r)^{\alpha}} - \frac{\mu_1}{1 + (C_s/N_A)^{\beta}}$$

$$\mu_{ph}(E_{eff}) = [\, B\,(T/E_{eff}) + C\,(1/E_{eff})\,]\,(1/T) , \quad \mu_{sr}(E_{eff}) - \delta/E_{eff}^{2} ,$$

where $\mu_{max}(T) = \mu_{max}(T/300)^{\gamma}$, α, β, γ, δ, A, B, C_r, C_s, μ_0, and μ_{max} are fitting parameters, and N_A is the dopant concentration in the substrate, and T is the absolute temperature.

In Shin *et al.* [13] the effective inversion layer electron mobility at room temperature is modeled by Mathiessen's rule:

$$1 / \mu = 1 / \mu_c + 1 / \mu_{ph} + 1 / \mu_{sr} , \text{ where,}$$

$$\mu_{ph}(E_{eff}) = [\, (1150\, T_n^{-2.5})^{-1} \left(\frac{0.0388\, T_n E_{eff}^{-1} + 1.73 \times 10^{5} E_{eff}^{-1/3}}{3.2 \times 10^{-9}\, p\, T_n^{0.5}} \right)^{-1}]^{-1} .$$

where

$$p = 0.09\, T_n^{1.75} + 4.53 \times 10^{-8} (N_1 / Z)^{-0.25}\, T_n^{-1} N_t , \quad T_n = T / 300 .$$

$N_1 - (2\, \varepsilon_{Si} / q)\,(E_{eff} + E_o)$, $N_t = 3 \times 10^{10}\ \text{cm}^{-2}$, E_0 is the transverse electric field at the inversion layer edge ;

$\mu_{sr}(E_{eff}) = 6 \times 10^{14}\ E_{eff}^{-1/3}$, Z = average inversion layer width.

$$\mu_c(E_{eff}) = \frac{1.1 \times 10^{21}\ T_n^{1.5}}{\ln(1+\gamma_{BH}^2) - \dfrac{\gamma_{BH}^2}{1+\gamma_{BH}^2}} \cdot \frac{1}{N_A} .$$

where $\gamma_{BH}^2 = [\,2 \times 10^{19} / (N_i / Z)\,] \,.\, T_n^2$.

Yamaguchi [14] modeled the effective inversion layer electron mobility at room temperature for use in SPICE circuit simulation by :

$$\mu_{eff} = \mu_0 [\ 1 + (\ N_A /(\ N_A/S + N_i\ ))^{-1/2} ,$$

where μ_0 , S and N_r are fitting parameters, determined experimentally.

In Yeric *et al.* [15] the effective inversion layer electron mobility at room temperature is modeled by :

$$\mu_{eff} = \mu_0 / [\ 1 + A\, E_{eff}^{\ 0.3} + B\, E_{eff}^{\ 2}\ ] ,$$

where μ_0 , A and B are fitting parameters.

Lee [16] modeled the effective inversion layer electron mobility at room temperature by :

$$\mu_{eff} = \mu_0 / [\ 1 + A\, E_{eff}\ ] ,$$

where μ_0 and A are fitting parameters.

In Liang *et al.* [17] the effective inversion layer electron mobility at room temperature is modeled by a simple empirical model:

$$\mu_{eff} = \mu_0 / [\ 1 + (E_{eff} / E_{crit}\ )]^{\ \alpha} ,$$

where μ_0 , α , and E_{crit} are fitting parameters.

Attempts were made to correlate empirical model parameters to physical models, Reichert *et al.* [18], Yamanaka *et al.* [19], Gamiz *et al.* [20], and Jungenman *et al.* [21]. Reichert *et al.* [18] showed that the 2-power factor in the mobility model was dependent not only on surface roughness scattering, but also on phonon scattering. Yamanaka *et al.* [19] showed that the inversion layer mobility of electrons due to surface roughness is independent of gate oxide thickness.

In order to compare the different inversion layer electrons mobility models we will take the recent empirical universal model of Chen *et al.* [11] as a reference. Room temperature operation ($T=300^{o}K$), and an acceptor concentration of 3×10^{16} cm^{-3} in the substrate was assumed. Figures (1) and (2) show a comparison of the calculated electron mobility at room temperature for inversion layer electrons, using the mobility models discussed above, as a function of the effective transverse electric field. In both figures Chen *et al.* [11] was taken as a reference. In Fig. 1, only the models of Yue *et al.* [10] and Yamaguchi [14] are in close agreement to the reference model of Chen *et al.* [11], above $E_{eff} > 5 \times 10^{5}$ V/cm. while in Fig. 2 only the model of Liang *et al.* [17] is in close agreement to that of Chen *et al.* [11], above $E_{eff} > 7 \times 10^{5}$ V/cm.

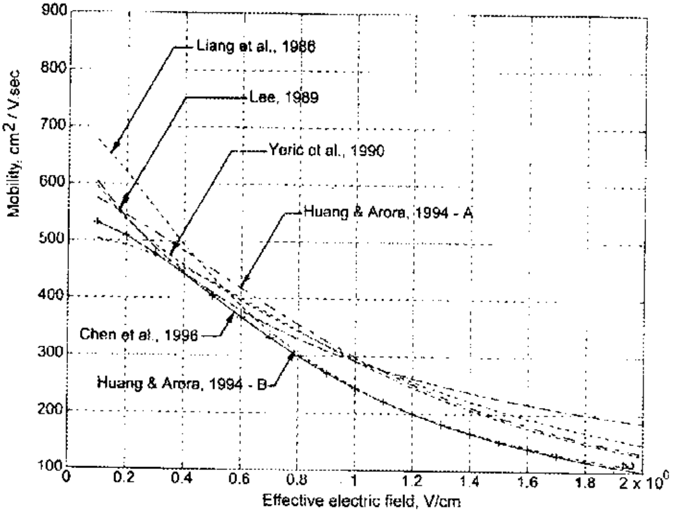

Fig. 1. Calculated effective inversion layer mobility for electrons in silicon using different models.

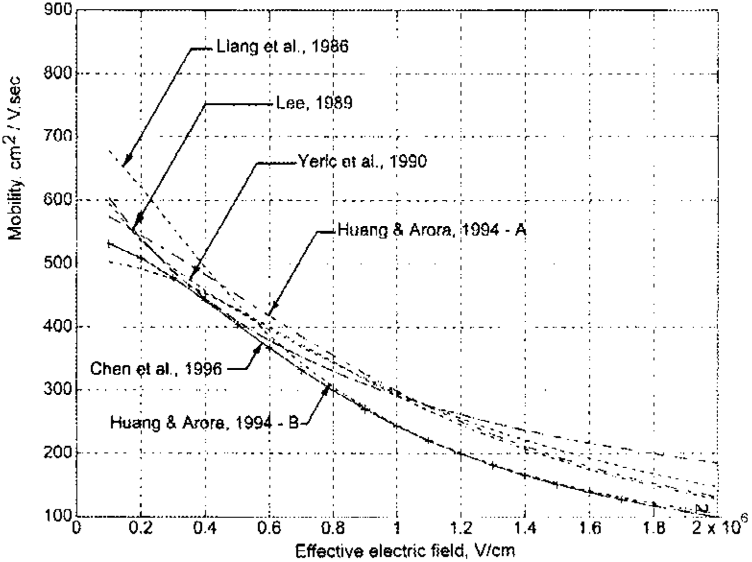

Fig. 2. Calculated effective inversion layer mobility for electrons in silicon using different mobility models.

Drift Velocity of Electrons

The drift velocity of electrons saturates at higher electric fields. A commonly used empirical model for the drift velocity vs. electric field characteristics is the piece-wise model Takeuchi *et al.* [22]

$$v_d = \mu_{eff} E \; / \; (1 + (E / E_{crit})) \;, \qquad E < E_{crit}$$

$$v_d = v_{sat} \qquad\qquad , \qquad E > E_{crit} \;,$$

where $E_{crit} = 2\, v_{sat} / \mu_{eff}$.

In Yamaguchi [14] the model was :

$$v_d = \mu_{eff}(N_A, E_{eff}) . E_t . f(N_A, E_{eff}, E_t),$$

where E_t is the tangential electric field, and

$$f(N_A, E_{eff}, E_t) = [1 + (\mu_{eff}(N_A, E_{eff}) . E_t / v_c)^2 (\mu_{eff}(N_A, E) . E_t / v_c + G)^{-1} + (\mu_{eff}(N_A, E_{eff}) . E_t / v_s)^2]^{-1/2} .$$

where G, v_c, and v_s are fitting parameters.

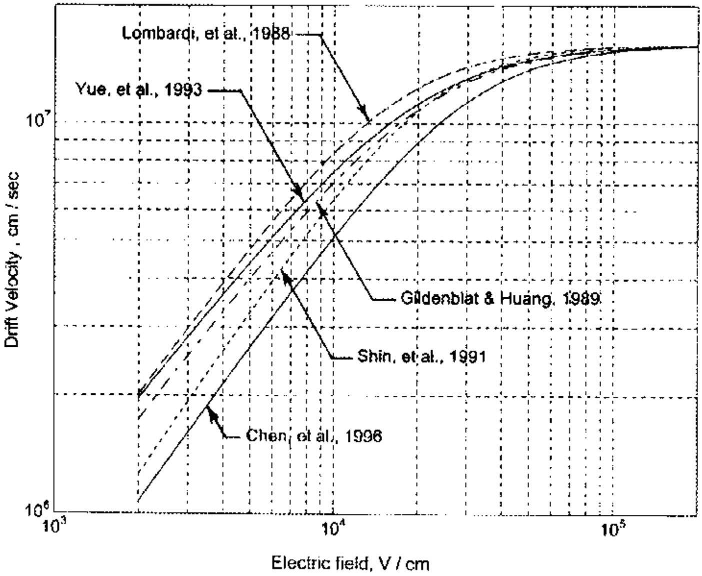

Fig. 3. Calculated drift velocity for electrons versus electric field.

The empirical model of Sodini *et al.* [23] shows a better fit to experimental results :

$$v_d = \mu_{eff} E \;/\; (1 + (E / E_{sat})^2)^{1/2},$$

where $E_{sat} = 2\, v_{sat} / \mu_{eff}$. We used this model to calculate the drift velocity dependence on electric field, using the various above-mentioned models for the effective mobility.

Figure 2 shows the calculated v_d vs E characteristics, as well as the piece-wise model of Takeuchi *et al.* [22] and the experimental results of Cooper *et al.* [24], for comparison. It is noticed from the figure that only the velocity model of Huang *et al.* [9] is very close to the reference model of Chen *et al.* [11].

MOSFET Saturation Drain Current

The MOSFET saturation drain current is the most important of the device parameters because of its effect on circuit speed. The model that has been used to model the effect of device fabrication parameters and bias conditions on the drain saturation current is :

$$I_{dsat} = \mu \frac{1}{2}_{eff} (W_{eff} / L_{eff}) (\varepsilon_{ox} T_{ox}) (V_{gs} - V_t) .$$

where T_{ox} is the oxide thickness under the gate, L_{eff} and W_{eff} are the channel length and width, respectively. This model not suitable for today's submicron devices, since it doesn't take into consideration such effects as mobility degradation due to the transverse electric field under the gate, velocity saturation, short-channel effect, and the source and drain series resistance of LDD structures, which have to be considered. When the effects of mobility degradation, velocity saturation, and series resistance are considered , the new model Chen *et al.* [11] for the saturation drain current, I_{dsat} , becomes :

$$I_{dsat} \quad \frac{[V_1 + 2(V_{gs} - V_t) R_s W v_{sat} C_{ox}] - \sqrt{V_1^2 + 4 (V_{gs} - V_t) E_{eff} L_{eff} W v_{sat} C_{ox} R_s}}{2 (R_s + W v_{sat} C_{ox} R_s^2)} .$$

where the saturation velocity of electrons, $v_{sat} = 8 \times 10^6$ cm/s, and the electric field corresponding to velocity saturation is $E_{sat} = 2 v_{sat} / \mu_{eff}$, C_{ox} is the oxide capacitance, and R_s is the series resistance of the device. We calculated the saturation drain current for an LDD MOSFET , Chen *et al.* [11] , using the above equation. We used $R_s = 38 \Omega$, $V_t = 0.836$ V. For the effective mobility of electrons in the inversion layer, μ_{eff} , we used the various mobilimodels discussed earlier. Figure 4 shows the saturation drain current calculated using the different mobility models, compared to the measured values of Chen *et al.* [11] . It is clear from the figure that the saturation drain current using the effective mobility models of Yue *et al.* [10] and Shin *et al.* [13] are the closest to that of Chen *et al.* [11], followed by the models using the effective mobility models of Lombardi *et al.* [12], and Gildenblat and Huang [7], and that the best fit for all models is for larger values of L_{eff} ; $L_{eff} \geq 0.65$ μm. The deviation for smaller values of L_{eff} is possibly due to the increased contribution of quantum effects.

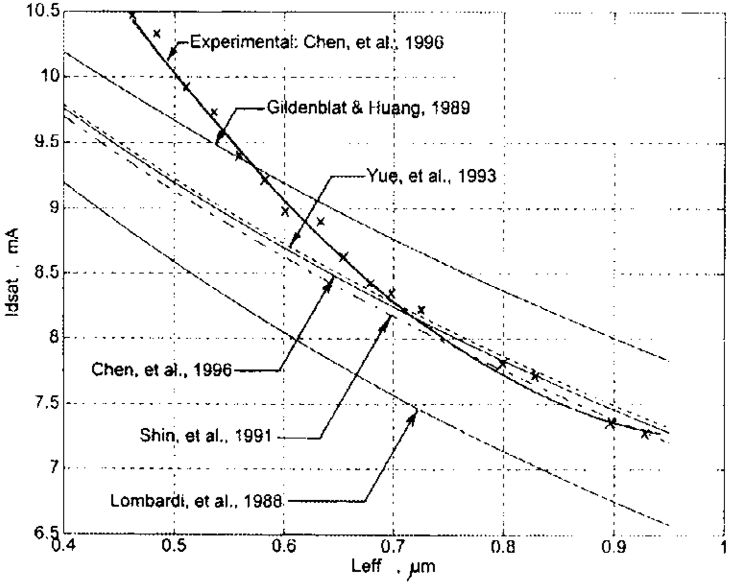

Fig. 4. Saturation drain current for LDD MOSFET. Comparison of different models with experimental results.

Conclusion

A detailed investigation of the different inversion layer mobility degradation models for electrons in silicon at room temperature is carried out in order to find out the simplest, most suitable, physically appropriate, and most accurate model to be used in CAD device simulation programs. An evaluation of the effective mobility of electrons comparing the various models is performed. The impact of the various mobility models on the drift velocity of electrons and on the calculated saturation drain current for a submicron MOSFET is studied. The effective mobility dependence on transverse electric field, electron drift velocity dependence on tangential electric field, and

MOSFET saturation drain current for s submicron device are studied for the different mobility models, and compared to recently reported models and experimental results.

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نماذج لتدهور حركة الإلكترونات في الطبقات المنعكسة بالسيليكون

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(استلم في ٢٦ /١١/ ١٩٩ م ، وقبل للنشر في ٢٢ /٤ / ١٩٩٨م)

ملخص البحث . تعتبر حركة حاملات الشحنة في طبقات السيليكون المنعكسة بارامترًا مهمًا يعكس آليات نقل حاملات الشحنة . ومن المهم جدا أن تتم نمذجة الحركة الفعالة لحاملات الشحنة بدقة حتى يتم حساب تيار المصرف بدقة باستعمال برامج محاكاة النبيطة . ويصبح هذا الأمر أكثر أهمية في حالة النبائط التحت - ميكرونية التي تستخدم إشابة أعلى للطبقة التحتية، عوازل أقل سمكا للبوابة ، ومجالات كهربية أعلى كثيرا جدا . وقد تم نشر عدة نماذج للحركة الفعلية لحاملات الشحنة في الطبقات المنعكسة داخل الـ " م.أ.س. " ، كدالة في المجال الكهربائي المستعرض الفعال ، واستخدم في ذلك استخلاص البارامترات التي تحقق أفضل ملاءمة لصيغة وضعية من المعطيات التجريبية . ومن أجل أن نحدد النموذج المناسب فيزيائيا لحركة الكترونات الطبقة المنعكسة عند درجة حرارة الغرفة قمنا بمقارنة الحركة الفعالة كدالة في المجال الكهربائي المستعرض باستخدام نموذج منشور حديثا لتدهور الحركة . وقمنا أيضا بحساب خصائص سرعة الانسياق كدالة المجال الكهربائي ، وذلك باستعمال نماذج مختلفة لتدهور الحركة ، وقارناها بالنتائج التجريبية التي سبق الإعلان عنها . وقمنا أيضا بحساب تيار التشبع للمصرف لنبيطة " ت.ت.م.م.أ.س. " ، وهو الذى يعتبر ذو الأهمية الأعلى نظرا لتأثيره على سرعة الدائرة ، وقمنا بحسابه باستعمال هذه النماذج وقارنا النتائج بالنتائج الحديثة النظرية والتجريبية التي تأخذ في الاعتبار تأثيرات تدهور الحركة ، تشبع السرعة ، و المقاومة على التوالي.