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Computer-generated Holograms: A Fast Ray-tracing Approach

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Abstract. A fast method that allows holograms to be efficiently generated by computer is described. The new method is about 42 times faster than the conventional ray-tracing algorithm with approximately the same resolution. The output of the new cut-shift algorithm-which we developed in this paper - has yielded excellent fidelity in the reconstructed images of two and three dimensional objects with simple and complex geometry.

1. Introduction

Ray-tracing is a convenient way to model optical systems using a computer. Computer-generate holograms (CGHs) potentially offer a new, valuable alternative method of data storage and display. Viewers gain the advantage of a three-dimensional (3D) data display which is useful in many fields, particularly those where 3D data sets are acquired by non optical methods, for example medical imaging, Computer-aided design (CAD) systems, information processing, etc. Computergenerated holograms are generated without neither a laser nor any associated optical instrumentation, thus escaping the constraints of conventional optical holography. The concept of using computers to define and generate hologram was introduced over twenty years ago. An important advantage of computer generated hologram is that the object to be reconstructed need not exist physically, only mathematically [1-3].

We aim to develop a complete holography model using computer only, including generation of holograms for 2D and 3D object models and reconstruction of their virtual images. In this paper we propose an efficient high speed method for CGH. Then we can input the 3D surface coordinates of a real object into computer memory. We may use these data as the boundaries to generate a body-fitted grid system for computer simulation, to modify these for computer animation, or to use these for pattern recognition of objects. What distinguishes the algorithm presented here is its simplicity and speed.

2. Methodology of Hologram Generation

In both holography and photography, light waves reflected from an object expose a photographic plate. Unlike conventional photography, however, the light wave in holography are coherent (generated by laser), and a portion of the light from the source is directed toward the film to serve as a reference beam. The resulting pattern on the plate is a set of fringes from the interference between the object and reference beams. These fringes contain not only information about the intensity of the light reflected from the object (as in regular photography), but also additional information enables one to use two-dimensional media (film) to record spatial information from 3D objects. The phase shifts correspond directly to path length differences of the lights, hence the added dimension of distance is captured. In computer simulations of holograms by ray tracing, the complex amplitudes that reach the plane of the photographic film from different directions are summed up [4].

The object beam is modeled as if the object were made of point sources. The light wave from a point source can be written as:

$$E_O = A_O \exp(ikR) / R, \tag{1}$$

where K is the wave number and A_O is the light wave amplitude which is real $exp\ (ikR)$ is the phase factor. The time-dependent phase $exp\ (-iwt)$ is ignored in this model since it does not change the final result. R is the distance between the hologram plane and the point $(x, y, z,)$ on the object.

$$R(x, y, z, \zeta, n) = [(x - \zeta)^2 + (y - n)^2 + z^2]^{1/2}, \tag{2}$$

where (ζ, n) is the coordinate on the hologram plane. The x - y plane is parallel to the ζ - n plane, the z- axis is chosen to be perpendicular to the hologram and the hologram is set at z = o . For simplicity we use a reference beam of normal incidence and described as:

$$E_R = A_R \exp(ikz) \tag{3}$$

So E_R is always a constant and at the hologram plane $E_R = A_R$.

The total amplitude of the light reaching any particular point on the plate may be found by summing all the rays reaching that point of the hologram plate as :

$$E_T = E_R + E_{o1} + E_{o2} + \ldots = E_R + E_{oT}, \tag{4}$$

where

$$E_{OT} = \sum_{j=1}^{N} \frac{Aj\exp(ikRaj)}{Raj}; \tag{5}$$

j is the index for each object point, a is the index for each hologram point, R_{aj} is the distance between the object and the hologram plate, R_j is the amplitude of each object point which may be computed from the placement of the object beam to the object and the reflectivity of the surfaces of the object and N is the number of object points.

The total intensity of the beam, with the asterisk denoting the complex conjugate, is :

$$I = E_T E_T^* \tag{6}$$

As the optical holography records only intensity on film, and as the intensity in the interference pattern depends on the phase distribution of the light from the object, computations are more efficient if the object beam is described in terms of only real numbers. From a single point in the object to a single point on the hologram plate, the beam intensity is described as :

$$I_\alpha = (A_{jcos}(kR_{\alpha j})) / R_{\alpha j}, \tag{7}$$

which may be arrived at by the following.

Since from eqs. (4 - 6) the total intensity of the beam at point α on the hologram plate is :

$$I_\alpha = \left(E_R + \sum_{j=1}^{N} \frac{A_j exp(ikR_{\alpha j})}{R_{\alpha j}} \right)^* \times \left(E_R + \sum_{j=1}^{N} \frac{A_j exp(ikR_{\alpha j})}{R_{\alpha j}} \right) \tag{8}$$

$$= \left(1 + \frac{1}{E_{R^*}} \sum_{j=1}^{N} \frac{A_j exp(-ikR_{\alpha j})}{R_{\alpha j}} \right) \times \left(1 + \frac{1}{E_R} \sum_{j=1}^{N} \frac{A_j exp(ikR_{\alpha j})}{R_{\alpha j}} \right) E_R^* E_R \tag{9}$$

In the worst-case, where all phases in the higher-order terms contribute maximally and R is the minimum distance between the object and the plate which is settled at z=0, the above equation can simply becomes:

$$I_\alpha \propto 1 + \frac{2}{A_R} \sum_{j=1}^{N} \frac{A_j cos(KR_{\alpha j})}{R_{\alpha j}} + O\left(\frac{A_0^2}{A_R^2 R^2} \right) N^2 , \tag{10}$$

where last term denotes the " order of " symbol. Note that the higher-order term has a maximum value of

$$(A_0^2 / A_R^2 R^2) N^2$$

In constructing a conventional hologram using a laser, we would use a reference beam much stronger than the object beam. This is mathematically modeled by the assumption

$$A_R >> \sum_{j=1}^{N} \frac{A_j exp(ikR_{\alpha j})}{R_{\alpha j}} \tag{11}$$

This assumption explains why object-object beam interference terms do not appear in the equations. Object - object beam interference contribution is far smaller than the object-reference beam interference that is used to generate the intensity fringe patterns on the hologram plate. Since the reference beam is far stronger that the object beam, the last term in eq. (10) can be dropped yielding the approximation

$$I_\alpha \propto 1 + \frac{2}{A_R} \sum_{j=1}^{N} \frac{A_j cos(KR_{\alpha j})}{R_{\alpha j}} \tag{12}$$

This equation is used for hologram generation by ray tracing using the computer simulation [4,5].

3. Fast Computation Algorithm for Hologram Generation

For accelerating the calculation of eq(12), we propose here a new algorithm consists of two simplifying procedures.

The first procedure is to save the time for calculating cosine function at all the points located in the same object plane that is parallel to the hologram plane and just to calculate it only at the central point of this plane. Then, simply by shifting indices of this point, we can calculate I_α for other points which are located in the same plane [6-8]. For example, as shown in Fig. 1, if (ζ_i, n_j) on the hologram plane is selected for object point (i, j), then $(\zeta_i + dx, n_j + dy)$ should be selected for another object point $(i + dx, j + dy)$. if dx and dy are chosen to be an integral number of the discrete grid spacing on the hologram, this procedure is quite simple as follows:

1- $(I(im), J(im))$ are the coordinates of (I, J) on the hologram plane selected as they are located in the circle of cosine function for one point (i, j) on the object plane. Where $im = 1, 2, \ldots\ldots, m$ and m is the total number of selected hologram point.

2- For another point $(i + dx, j + dy)$, $(I(im) + dx, J(im) + dy)$ will be selected.

This procedure is repeated for all object planes parallel to the hologram plane. By this procedure, we practically reduce calculation time by about 2.6 times less than the conventional method.

The second simplifying procedure is to reduce the spatial range of the hologram points into a size sufficient to generate the hologram pattern for a required object. The hologram generated by one point shows Fraunhoufer diffraction pattern. Its part of larger radius possesses the information of small scale. Therefore, neglect of that part will not harm the overall image of reconstructed object. This justifies the use of only a central part of Fraunhoufer diffraction pattern. Thus, even if the hologram size is 512 x 512 points, we can select the points of only 128 x 128 as the "stencil" and

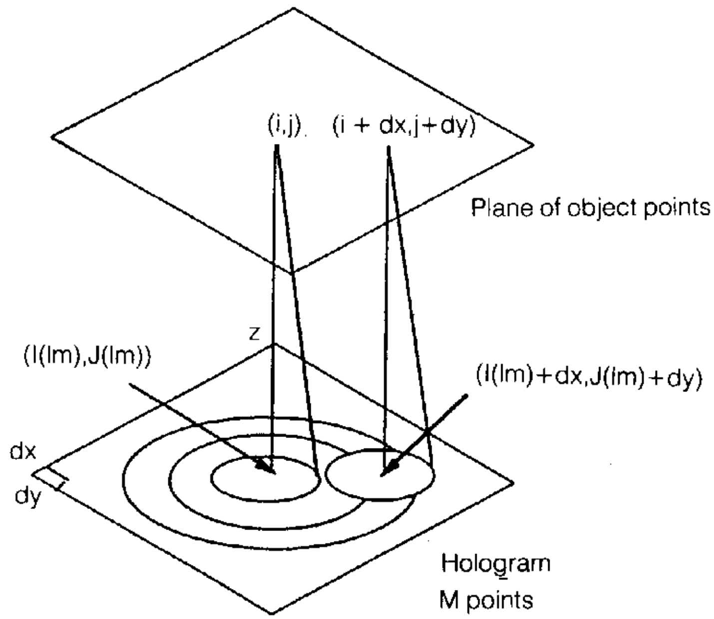

Fig. 1 . Algorithm for selecting pointx on the plane of hologram

successfully reconstruct the object, then reducing the calculation by 16. This procedure depends on the object size and the distance between the object and the hologram.

In the following we summarize the improvement in the speed by using the new algorith relative to eq. (12):-

Number of calculation using eq. (12) = N x M

Number of calculation using first simplifying procedure (shift - procedure) only ≈ (N x M) / 2.6

Number of calculation using the two simplifying procedures (cut and shift procedures) ≈ (N x M) / 2.6 x (*cut_ratio*),

where M is the total number of hologram points and *cut_ratio* is the ratio between the size of the "stencil" to the total size of the hologram pattern. For example if the hologram size is 512 x 512 but the size of "stencil" 128 x 128, then the *cut_ratio* = 1/16. It is clear that our method in this example will decrease number of calculation by about 2.6 x 16 ≈ 42 times less than the full calculation of eq. (12). We shall call this fast technique as a " cut-shift" algorithm.

4. Performance Evaluation

We can characterize the performance of methods used for CGHs by two factors. First factor is the resolution (R) of the virtual image reconstructed from hologram generated by these methods. and second factor is the total computational time (T) used to generate this hologram pattern. The factor (R) is defined as the ratio of the similar pixels in both the object model and the reconstructed image relative to the total number of pixels in the reconstruction area (where reconstructed area is the selected area for searching the object points in the reconstruction procedure).

In order to evaluate the performance of the new method, we have generated hologram patterns for many 2D and 3D object models with the conventional method (which is fully calculate eq.(12)) and with the new method. We will discuss here three cases. First and second case the object model is in 2D plane perpendicular to the z-axis. Firstly, the object is a circle model which consists of 26 separated points and secondly, the object is a car model with size 138 x 42 in x, y directions respectively consisting of 493 points. The third case is 3D object model, as a pyramid made of 7 frames consisting of 529 points whose size gradually shrinks in the z-direction. Fig.2 shows the plot of hologram patterns for the pyramid, as a example for the above three cases. Fig.2 (a) and (b) show the plot of hologram patterns using full calculation and new methods respectively. The hologram size, for the three cases, is 512 x 512 and the size of "stencil" in the new procedure is 128 x 128. From Fig.2, we notice that the two hologram patterns have approximately the same resolution, except at the edges of the pattern of the new method. The same observation holds for the hologram patterns of the circle model and the car model. However, these changes in edges have no effect in the reconstruction of virtual images as shown latter.

In order to evaluate the performance of the two methods depending on factor (R), we have reconstructed the holograms thus obtained using the equation [5-8]:

$$\phi_i = \sum_{\alpha} \phi_{i\,\alpha} = \sum_{\alpha} I_{\alpha} \cos\left(kR_{i\,\alpha}\right) / R_{i\,\alpha}, \tag{13}$$

where ϕ_i is the reconstructed image. This reconstruction method, using the eq. (13), has been proved to have much better resolution than the FFT method [8].

We applied this reconstruction method for the holograms obtained from the above three object models and have the results which are shown in Fig. 3, 4 the resolution

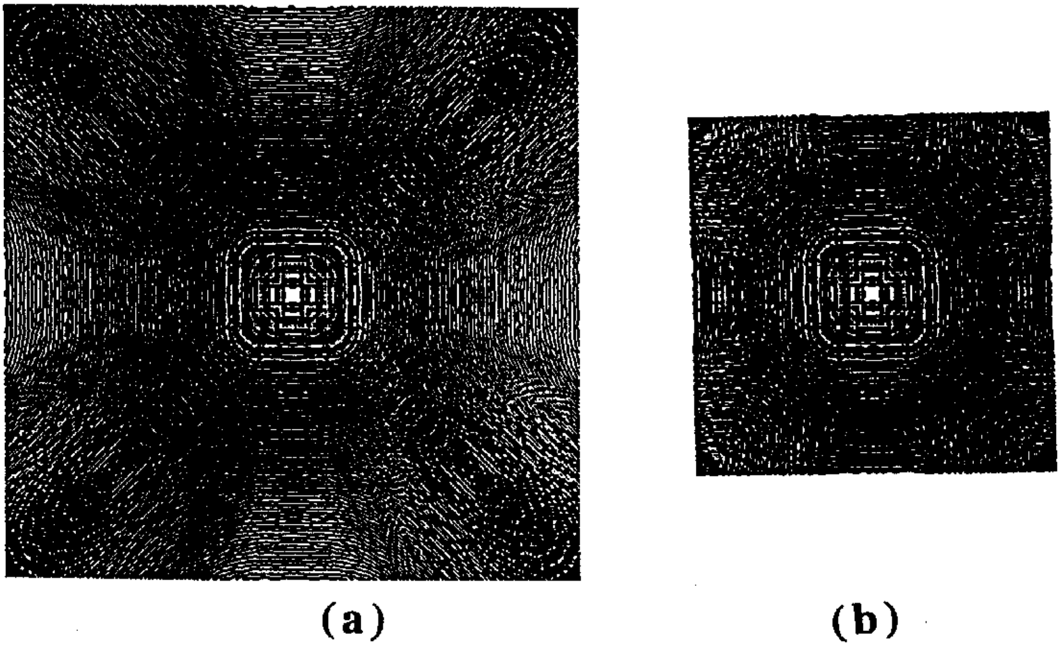

Fig. 2 . The plot of computer-generated holograms for pyramid model with (a) conventional method and (b) present method.

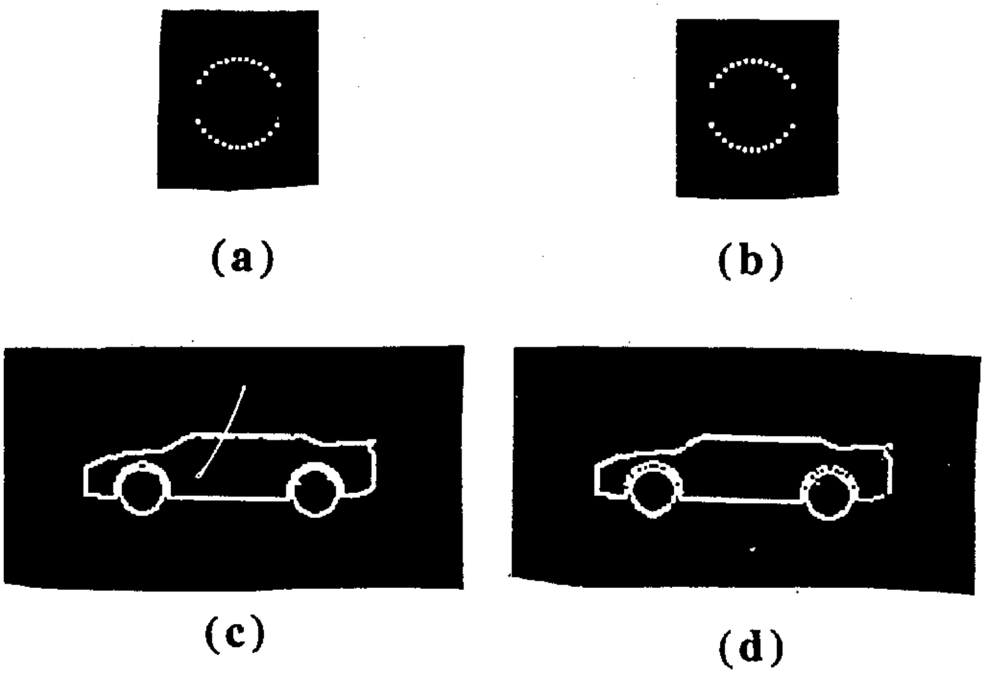

Fig. 3 . The virtual images for circle model, using the hologram patterns generated by (a) conventional method and (b) present method, the virtual images for car model, using the hologram patterns generated by (c) conventional method and (b) present method

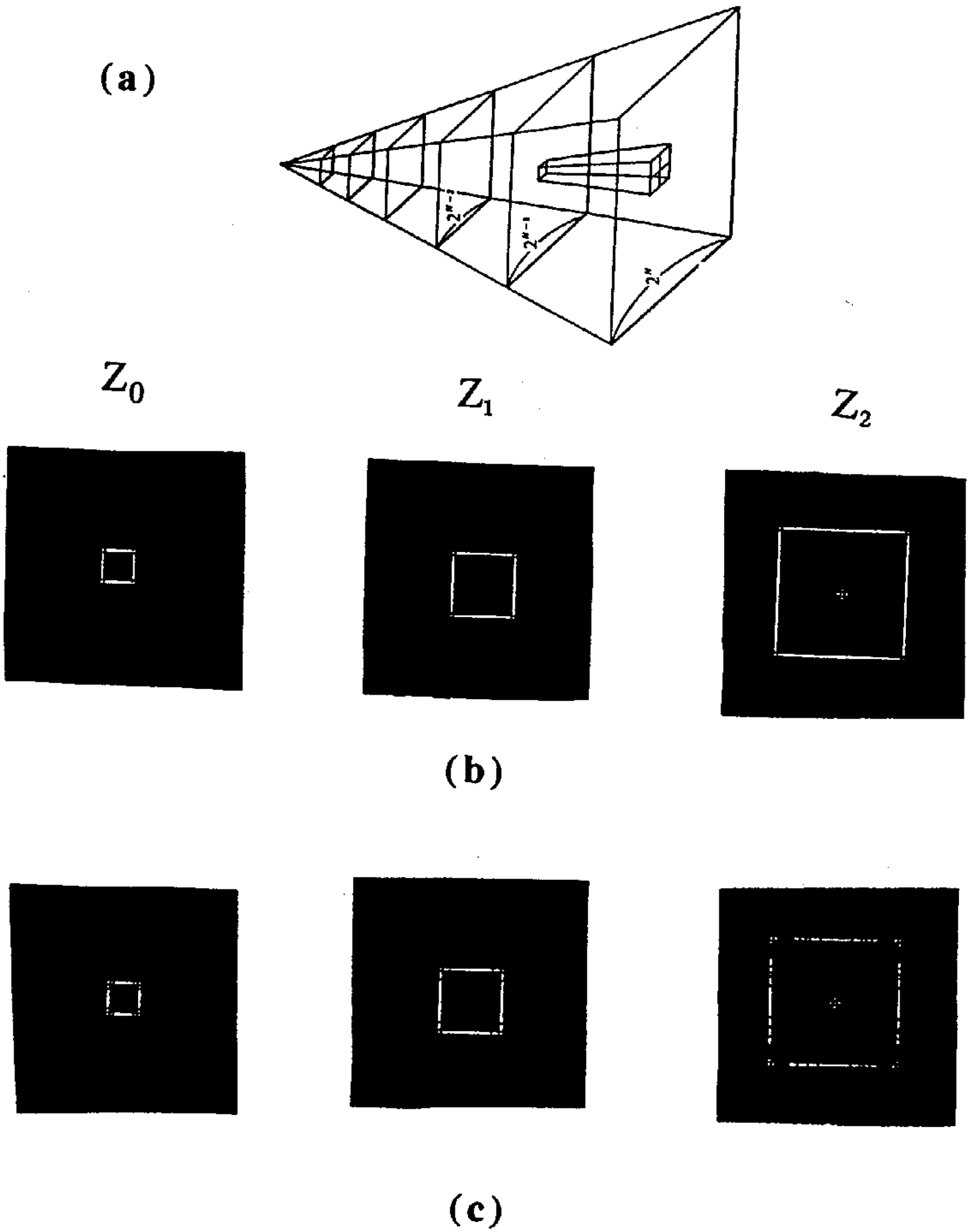

Fig. 4 . (a) 3D pyramid model and its virtual images using the hologram patterns generated by (b) conventional method (c) present method with different cross-sections planes in direction

characteristic (R) is listed in Table 1. From Fig.3 (a), (b), and Table 1 the results whow that the two CGH methods give the same resolution (R) for the circle model, although we use two different hologram patterns. The same notice approximately exists in Fig.3 (c), (d), and Table 1 for the car model. Since the virtual image is purely 3D for the pyramid model (as shown in Fig.4 (a)), cross-sections of the object are retrieved on the planes placed at several points Z_0, Z_1 and Z_2, as shown in Fig.4 (b) and (c). The same notice for resolution factor (R) approximately exists in Fig.4 and Table 1 for the pyramid model.

Table.1. The list of the resolution factor (R) for the threestudies cases.

Object mode	Reconstruction area	R (CON)	R (NEW)
Circle model	50 × 50	100 %	100 %
Car model	256 × 256	96.7 %	96.15 %
Pyramid model	256 × 256	93.77 %	92.61 %

This results proved that our method succeeded in generating holograms for any object in 2D or 3D having simple or complicated geometry, with approximately the same resolution as conventional method.

Since the calculation speed, for our method, has been greatly improved, it would be useful to compare the computational time characteristic (T) between the two methods on scalar computer HP730. Fig.5 shows the computational time (T) with different hologram sizes generated by the conventional method and the new method, applied on the circle model as an object for simplicity. It is clear from Fig.5 that the conventional method has for longer computational time (T) than the new method, specially at large sizes of the hologram pattern.

As a performance evaluation view, this great gain in time (T) with approximately the same reconstructed image resolution (R) as the conventional one, that makes our method to be practically acceptable as CGH fast efficient method.

5. Conclusion

We have provided a high-speed method for computer-generated hologram with about 42 times faster than the previous method. The speed will increase further when the object size increases. Combined with the correlation method [5- 8] in reconstruction, the new method has approximately the same resolution as the conventional one for

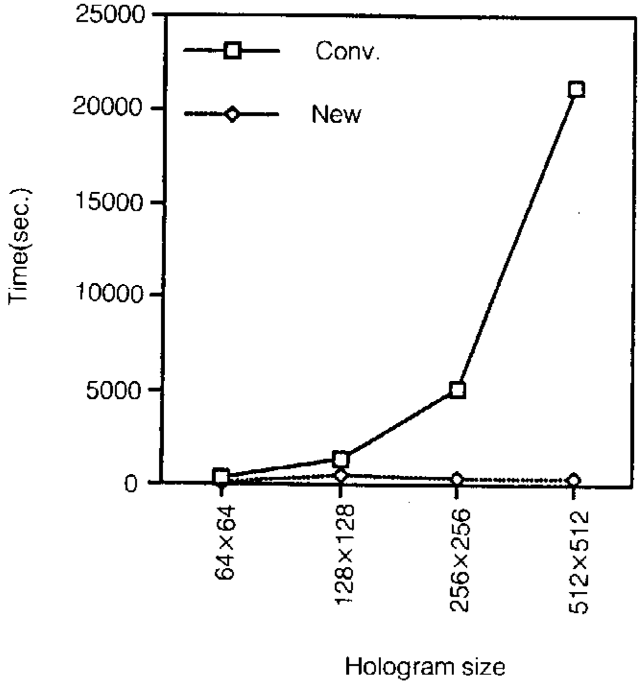

Fig. 5 . A comparison of computational time (T) between conventional method and the present method for hologram generation

reconstructed images and provides a highspeed, complete 3D holographic system allowing flexible viewing angles, immediate reconstruction with no special hardware requirements and absolutely none of the noise problems associated with optical holography arrangements.

The most important point is that our method can used as massively j parallel system without any difficulty and the increase in speed is linearly proportional to the number of machines.

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توليد الهولوجرامات بالحاسوب : طريقة متابعة بالشعاع سريعة

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ملخص البحث . تصف هذه الورقة طريقة سريعة وفعالة لتوليد الهولوجرامات بالحاسوب. الطريقة الجديدة أسرع ٤٢ مرة من خوارزمية المتابعة بالشعاع التقليدية وتعطى نفس الدقة تقريباً. خوارزمية القطع والإزاحة التي نصفها في هذه الورقة أعطت تطابقاً ممتازاً مع أصل صور الأشياء ثنائية البعد وثلاثية البعد ذات الهندسة البسيطة أو المعقدة والتي أعيد رسمها بالخوارزمية المقترحة.