*J. King Saud Univ.*, Vol. **2**, *Eng. Sci.* (1), pp. 99-114, (1410H/1990)

# CIVIL ENGINEERING

## Flexural Behaviour of Beams Externally Reinforced Through Shot-Pin Shear Connectors

**Ghulam Husain Siddiqi and Jedaia Nahar All-Qahtani**
*Department of Civil Engineering, College of Engineering, King Saud University,*
*P.O. Box 800, Riyadh 11421, Saudi Arabia.*

**Abstract.** A study on flexural behaviour of test beams which are externally reinforced with plates through shot-pin connectors is presented here. The study formulates equations for design and investigation, for strength and servicibility, of externally reinforced beams in accordance with ACI-Code procedure. On the experimental side the study presents test results of nine beams which were tested to assess the strength performance and ascertain deflection and strain behaviour. Comparison between the calculated and measured values is also made which validates the theoretical formulations and the composite action tacitly assumed.

### 1. Introduction

An under reinforced beam when loaded beyond its design capacity shows its distress by vertical flexural cracks. Such a beam (which is already in service) may be reinforced externally to develop additional strength to sustain the overload. This treatment may be applied to the beam in pre- or post-cracked state. However, the additional strength that can be developed is dependent upon the existing concrete strength and the extent to which the steel ratio may be augmented before maximum steel ratio is reached.

The method of bonding the external reinforcement to a beam is of crucial importance. The effectiveness of achieving this 'bond' through shot-pin shear connector is reported in Ref. [1,2]. Besides explaining the mechanics of bonding by shot-pins it is also established in these references, through experimental study, that a shot-pin connection made by a 27 mm long pin (manufacturer designation NK and of diameter 3.65 mm) can transfer a minimum of 4.965 tons, of longitudinal shear.

This study presents a theoretical formulation to evaluate the strength and deflections of an externally reinforced (ER) beam and to fix the maximum and minimum steel ratios to ensure ductility. On the experimental side it reports the test results of nine beams to assess the strength, the strain-pattern and the deflection behaviour of such beams. The experimental study also affords to validate the assumption of composite action made in the theoretical formulation.

## 2. Research Significance

The application of external reinforcement, in the form of plates, is usually made through epoxy resins. Such applications are reported in literature [3,4,5]. The details of application procedure and the long term behaviour of the resin are the matters of concern. This study explores an alternative method of application of external steel which is practical, fast and effective.

## 3. Theoretical Formulation

Figure 1 shows the section of a rectangular beam reinforced both internally and externally. b, h and d are the width, the overall depth and depth to the centroid of internal tension steel while $A_s$ is the area of this steel. Subscript e to d and $A_s$ refers these symbols to external tension steel while a bar over these symbols refers them to equivalent values.

The yield stresses of internal and external steels are denoted by $f_y$ and $f_{ye}$ respectively and their ratio $f_{ye}/f_y$ by $\alpha$. The ratio $d_e/d$ is denoted by r. The depth $d_e$ may be taken approximately equal to h. The following formulation is made for the equivalent section of Fig. 1(b) and its stress diagram of Fig. 1(c).

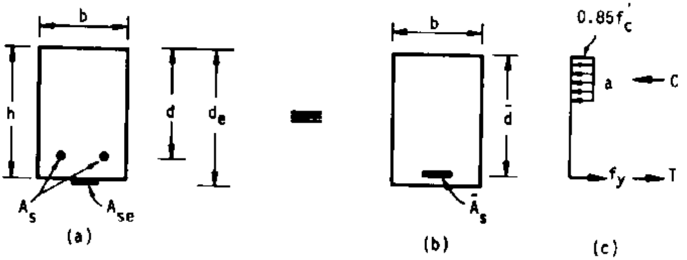


**Fig. 1. Rectangular Rectangular reinforced concrete beam**
**(a) with internal and external reinforcement,**
**(b) with equivalent reinforcement,**
**(c) Stress block at nominal capacity**

### 3.1. Equivalent Steel Area and Depth

Equating the yield force in tension steels of Fig. 1(a) and (b) yields equivalent steel area as

$$\bar{A}_s = A_s + \alpha A_{se} , \tag{1}$$

and equating moments of these forces about the top fiber gives equivalent depth as

$$\bar{d} = \frac{A_s + \alpha\, r\, A_{se}}{\bar{A}_s}\, d \tag{2}$$

### 3.2. Steel Ratio

Steel ratio ($\bar{\rho}$) of equivalent section is defined as

$$\bar{\rho} = \bar{A}_s / b\bar{d} \tag{3}$$

### 3.3. .Internal Forces on Equivalent Section

Resultant of stress block, C, is

$$C = 0.85\, f'_c\, a\, b \tag{4}$$

where $f'_c$ is the cylinder strength of the concrete and a, the depth of stress block of the equivalent section.

The resultant tensile force (T) in the steel is

$$T = \bar{A}_s\, f_y \tag{5}$$

### 3.4. Depth of Stress Block

Equilibrium of internal forces yields

$$a = \frac{T}{0.85\, f'_c\, b} \tag{6}$$

### 3.5. Nominal Moment Capacity

The nominal moment capacity ($M_n$) is obtained from moment equilibrium as

$$M_n = C\,(\bar{d} - (a/2)) \tag{7}$$

It may be pointed out that the constant $\beta_1$, and the balanced, the maximum and minimum steel ratios are functions of $f'_c$ and $f_y$ and may be determined from the code procedure [6,7].

### 3.6. Serviceability Condition

Under service loads, when the stress-strain relation in concrete is still linear the depth of neutral axis, $k\bar{d}$, of the transformed cracked section may be obtained from,

$$k^2 + 2n\bar{\rho}k - 2n\bar{\rho} = 0 \tag{8}$$

where n is the modular ratio, and the moment of inertia of this section $I_{cr}$ may be obtained from,

$$I_{cr} = \frac{1}{3} b\,(k\bar{d})^3 + n\bar{A}_s\,(\bar{d} - k\bar{d})^2 \tag{9}$$

The gross moment of inertia of an uncracked section and the cracked moment may be obtained as for any rectangular section.

## 4. Test Program

### 4.1. Objectives

Objectives of the test program are to investigate the degree of composite action that is developed when a concrete beam is externally reinforced, in pre-and post-crack stages, by shot-pins. The effectiveness of this treatment is assessed from strength and deflection point of view. The kinematics of plane sections is also studied.

### 4.2. Material Strength

Concrete used has an average 28 day cylinder strength of 325 kgf/cm$^2$, while the internal and external steel used have yield strengths of 4636 and 3310 kgf/cm$^2$ respectively.

### 4.3. Test Specimens

All the test beams were of size 20x20x140 cm and were reinforced with 2 $\phi$14 bars providing a steel ratio of 0.0091 and $\phi$6 stirrups at spacing of 8 cm. The beam was designed to be under reinforced and an adequate amount of shear reinforcement was provided to sustain the nominal loads after external reinforcing.

### 4.4. Test Set-up and Instrumentation

Figure 2 shows the test set-up and the instrumentation employed. The strains in internal steel and slippage of the steel plate at the connection were monitored by electrical resistance (ERS) gauges (not shown in the figure).

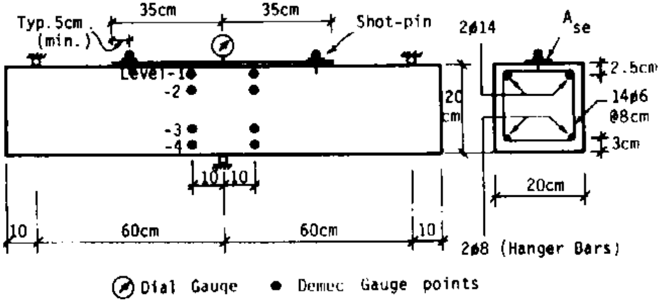


Fig. 2. Test set-up and instrumentation

### 4.5. Test Program

Table 1 describes the details of reinforcing used, the ERS gauges applied, and mode of loading used for each test beam.

## 5. Predicted Behavior

Table 2 presents the calculated values of the strength variables of the test beams. M and P are moment and transverse load respectively. Subscripts cr, d, f and n denote cracking (dead) failure and nominal values respectively of these symbols.

Table 3 records the predicted values of deflections at load steps and the associated variables of the test beams.

## 6. Experimental Results

Two beams, B1-00-LL and B6-00-LL (without external reinforcement), were tested to establish the basic characteristics of the test unit. Five beams, B3-02-LL, B5-02-LL, B7-06-LL, B11-10-LL and B12-10-LL (with external reinforcement) were tested to assess the effect of number of shot-pins used and area of external reinforcement. Beam B8-02-LU was loaded to service load level, unloaded and reloaded to verify elasticity of its behaviour. Beam B4-02-LP was loaded to failure before plating, unloaded, plated and reloaded to verify the effectiveness of this treatment to a distressed beam.

**Table 1. Identification, instrumentation and mode of loading of test beams**

| Beam identification | | B1-00-LL | B3-02-LL | B4-02-LP | B5-02-LL | B6-00-LL | B7-06-LL | B8-02-LU | B11-10-LL | B12-10-LL |
|---|---|---|---|---|---|---|---|---|---|---|
| External reinforcement | | – | | ---- | | – | / | / | / | / |
| Size (cm$^2$) | | | 0.3×5 | 0.3×5 | 0.3×5 | | 0.3×5 | 0.3×5 | 0.3×5 | 0.3×10 |
| No. of shot-pins | | – | 2 | 2 | 2 | – | 6 | 2 | 10 | 10 |
| Electrical | External steel | – | / | – | / | – | / | / | / | / |
| strain | Internal steel | – | – | – | – | – | / | / | – | / |
| Guages | Slippage monitoring | – | / | – | / | – | – | – | – | – |
| Crack observations | | – | – | – | / | / | – | / | / | / |
| Mode of loading | LL | / | / | – | / | / | – | / | / | / |
| | LU | – | – | – | – | – | – | / | – | – |
| | LP | – | – | / | – | – | – | – | – | – |

**Notes:**

(1) In Bs-nn-LX numerals s is serial number. nn is the number of shot-pins employed and character X codifies the loading procedure.
(2) Crack observations are made for appearance of first crack.
(3) LL: Loading continued to failure.
(4) LU:Loading to service load level. unloading and reloading to failure (ER test beam during loading-reloading stages).
(5) LP: Loading to failure, unloading, application of external reinforcement. reloading to failure. (Post-crack ER test beam).

**Table 2. Calculated values of strength variables of test beams**

| Variables | | IR Test Beams | ER Test Beams |
|---|---|---|---|
| $M_{cr}$ | t-cm | 48.10 | 48.10 |
| $P_{cr}$ | t | 1.66 | 1.66 |
| $M_d$ | t-cm | 1.73 | 1.73 |
| $M_n$ | t-cm | 224.29 | 309.30 |
| $M_f$ | t-cm | 226.02 | 311.03 |
| $P_f$ | t | 7.53 | 10.37 |

**Note:** IR internally reinforced, ER externally reinforced.

**Table 3. Calculated values of midspan deflections of the test beam**

| Load | | IR test beams | | ER test beams | | |
|---|---|---|---|---|---|---|
| t | $M_a$ t-cm | $I_e$ cm$^4$ | ∧ mm | $I_e$ cm$^4$ | ∧ mm | Remarks |
| 0.8 | 22.27 | 13333.3 | 0.07 | 13333.3 | 0.07 | |
| 1.6 | 46.27 | 13333.3 | 0.15 | 13333.3 | 0.15 | First crack |
| 2.4 | 70.27 | 7005.3 | 0.42 | 8536.0 | 0.35 | |
| 3.2 | 94.27 | 5259.4 | 0.76 | 7213.0 | 0.55 | |
| 4.0 | 118.27 | 4650.1 | 1.07 | 6751.0 | 0.75 | |
| 4.8 | 142.27 | 4384.1 | 1.37 | 6549.0 | 0.92 | |
| 5.6 | 166.27 | 4250.0 | 1.65 | 6277.0 | 1.13 | |
| 6.4 | 190.27 | 4025.0 | 2.00 | 6277.0 | 1.29 | $A_{sc}$ yields @ 6.68 t |
| 7.2 | 210.27 | 4025.0 | 2.25 | 6277.0 | 1.44 | |
| 7.34 | 218.32 | 4025.0 | 2.30 | – | – | $A_s$ yields |

**Notes:**

(1) $I_g$ = 13333.3 cm$^4$ for both sections.

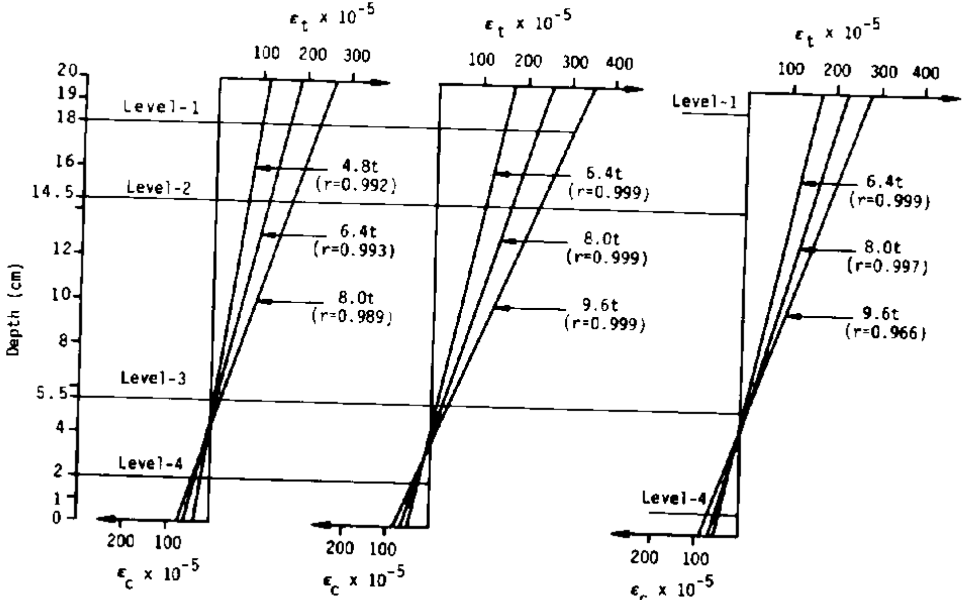
(2) $I_{cr}$ = 4025 and 6277 cm$^4$ for sections of IR- and ER-test beams respectively.

(3) $M_a$ is applied moment at the load step.

(4) $I_e = [M_{cr}/M_a]^3 I_g + [1-(M_{cr}/M_a)]^3 I_{cr} \leq I_g$.

## 6.1. Kinematics of Plane Section

Figures 3 and 4 present plots of strain over the depth at selected loads of a beam without external reinforcement, two beams with two-pin connection and three beams with multiple pin connection. The plots are linear regressed lines and their correlation factors are recorded. Multiple pin connections introduce non-linearity due to slippage at higher load levels.



(a) B6-00-LL (b) B3-02-LL (c) B5-02-LL

**Fig. 3.** IR and Two Pin Connected ER Test Beams. Midspan Strain Diagram at various levels over the depth at selected loads.

*Note:* Level-1, Level-2, etc. refer to Demec point-levels on the beam. r is correlation factor of the regressd curve.

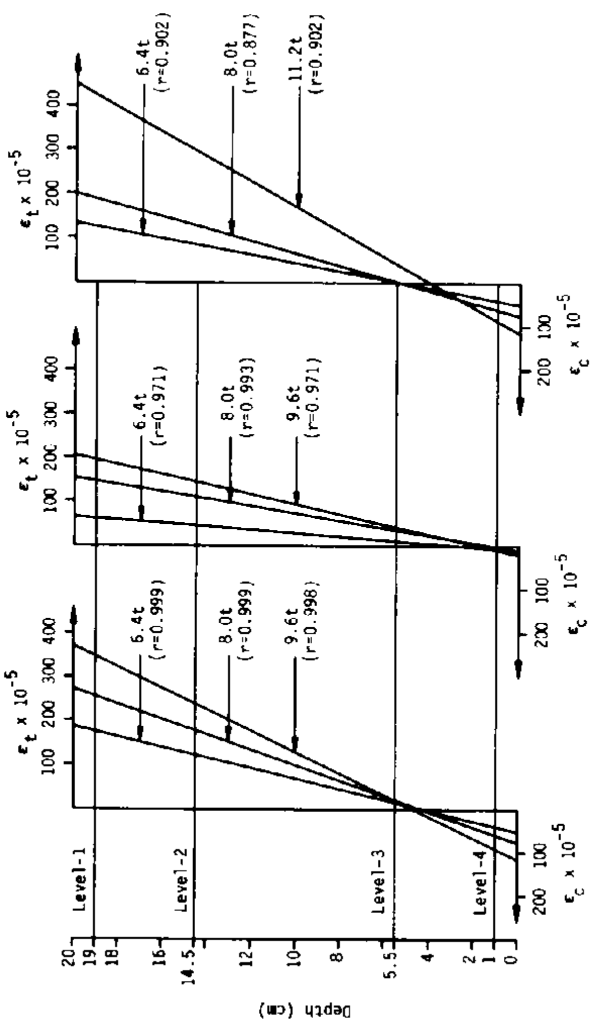


**Fig. 4.** ER Test Beams with Multiple Pin Connection. Midspan Strains at Various Levels over the Depth at Selected Loads.

*Note:* r is correlation factor of th eregressed curve.

### 6.2. Midspan Deflecftions

The uncertainty of the extent of cracking over a section of reinforced concrete beam makes the effective moment of inertia of a section difficult to estimate. According to Ref. [8, pp. 465-468] it is possible to estimate the deflection with a margin of error of ± 20%

Table 4 presents the measured midspan deflections of the test beams. In the case of the IR test beam the measured deflections differ by 5.8 to 16.0% from the calculated values. The calculated versus measured deflecltions of ER test beams, at various load steps, are plotted in Fig. 5. In their case 57.5% of the measured values fall outside the accepted ± 20% error margin.

Figure 6 presents plots of measured deflections versus loads of five test beams. Compared to the internally reinforced (IR) test beam, the ER beams are stiffer.

### 6.3. Loading Unloading Behavior

Test beam B8-02-LU is loaded to service load level, unloaded and then reloaded to failure. The deflection versus load graph is plotted on Fig. 7. The beam retains a permanent set at unloading because of lack of recovery time allowed.

### 6.4. Post-Crack Plating

Figure 8 presents two plots of load-deflection values of test beam B4-02-LP. One is before plating and the other after it. The failure loads in pre- and post-crack reinforcing stages were 8.8 and 10.0 tons respectively, while the measured deflections in the two stages, respectively, were 4.48 and 4.8 mm. Thus an increase of 13.6% in failure load and 7.1% in ductility of the beam is realized by the post-crack plating of the beam.

### 6.5. Failure Load

Table 5 presents failure loads of the test beams. The measured values, of ER beams differ from the calculated ones by +0.29 to −5.0%. The percentage increase in failure load of ER beams over the IR beam is also recorded.

## 7. Observations and Conclusions

The study presented is the first one on the performance of test beams externally reinforced by shot-pin shear connectors. Within the limitations of this study the following observations may be made:

The application of 50 × 3 mm plate to a test beam resulted in increase of failure load by 25 to 30%, increase of stiffness and consequent reduction of 40-60% in midspan deflections, increase in the range of elastic behaviour, some increase in duc-

**Table 4. Measured midspan deflections of test beams (mm)**

| Load (tons) | IR test beam | ER test beams | | | | |
|---|---|---|---|---|---|---|
| | B6-00-LL | B3-02-LL | B5-02-LL | B8-02-LU | B7-06-LL | B11-10-LL |
| 1.6 | 0.29 | 0.37 | 0.08 | 0.31 | 0.11 | 0.12 |
| 2.4 | 0.47 | 0.53 | 0.15 | 0.40 | 0.22 | 0.19 |
| 3.2 | 0.80 | 0.75 | 0.25 | 0.56 | 0.49 | 0.26 |
| 4.0 | 1.18 | 0.91 | 0.47 | 0.78 | 0.84 | 0.35 |
| 4.8 | 1.49 | 1.12 | 0.83 | 1.00 | 1.20 | 0.49 |
| 5.6 | 1.86 | 1.52 | 1.20 | 1.22 | 1.63 | 0.71 |
| 6.4 | 2.23 | 1.81 | 1.61 | 1.48 | 2.08 | 1.16 |
| 7.2 | 2.61 | 2.22 | 1.90 | 1.79 | 2.40 | 1.76 |
| 8.0 | 3.23 | 2.63 | 2.27 | 2.00 | 2.80 | 2.30 |
| 8.8 | 6.00 | 3.12 | 3.03 | 2.65 | 3.30 | 2.73 |
| 9.6 | | 3.30 | 3.20 | 3.13 | 3.76 | 3.08 |
| 10.6 | – | 5.80 | 5.40 | 4.60 | 5.70 | 3.93 |

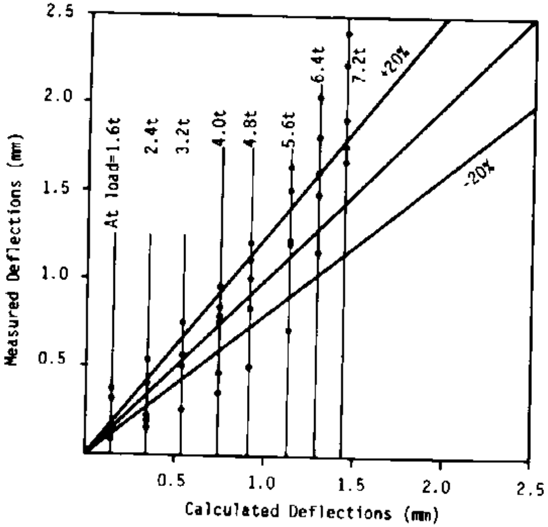


Fig. 5. Comparison of Calculatked and Measured Deflections of Externally Reinforced Beams at Various Load Levels

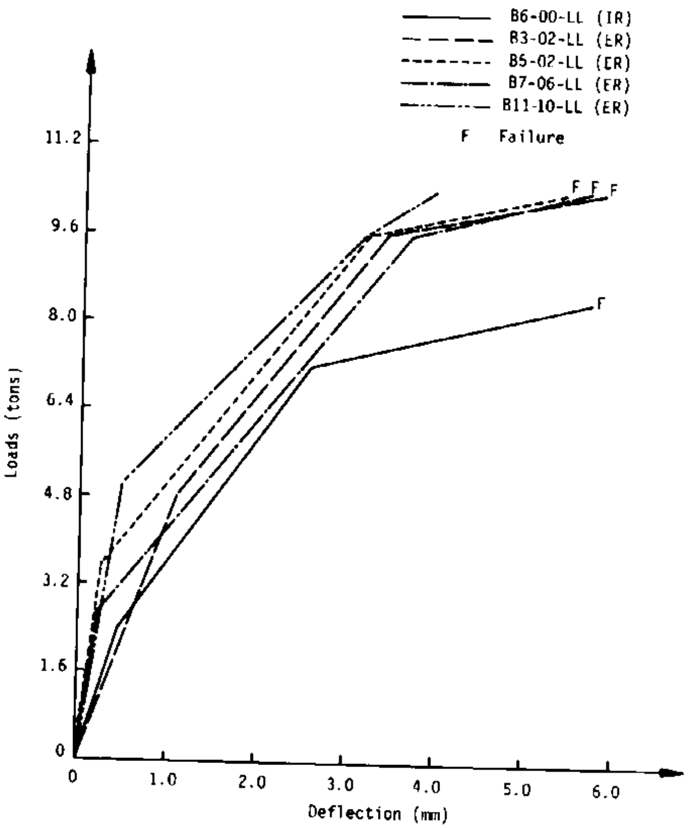


Fig. 6. The Measured Midspan Deflections of the Test Beams

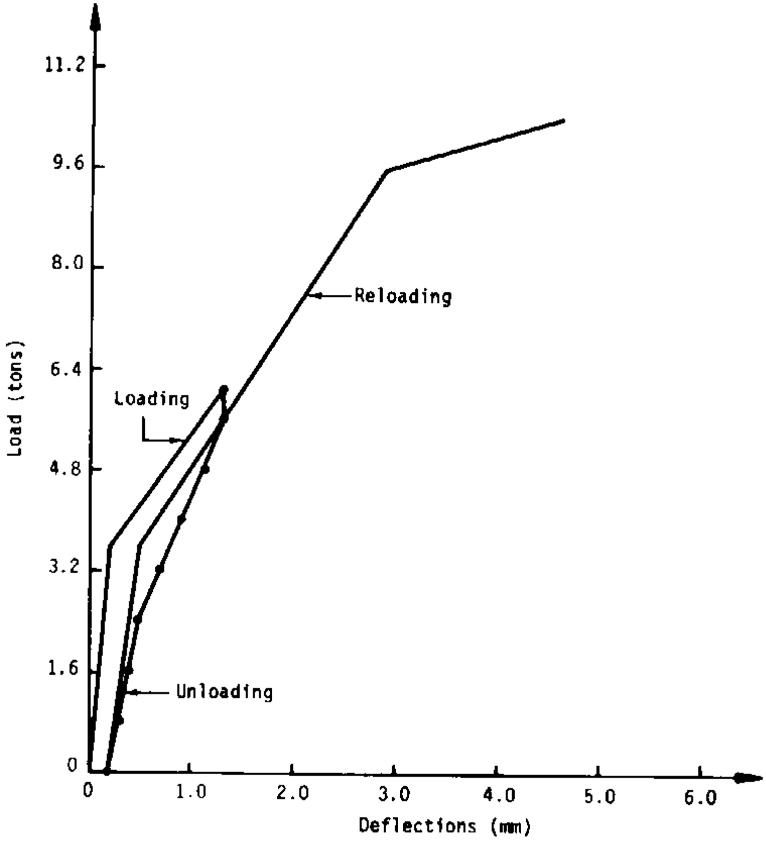


Fig. 7. The Measured Midspan Deflections of B8-02-LU During Loading, Unloading and Reloading Stages

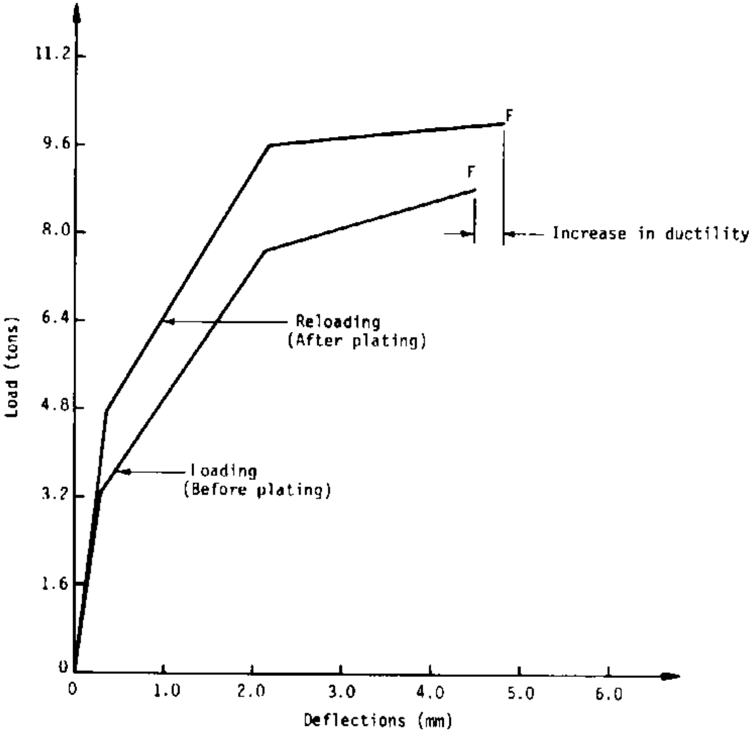


**Fig. 8. The Measured Midspan Deflections of B4-02-LP During Loading and Reloading Stages**

**Table 5. Measured and calculated failure loads of test beams**

| Test beam | $A_{se}$ (cm²) | Failure loads Measured | Failure loads Calculated | Percent increase |
|---|---|---|---|---|
| B6-00-LL | – | 8.4 | 7.53 | |
| B3-02-LL | 0.3 × 5 | 10.4 | 10.37 | 23.8 |
| B5-02-LL | 0.3 × 5 | 10.4 | 10.37 | 23.8 |
| B7-06-LL | 0.3 × 5 | 10.4 | 10.37 | 23.8 |
| B8-02-LU | 0.3 × 5 | 10.4 | 10.37 | 23.8 |
| B11-10-LL | 0.3 × 5 | 11.0 | 10.37 | 31.0 |
| B12-10-LL | 0.3 × 10 | 12.4 | 13.05 | 47.6 |

**Note:** Percent increase is in measured failure load over that of B6-00-LL.

tility, and delay in appearance of the first crack. The plane section behaviour is maintained within a correlation factor of 0.877 to 0.999.

Increase in the number of shot-pins (above minimum of two) does not significantly increase the load capacity but helps control crack-widths.

Behaviour of B4-02-LP during reloading after post crack application of external reinforcement is satisfactory both from load and deflection points of view. On the basis of this behaviour it is recommended that full-size beams treated for restoration and improvement of strength in this manner are tested. Exploration of the possibility of application of external compression and shear reinforcement by shot-pins is recommended.

## References

[1] Al-Qahtani, Jedaia, N.B. "External Reinforcement of Concrete Flexural Elements Through Shear Connectors." *Master's Thesis,* Department of Civil Engineering, King Saud Univ., May (1987).

[2] Siddiqi, G.H. and Al-Oahtani, J.N. "Test Performance of Shot-Pin Connected External Reinforcement." *Symposium on Concrete and Concrete Structures in the Middle East,* King Saud University, Riyadh, April 25-29 (1987).

[3] Macdonald, M.D. "The flexural behaviour of concrete beams with bonded external reinforcements." *TRRL Supplementary Report 415,* Transport and Road Research Laboratory, Dept. of the Environment, Crowthorne (1978).

[4] Macdonald, M.D. "The flexural performance of 3.5 m concrete beams with various bonded external reinforcements." *TRRL Supplementary Report 728.* Transport and Road Research Laboratory, Dept. of the Environment, Crowthorne (1982).

[5] Jones, R.N.; Swamy, Bloxham, J. and Bouderbalah, A. "Composite behaviour of concrete beams with epoxy bonded external reinforcement." *The International Journal of Cement Composites,* 2, No. 2 (1980).

[6] American Concrete Institute. *Building Code Requirements for Reinforced Concrete.* (ACI 318-83), Michigan: Detroit, Sept., 1983.

[7] American Concrete Institute. *Commentary on Building Code Requirements for Reinforced Concrete.* (ACI 318R-83), Michigan: Detroit, Sept. 1984.

[8] Park, R. and Paulay, T. *Reinforced Concrete Structures.* New York: John Wiley & Sons, 1975.

(*Manuscript Received:* 23.12.1987; *Accepted:* 18.4.1988)

# تصرف الانحناء للجسور المسلحة خارجيًا بواسطة روابط المسمار المقذوف

**غلام حسين صديقي وجديع نهار القحطاني**

قسم الهندسة المدنية، كلية الهندسة، جامعة الملك سعود، ص.ب ٨٠٠، الرياض١١٤٢١،
المملكة العربية السعودية

**ملخص البحث.** سوف تقدم هنا دراسة عن تصرف الانحناء للجسور المسلحة خارجيًا بواسطة صفائح مثبتة بإستخدام المسامير المقذوفة. وتتضمن الدراسة اشتقاق معادلات لتصميم وتحليل قوة صلاحية استخدام الجسور المسلحة خارجيًا وذلك بناء على طريقة معهد الخرسانة الأمريكي. يتضمن البحث نتائج اختبارات تسعة جسور قد اختبرت لتقويم أدائها من ناحيتي قدرتها وانحرافها وقد أجريت أيضًا مقارنة بين القيم المحسوبة والمقاسة والتي قد أثبتت صحة القوانين النظرية.