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Correction of Heights from Parallax-bar Using Finite Element Approach

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Abstract. The finite element approach has been used to correct heights obtained from parallax bar measurements and conventional parallax-height formula based on vertical aerial photography. Experimental tests show that this approach could give higher accuracy than the conventional second degree polynomial correction, while using the same number of control points and involving less computational effort.

Nomenclature

$a_1, a_2 \dots a_5$ polynomial constants

H_0 flying height above datum

h_A height of ground point (A) above datum

X axis of the model system in the flight direction

Y axis of the model system across flight direction

σ standard error in height

P_a Stereoscopic parallax of point (a)

dp_{ia} difference in parallax between reference point (a) and any other point (i) in the model

h_{IA} difference in height between ground point (A) and ground point (I).

1. Introduction

The finite element method has been applied in Photogrammetry to solve problems like height interpolation for digital terrain modelling [1,2] and for camera calibration [3,4].

The correction of systematic errors in height determined by the conventional parallax-height formula based on vertical aerial photography and parallax bar measurements introduced by different camera tilts which are not catered for in the formula have already been treated by Thompson [5] and Methley [6]. The height correction used by Thompson [5] and given as:

$$dh = a_1 + a_2X + a_3Y + a_4XY + a_5X^2 \tag{1}$$

is based on analysing the tilt effects and formulating a polynomial of second degree in X to correct the x-parallaxes that lead to the errors in heights. This is still the most popular solution for this problem. Methley [6] used the same principle, adding a term including the second degree in Y which was assumed to be of negligible effect by Thompson and hence used the formula:

$$dh = a_1 + a_2X + a_3Y + a_4XY + a_5X^2 + a_6Y^2 \tag{2}$$

The author [7] has also used Sheperd's interpolation given by:

$$f(x,y) = \left(\sum_{i=1}^{n} \frac{F_i}{r_i^{\mu}} \right) / \left(\sum_{i=1}^{n} \frac{1}{r_i^{\mu}} \right) \qquad \text{for } r_i = 0 \tag{3}$$

$$= F_i \qquad \text{for } r_i = 0$$

where $r_i = \sqrt{\left[(X_i - X)^2 + (Y_i - Y)^2 \right]}$

and μ is factor ≥ 0 in which weights of distances to control points are used in the interpolation.

Although the parallax bar is an approximate method for height determination the case might arise where one would have no alternate method to determine ground height since this method uses the cheapest instrumentation in the field of photogrammetry. Also, the erroneous model obtained from parallax bar measurement in the presence of tilt effect may represent the shape of erroneous model^ that could be obtained using other nonconventional photography and the success of using the finite element technique in correcting this model would indicate the possibility of its use with other types of photography. In this paper, the finite element approach would be applied for the correction of heights obtained by the conventional parallax-height formula.

In this case, a model domain is divided into sub-domains or finite elements and a linear polynomial will be used to describe the corrected surface in a piecewise fashion where each piece is composed of a triangle, thus reducing the number of unknown parameters and consequently the number of the required control points (three control points for a triangular piece).

2. The Finite Element Technique

The basic principle is to represent approximate solutions F_h and test functions f_h by polynomial defined piecewise over geometrically simple subdomains of some region R_h, with R_h in the X-Y plane. The first target is to choose a function that will be general enough to model irregular domains but consists of elements (δR) simple enough to minimise computational effort.

As shown in Fig. 1 simple triangles and/or quadrilaterals can be used for this purpose. A linear polynomial function in two dimensions is of the form:

$$f_h(X,Y) = a_1 + a_2X + a_3Y \tag{4}$$

where a_1, a_2 and a_3 are the three polynomial constants and X,Y are the point coordinates in the model system. Thus, three independant values of f_h must be specified to determine these constants which means that the elements should have three nodes, suggesting a triangle with nodes at the verticles. It also means that three height control points at the triangle verticles should be known to determine the three polyno-

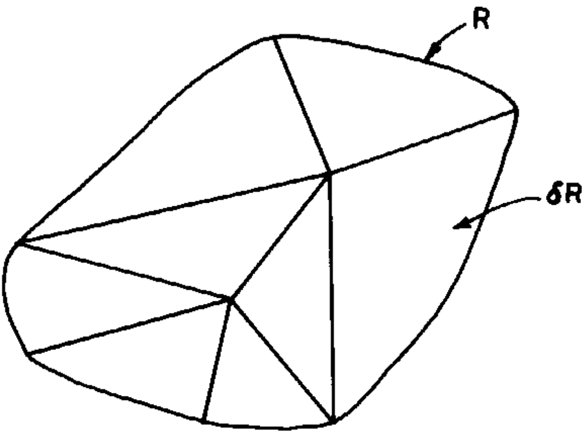

Fig. 1. Region R divided into subdomains δR

mial constants. Moreover, if two adjacent triangles in the mesh share one side and hence share two nodes a function can be produced if the functions f_i and f_{i+1} on the two triangles have the same values on the boundary between the two triangles. This, of course can be achieved when the two functions have the same values at the common nodes, Becker *et al.* [8], or the two common height control points in our case.

3. Practical Tests

Two stereomodels formed by photography taken by the Wild RC10 camera of focal length 153.03 mm, over an area on Appezell, Switzerland were used in this project to check the applicability of the finite element approach and to compare the results obtained to those achieved by the conventional tilt correction.

The first model, called model 1, is composed of an overlapping pair taken from an altitude of 2910 meters above average ground surface; while the second model, called model 2, is composed of an overlapping pair taken from an altitude of 2130 meter above average ground surface.

Each of the two models has been oriented in the laboratory and stereometer readings were recorded for 19 points in model 1 Fig. 2 and 15 points in model 2 Fig. 3.

The basic parallax height relation given by Thompson [9]:

$$h_{IA} = \frac{(H_0 - h_A)\, dP_{ia}}{P_a + dP_{ia}} \tag{5}$$

has been used to determine crude heights for all points in each model, with one central point selected as reference (control) point.

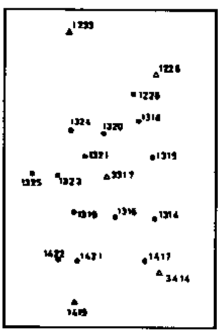

Fig. 2 . Model 1, distribution of control (△) and check points (•)

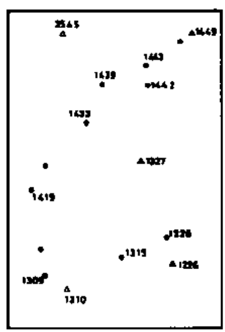

Fig. 3 . Model 2, distribution of control (△) and check points (•)

Conventional polynomial correction given in equation (1) has been used to correct the crude heights in which case the control points distribution shown in Fig. 2 and Fig. 3 were adopted for models 1 and 2 respectively.

Each model was then divided into piecewise triangles the vertices of each triangle are three of the mentioned control points, and the correction linear polynomial formula given in (4) involving only three polynomial constants have been used. All points within each triangle were used as check points. The heights of all points used in these tests (check and control points) were provided by the Wild Company together with the photography for the purpose of training and research. These heights have standard errors of ±0.02 m on the ground and are thus used as true heights to analyse the results. Figs. 4 and 5 show the piecewise triangles adopted for models 1 and 2 respectively.

The results of the tests described above are given in Table 1. For each solution the standard error of the computed heights of the check points is given as per thousand of flying height for the corresponding model.

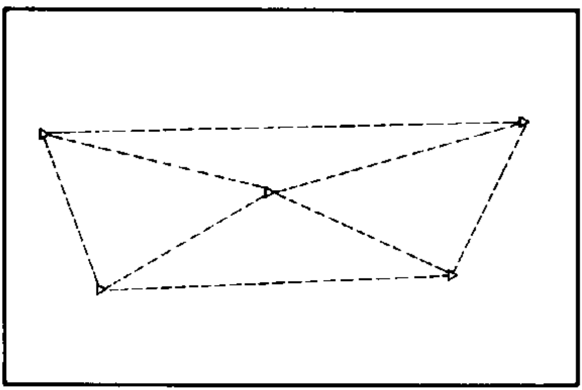

Fig. 4 . Model 1, piecewise triangles (*5 control points*)

Table 1. Standard errors (σ) ‰ H in height for models 1 and 2

Solution / Model	Conventional polynomial 5 parameters	Finite Element 3 parameters	Number of check points
Model 1	0.630	0.330	14
Model 2	1.005	0.745	10

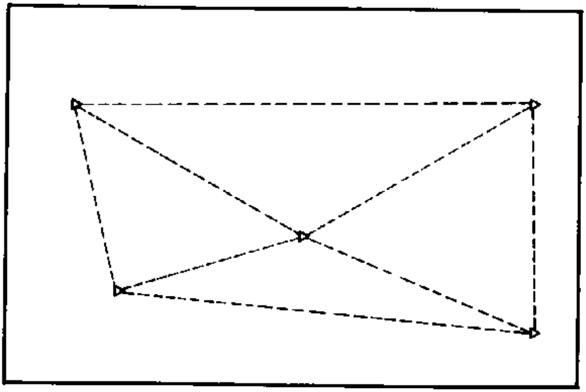

Fig. 5 . Model 2, piecewise triangles *(5 control points)*

From this table it is clear that the finite element approach could improve the results of both models. For Model 1, the standard error was improved from 0.63 ‰ H to 0.33 ‰ H that corresponds to about 43 % improvement. For Model 2, the standard error has been improved from 1.005 ‰ H to 0.745 ‰ H which implies an improvement of 26 %. Although, the number of check points tested in Model 1 is greater than that for Model 2, the results of Model 1 show very high improvement when the finite element approach is used.

The number of available ground control for the models under test is quite limited to use more than the minimum number since some of these points are to be used as check points. It is also true that the use of more control points is not economical since that would add more ground survey work for establishing control. However, one test has been carried out for each model using seven control points and applying the finite element method. Figures 6 and 7 show the pattern of the piecewise triangles for models 1 and 2 respectively when 7 control points are used. The results of the test are given in Table 2.

Table 2. Standard errors (σ) ‰ H in height using finite element method and 7 control points

Model	Standard Error in Height ‰ H
Model 1	0.346
Model 2	0.659

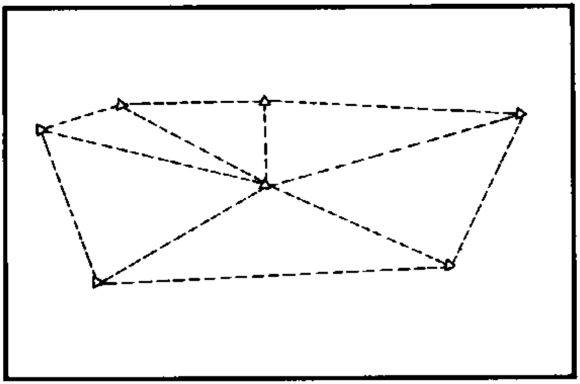

Fig. 6 . Model 1, piecewise triangles (*7 control points*)

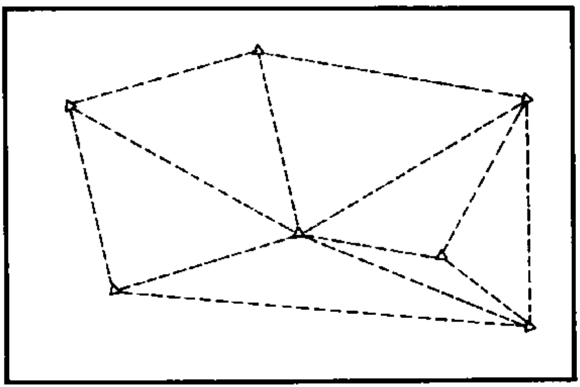

Fig. 7 . Model 2, piecewise triangles (*7 control points*)

When these results are compared with those in Table 1, where 5 control points are used, the addition of two more control points would improve the result of model 2 from 0.745‰ H to 0.659‰ H which corresponds to 11.5% improvement. While model 1 was not highly affected and the result even deteriorates by 4.5%, from 0.330‰ H to 0.346‰ H. This could only be justified by the fact that the optimum result have been reached, using the minimum number of control points.

4. Comparison with Other Results

Some previous work on heighting accuracy from parallax bar measurements would be reported here for comparison. The results of the work done by Methley using large scale photography (1:500) and reported by Cheffins [10], is summarised in Table 3.

Table 3. Standard height errors σ ‰ H - Methley

No. of control points	No. of check points	Standard error ‰ H
5	19	1.29
7	17	0.38
9	15	0.29
10	11	0.31

The standard height error obtained by Lo [11] after carrying out four tests on building height determination from 1:10000 scale aerial photography using parallax bar measurement and the direct parallax-height relation of equation (5), was 1.17‰ H. While the work done by the author [7] on a 1:12000 scale photography gave standard height error of 0.7‰ H after applying Shepard's interpolation using five control points and 14 check points.

Since two models have been tested in this paper, giving standard errors of 0.33‰ H and 0.745‰ H, the weighted standard error for the two models can be determined to be 0.503‰ H.

In comparison with the results of other tests where only five control points were used, the finite element appraoch gives the best result (0.503‰ H) followed by Shepard's interpolation (0.70‰ H) and then Methley's test (1.29‰ H). However, when the number of control points is increased to 9 Methley's results showed the best accuracy improvement, from 1.29‰ H to 0.29‰ H.

5. Conclusion

The finite element technique has been adopted to correct the crude heights obtained from the conventional parallax height relation which is subject to tilt effects. Practical tests have shown that the standard error in height after applying this technique varies from 0.33‰ H to 0.745‰ H when using 5 control points, introducing an improvement varying from 26 % to 48 % to results obtained by conventional method of height correction.

References

[1] **Ebner, H.** and **Reiss, P.** "Height interpolation by the method of finite elements." Presented Paper ASP, *Digital Terrain Symposium*, St. Louis, (1978).

[2] **Ebner, H.** and **Reiss, P.** "Experiences with height interpolation by finite elements." *ASP-ACSM Fall Technical Meeting*, San Francisco-Honolulu, (1981).

[3] **Munjy, R.A.H.** "Calibrating non-metric cameras using the finite element method." *Photogrammetric Engineering and Remote Sensing*, **52**,8 (1986), 1201–1205.

[4] **Munjy, R.A.H.** "Self calibration using the finite element approach." *Photogrammetric Engineering and Remote Sensing*. **51**,3 (1986), 411–418.

[5] **Thompson, E.H.** "Corrections to X-parallaxes." *Photogrammetric Record*, **6**,32 (1968), 202–210.

[6] **Methley, B.D.F.** "Heights from parallax bar and computer." *Photogrammetric Record* **8**,47 (1970), 563–582.

[7] **El-Hassan, I.M.** "A Photogrammetric Application of Shepard's Interpolation." *The Sudan Engineering Society Journal*, **28** (1985), 5–8.

[8] **Becker, E.B.**; **Carey, G.F.** and **Oden., J.T.** *Finite Elements*, Vol. 1. London: Prentice-Hall, 1981.

[9] **Thompson, E.H.** "Heights from parallax bar measurements." *Photogrammetric Record*, **1**,4 (1954), 38–49.

[10] **Cheffins, O.W.** "Accuracy of heighting from vertical photography obtained by helicopter." *Photogrammetric Record*, **6**,34 (1969), 379–381.

[11] **Lo, C.P.** "Determining and Presenting the Third Dimension of a City Centre." A Photogrammetric approach, *Photogrammetric Record* **6**,36 (1970), 625–639.

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استخـدام طريقة العنصر المحدود لتصحيح الارتفاعات الناتجة من استعمال ذراع القياس

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ملخص البحث: لقد تم في هذا البحث استخدام طريقة العنصر المحدود لتصحيح الارتفاعات التي يتم إيجادها عن طريق العلاقة الهندسية بين ارتفاع الجسم وإزاحة صورته الناتجة من تغير مكان التقاط الصورة والمقاسة بذراع القياس، تلك العلاقة التي تعتمد على افتراض أن الصورة رأسية تماما.

وقد دلت الاختبارات المعملية التي أجريت خلال البحث على أن استخدام طريقة العنصر المحدود تعطي نتائج أدق وعمليات حسابية أقل من طريقة التصحيح المعروفة والتي تستخدم المعادلات متعددة الحدود وذلك مع استخدام العدد نفسه من نقاط التحكم.