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CIVIL ENGINEERING

A Parallax Bar Heighting Experiment with Metric Camera and Large Format Camera Photography

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Abstract. A photographic stereopair of the Metric Camera (MC) and another one of the Large Format Camera (LFC) covering parts of the Red Sea Hills in Sudan were evaluated for height accuracy using the parallax bar heighting technique. X-parallaxes of a number of points on the two stereomodels were measured and used to compute the crude heights of the points. Corrections to the crude heights were then computed using different mathematical algorithms and a set of well-defined control points. Discrepancies between corrected (computed) heights and known heights of another set of points were used to compute the root-mean-square errors of height measurement. The results show that Shepard's interpolation or a higher degree polynomial can produce height errors of the order of ± 53 m. This error value may serve a useful purpose in some basic thematic applications where high height accuracies are not of paramount importance.

Introduction

The use of the parallax bar in conjunction with a mirror stereoscope in obtaining object heights using x-parallax measurements on a stereopair of near vertical photography is one of the basic practices in photogrammetry. It has long been acknowledged as a useful and inexpensive heighting technique. However, its application outside the educational environment has been surprisingly limited. This is because, this technique is capable of producing heights of only discrete points on the models which means that an additional interpolatory stage is essential if any sort of contour plotting is to be carried out. Further, parallax bar heighting has been accorded a reputation of low accuracy mainly because it is usually carried out with prints rather than film or glass diapositives. In many application fields, however, stringent accuracy figures are not of paramount importance. Areas such as geology, forestry, hydrology, geographic and reconnaissance surveys are examples in which parallax bar heighting is often used to determine object heights to a fairly modest degree of accuracy using stereoscopic pairs of aerial photography.

In the area of remote sensing, although satellite images have already played an important role in providing planimetric image maps and photomaps for many application areas, parallax bar heighting had been virtually ignored due to the fact that these satellite images lack some basic metric properties most favoured by photogrammetrists. With the launch of the European Metric Camera (MC) and the NASA Large Format Camera (LFC) in 1983 and 1984 respectively, it was possible to explore the potential of metric photographs taken from space platforms for heighting using the parallax bar. A modest research experiment had, therefore, been initiated in the Department of Surveying Engineering of the University of Khartoum, Sudan, to investigate the accuracy of parallax bar heighting using stereopairs from MC and LFC photographic coverage over Sudan. The present paper reports the results of this experiment.

Description of Test Materials, Areas and Instruments

The MC stereopair used in this experiment was selected from a MC strip covering parts of the Red Sea Hills in eastern Sudan. The imagery was acquired in November 1983. The stereomodel was formed from photos 91 and 92 of the strip and the two photos were in the form of third generation black and white positive films at a scale of 1:820000. The B/H ratio for this photography was in the order of 0.3.

The LFC stereopair covered another part of the Red Sea Hills region. It was formed from photos number 110 and 111. The photography was acquired during the Space Shuttle mission No. ST-41G in October 5th 1984. The scale of the photos was 1:780000 and the B/H ratio was in the order of 0.6. This stereomodel was formed from 23 cm × 23 cm format photos (centre half frames).

The topography on the test areas is rather varied in nature. There are numerous hills and rock features, a good deal of depressions (wadis) and dry streams and some rather extensive alluvial plains with low seasonal vegetation growing on them. Terrain elevations range from around 610 m at the bottoms of depressions to nearly 1300 m, and from around 600 m to well above 1650 m for the test areas covered by the MC model and the LFC model respectively.

The instrument used for observation was a standard Wild ST4 mirror stereoscope with a 250 mm fixed stereoscopic distance, 8X magnification oculars and pentaprisms. The stereoscope was equipped with a cantilever stand to allow x-parallax observations in an unobstructed space below the stereoscope. The parallax bar used in conjunction with this stereoscope reads to 0.01 mm and had a maximum range of

270 mm. In addition, the stereoscope was equipped with a board of transparent material which allows observation of x-parallax on films or diapositives, a property most favourable for the circumstances of the present experiment. The board had a parallel guidance mechanism so that stereoscopic observation was not disturbed by movement.

Mesurement of Coordinates and X-parallaxes

To aid incorrect interpretation of points to be measured, a 5X enlargement of each photo of the two models was prepared. A field trip of three days duration was made to the two test sites to resolve some ambiguities in interpretation. A photo interpreter with nearly 23 years experience carried out the measurements of this experiment. Actually, he was the person who originally derived photogrammetrically the coordinates of some of the points used in this experiment many years before. Furthermore, he is a native of the province containing the two test areas.

On the MC stereomodel, initially 23 points were selected. Some of the terrain heights of these points were interpolated from 1:100000 scale maps of the area; (this is the largest coverage scale available in that part of the country); others were well-defined points whose heights were derived photogrammetrically. A well-defined and targeted trigonometric point was chosen as a reference point relative to which all parallax measurements were referred.

The corresponding number of points selected on the LFC stereomodel was 28. Heights were also either interpolated from 1:100000 scale maps or derived photogrammetrically in a separate operation many years before. The reference point on this model was not as sharp as that on the MC stereopair but was believed to be reasonably defined (it was the centre of a village school with white metal roofs and white-painted brick walls surrounding it).

Each photograph employed in this test had to be baselined and then properly aligned under the mirror stereoscope before accurate x-parallax measurements could be obtained. This process had been carried out with as utmost care as possible. Photographic coordinates of control and check points were measured using a Houston Hipad digitizer reading to 10 micrometers. The measured photographic coordinates of the points were refined with the application of an affine transformation based on the known coordinates of the calibrated fiducial marks of the space photos. The parallax of each point was measured six times. The standard deviation from the mean was then computed for each point. Any observation showing a discrepancy greater than $\pm 3e$, where e is the standard deviation from the mean, was rejected. The mean of the accepted parallax readings was then taken to represent the parallax reading at that particular point (in fact only 3 points on the MC model and one point on the LFC model were rejected on this basis making the number of points showing acceptable parallax readings on the two models 20 and 27 respectively).

Mathematical Formulations Used

Conventional methods

The basic equation for height differences using changes in the x-parallax measured on a photographic stereopair is given by:

$$\triangle h_{AB} = P_{AB}(H - h_A) / (P_A + \triangle P_{AB}) \quad (1)$$

In equation (1), A is a reference point of known height h_A. B is a point whose height h_B is to be determined, $\triangle h_{AB}$ is the height difference between points A and B, H is the flying height above a certain datum, P_A is the absolute parallax of reference point A and $\triangle P_{AB}$ is the difference in measured parallaxes of points A and B. Equation (1) is used to obtain what is called *crude height* which must be refined to give *corrected height*. The following adjustment procedure was developed by Thompson [1]:

$$dh = a_0 + a_1x + a_2y + a_3xy + a_4x^2 \quad (2)$$

to correct for displacements due to tilt, relief and earth curvature. In equation (2), dh is the correction to the crude height; x, y are model coordinates (in mm) with the origin at the centre of the base-line; and a_i are unknown parameters to be computed from a set of known height points. It is clear that a minimum of 5 well-distributed control points is required to solve for the parameters of equation (2).

Methley [2] modified Thompson's formula by adding an extra term to reduce the effect of differential ω; thus:

$$dh = a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2 \quad (3)$$

Equations (2) and (3) are most familiar in the educational environment where they appear in numerous courses and texts on photogrammetry. They are used in this experiment, to correct computed crude heights, together with the following algorithms.

The shepard interpolation method

Basically, this is an interpolation method for fitting a surface to a given data. In this method, if G is a domain in the (x,y) plane and F is a real function defined on G, and if values of $F_j - F(x_j,y_j)$ of F are known at some set of points (x_j, y_j) located in G(j = 1,2, ... ,n), then the problem is to find a function f defined on G which reasonably approximates F. If, furthermore, R is the usual distance metric in the plane (x,y) and given a point (x,y) and $r_j = R[(x,y), (X_j,Y_j)]$ for j = 1,2, ... ,n and for $0 < \mu < 1$, then Shepard's interpolation for the problem is:

$$f(x,y) = \frac{\left[\sum_{j=1}^{n} \frac{F_j}{r_j^{\mu}}\right]}{\left[\sum_{j=1}^{n} \frac{1}{r_j^{\mu}}\right]} \qquad \text{when } r_j = 0$$

$$= F_j \qquad \text{for } r_j \neq 0 \qquad (4)$$

The value of $f(x,y)$ at the nondata points is obtained as a weighted average of all the data values, where the j^{th} measurement is weighted according to the distance of (x,y) from point (x_j, y_j). The value of μ is normally taken less than 1 to prevent cusps at data points [3]. The method is, therefore, a simple arithmetic mean interpolation approach which does not require many control points for the solution of a problem.

Higher degree polynomial

This algorithm has been used by the present author in an other investigation concerned with evaluating the efficiency of different mathematical interpolations for heighting using the parallax bar technique and conventional aerial photos [4]. It has the following form:

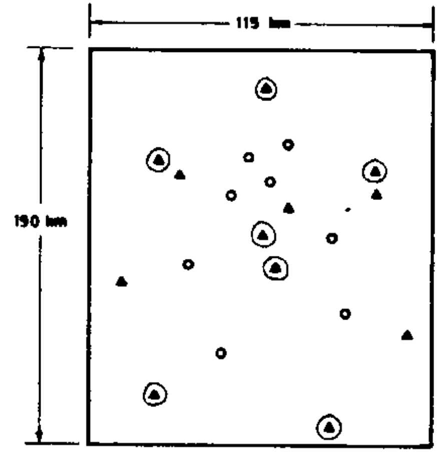
$$dh = a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2 + a_6x^2y + a_7y^2x + a_8x^2y^2 \qquad (5)$$

where dh, x, y and a_i are as before.

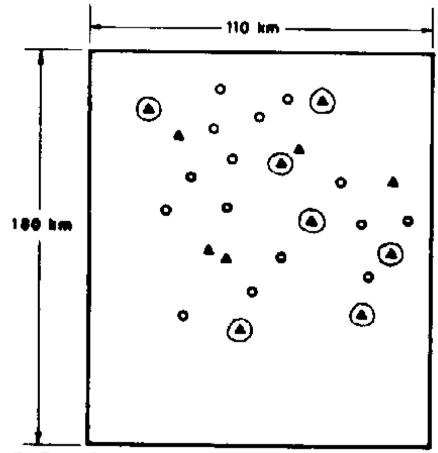
It was hoped that this extended polynomial would minimize, in addition to the effects of tilts and relief, the appreciable effects of earth curvature and atmospheric refraction to be anticipated in such metric space photography.

Method of Computation, Results and Analysis

At the initial stage of computation, and because of the large coverage of the two stereomodels, a zone-to-zone transformation was carried out on measured terrain coordinates. It is clear that at least nine vertical control points are required to solve for the parameters of equation (5). For both stereomodels, twelve points were used as data points and the parameters were computed using least squares techniques. The computed parameters were then substituted back into equation (5) to compute height corrections for the check points. The latter were chosen in such a way that, as best as possible, they always remained between control points so as to avoid many check points being extrapolated (Fig. 1). Further, the terms of equation (5) were truncated one by one proceeding from highest to lowest order and the program was run again to compute the corrections for the crude heights of the check points.



(a) Control distribution on the MC model



(b) Control distribution on the LFC model

Fig. 1. Control distribution used in the test
- Control point used with shepard's solution
▲ – Control (or data) point
○ – Check point

The height corrections of the check points were applied to the corresponding values of the computed crude heights to obtain the corrected heights as derived from the parallax bar experiment. These were then compared with their known equivalents and the discrepancies used to compute the root-mean-square errors of height measurement for each mathematical model used. Table 1 presents the results of the experiment.

Table 1 shows that for the MC model, the best result was obtained when the 9-term polynomial was used. Elimination of the last two terms reduces the heighting accuracy but rather insignificantly. When the sixth term suggested by Methley was truncated, thus limiting the polynomial to the form developed by Thompson, a nearly two-times drop in heighting accuracy occured. This suggests the importance of the differential ω-term which should always be added to the standard adjustment equation developed by Thompson [1].

On the LFC model, the best result was again obtained when the full 9-term correction polynomial was used. Again, when the differential ω-term (a_5y^2) was removed, the heighting accuracy deteriorated very badly (more than two times). On both models, Shepard's interpolation technique, although a simple algorithm, gave height accuracy values that approximate those obtained with the higher degree polynomials. Thus, values of ±62.2 m(±0.27 0/00 of flying height) and ±53.3 m(±0.23 0/00 of flying height) were obtained. The use of this simple technique is, therefore, recommended for computing corrections for crude heights in parallax bar heighting using metric space photography.

Comparison with other Tests

A similar work to the present experiment was carried out by Lo [5] on 3X enlarged pair of LFC photos of Boston and Providence areas in U.S.A. A specially designed precison parallax bar (the zoom height finder) with an enlarged micrometer head reading to 1 micron was used in conjunction with a conventional stereoscope with 6X magnification oculars. A number of control distribution patterns were used. With 6 control points and 13 check points, the accuracy obtained was ±63.0 m (±0.27 0/00 H) using Methley's form of the correction polynomial. With 9 control points, and using least squares, Methley's form gave a poorer accuracy value of ±91.1 m (equivalent to ±0.40 0/00 H). The conventional correction equation developed by Thompson [1] gave a much poorer r.m.s.e. value (±310.9 m equivalent to ±1.35 0/00 H). However, in this case, Lo [5] used only five control points to solve for the parameters of the equation, thus contributing to this large error value. It is, therefore, evident that the results obtained in this study compare favourably with those obtained by Lo [5] when using Methley's form of the correction equation. The superiority of the results obtained with other solutions of this experiment is clear from Table 1. Of course, all these results are inferior to those obtained by Rackham

Table 1. Heighting accuracy values obtained after different solutions

	MC Model		No. of control points	No. of check points	LFC model		No. of control points	No. of check points
Mathematical algorithm	r.m.s.e. m	0/00 H			r.m.s.e. (m)	0/00 H		
Shepard's formula	± 62.2	± 0.27	7	13	± 53.3	± 0.23	7	20
9-term polynomial	± 60.2	± 0.26	12	8	± 50.9	± 0.22	12	15
8-term polynomial	± 69.1	± 0.30	12	8	± 50.8	± 0.22	12	15
7-term polynomial	± 65.7	± 0.29	12	8	± 62.1	± 0.27	12	15
6-term poly (Methley's)	± 77.9	± 0.34	12	8	± 58.3	± 0.25	12	15
5-term poly. (thompson's)	± 138.5	± 0.60	12	8	± 119.1	± 0.52	12	15

[6], Dowman [7], Davidson [8], Elniweiri [9], Derenyi [10] and Murai [11] who used much more advanced instruments and sophisticated analytical control extension procedures.

Conclusion

The paper reported the results of an experiment concerned with deriving height information from metric space photography using the simple and inexpensive technique of the parallax bar. The results of the experiment have shown that MC and LFC photography can produce stereoscopic images of sufficient quality which allows use of a standard mirror stereoscope of 8X magnification oculars and standard parallax bar. Height information to an accuracy of around ±50 m for the LFC model and around ±65 m for the MC photography could easily be obtained depending on the correction polynomial used to refine crude heights. This means that the simple analogue mapping technique of using a mirror stereoscope and a parallax bar can be utilized economically for mapping at 1:100000 scale with a contour interval of 100 m to 130 m. This may be an important conclusion for those earth scientists concerned with basic thematic mapping applications such as geology, hydrology, geography, etc. where stringent height accuracy values are not of paramount importance. The results of the present experiment may also serve a useful purpose in some developing countries where topographic coverage is either very poor and outdated or simply absent.

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تجربة في حساب ارتفاعات النقاط باستعمال ذراع القياس وصور الكاميرا المترية والكاميرا ذات التغطية الواسعة

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ملخص البحث. نموذجان صوريان من الكاميرا المترية الفضائية الأوربية والكاميرا المترية الأمريكية ذات الحجم الواسع يغطيان أجزاء من منطقة تلال البحر الأحمر في السودان أخضعا لتجربة علمية لتقويم استعمالها في حساب ارتفاعات النقاط باستعمال جهاز ذراع القياس. قيست الإزاحة الصورية السينية لعدد من النقاط التي تظهر على النموذجين واستعملت قيمها لاستنباط الارتفاعات التقريبية لهذه النقاط. برمجت معادلات رياضية مختلفة واستعملت مع بعض نقاط التحكم الرأسية في إيجاد ارتفاعات نقاط أخرى معلومة الارتفاع على النموذجين. تم تقويم دقة صور الكاميرتين في إيجاد ارتفاعات النقاط بحساب الانحراف المعياري بين الارتفاعات الحقيقية للنقاط والارتفاعات المحسوبة والمستنبطة من التجربة. وقد أظهرت النتائج أنه بالإمكان استعمال هذه الصور الفضائية في تحديد الارتفاعات بدقة تصل إلى ± ٥٣ مترًا. وهذه الدقة قد تكون مفيدة جدًا في بعض الاستعمالات الموضوعية والتي لا تتطلب دقة عالية.