

## **Solving Stochastic Machining Economics Problem Using Simulation Optimization Approach**

**Abdulrahman. M. Al-Ahmari**

*Industrial Engineering Department, College of Engineering, King Saud University  
P.O.Box 800, Riyadh 11421, Saudi Arabia  
alahmari@ksu.edu.sa*

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**Abstract:** The emerging of simulation and optimization approaches achieves a rapid growth in recent years. In this paper, the new concept of simulation optimization is applied to solve machining economic problem with stochastic tool life for turning operations. The developed simulation optimization model determines the optimal values of process parameters and tool replacement interval with practical constraints when the objective is to minimize the total production cost or total production time per workpiece taking into consideration the maintenance and quality costs. In practice, stochastic models of machining systems are more applicable than deterministic models due to the probabilistic nature of tool life. The developed model is validated and demonstrated using numerical examples. Extensive sensitivity analysis is provided to consider the effect of model inputs on the optimal results.

**Keyword:** Machining Economic, OptQuest, Simulation Optimization, Stochastic tool life, Scatter Search, Arena.

### **Introduction**

In most of manufacturing organizations, optimization of manufacturing operations is one of the greatest targets of Computer Aided Process Planning (CAPP) which is major component of Computer Integrated Manufacturing (CIM). Numerous variables are affecting the economics of manufacturing operations (particular machining operations) such as tool materials, machine tool capability, cutting conditions, and specifications of workpiece material. With the advent of sophisticated and highly expensive Numerical Control (NC) machines and FMS into the shop floor, the need for the optimal utilization of these resources is necessary. This requires not only the consideration of the machining economics in the selection of cutting conditions but other cost factors should be considered such as maintenance and quality costs. Thus the optimal process parameter selection becomes one of the important functions of the process planning activity in manufacturing systems.

The level of tool maintenance can have a significant impact on both the efficiency of the cutting process and the likelihood of tool failure; this constitutes the incorporation of process control in

machining economics (Hui et al. 2001). In addition, the absence of proper consideration of quality cost when determining optimal machining conditions is likely to result in inappropriate decisions.

Various optimization techniques for selecting process variables were developed. Lambert and Walvekar (1978) developed a dynamic programming model for the multipass turning operation under constraints of force; cutting power and surface finish to determine values of machining variables that give minimum production cost. They considered two-pass turning operations. Ermer and Kromodihardjo (1981); and Gopalkrishnan et al. (1991) considered the single and multipass turning operations based on some practical constraints related to process variables (speed, feed, cutting tool life, cutting force and surface roughness) to find the minimum production cost or time. Shin and Joo (1992) presented a model for the multipass turning operation using a fixed machining interval. They used dynamic programming for the selection of depth of cut for individual passes. Gupta et al. (1995) considered the optimal subdivisions of depth of cut in machining economic problem using two steps. The first step is the minimization cost for rough and finish passes for various fixed depth of cut.

In the second step, an optimal combination of depths of cut for rough passes and the finish pass, the optimal number of passes and the minimum total cost are determined using an integer programming model. Lee et al. (1999) developed a fuzzy non-linear programming model to optimize machining operations. Arezoo et al. (2000) developed an expert system for selection of cutting tools and conditions of turning operations to select the toolholder, insert and cutting conditions (feed, speed and depth of cut). They used dynamic programming to optimize cutting conditions. Al-Ahmari (2001) presented a mathematical programming model for optimizing the process parameters and subdivisions of depth of cut in multipass turning operations. The model is a direct non-linear mathematical model that solves the optimization problem of multipass turning operations providing the considered decision variables (cutting speed, feed rate, depth cut, subdivision of depth of cut, and number of passes) for both finishing and rough cutting.

Very few studies considered the tool life variability in the machining economic problem. Iakovou et al (1996) proposed analytical models and numerical procedures for simultaneously determining the optimal cutting speed and tool replacement policy in machining economics problems with stochastic tool lives. Their model is an unconstrained optimization model and is based upon the basic Taylor tool life equation. Wysk et al (1978) presented a mathematical model for the selection of optimal machining parameters taking into account the probabilistic nature of the tool life and the probability of catastrophic tool failure. They used the minimum production time criterion and a search technique to find solution. Jianqiang and Keow (1997) used a lognormal distribution to fit the tool life data by wear and derived a model for determining optimal tool replacement intervals coupled with a forecasting tool replacement strategy. Doyle (1973) presented a theoretical analysis to determine the optimum cutting speed, considering variable tool life for each of three common tool replacement policies. Kaspi and Shabatay (2003) minimized the expected cost per unit under three different tool replacement strategies: Failure Replacement, Opportunistic Replacement and Integrated Replacement. For the first strategy, they used a one-dimensional search algorithm to find the optimal solution. For each of the other two strategies, they developed approximate models using simulation. Shabatay and Kaspi (2003) also presented models for calculating the optimal cutting feed rate, spindle speed, and age of preventive tool replacement for a standalone cutting machine. The optimal cutting

conditions are determined and analyzed for three different objective functions: minimum expected cycle time, minimum expected cost per unit, and maximum expected profit-rate, under the Age Replacement Strategy (ARS) and assuming that the tool-life distribution function is normal. They presented a one-dimensional search procedure for the optimization. Hui et al (2001) developed a time dynamic economic model for a single-pass turning. The model incorporates considerations on the stochastic nature of tool-life and some tool maintenance activities such as tool replacement and tool regrinding. They also modeled the quality cost of tool-cutting in terms of deviation from target roughness and deviation from target dimension.

The tool-life distribution function is a major factor that needs to be determined in order to construct an analytical model when considering the possibility of a preventive tool replacement strategy. Rosetto and Levi (1977) argued that wear processes cause failures according to normal or lognormal distribution functions, while the distribution function of a sudden failure by fracture and chipping is exponential. Their results indicated that the use of a Normal distribution function seems to be reasonable for the range of economic cutting speeds since, in this range, tool failure is primarily due to a conventional accelerated wear process in which the probability of tool failure usually increases with the accumulated tool cutting time. Sheikh et al. (1999) determined the optimal age of tool replacement for predetermined cutting conditions. LaCommare et al. (1983), Koulamas et al. (1987); Fenton and Joseph (1993) and Al-Ahmari (2004) used simulation, while Koulamas (1991) used geometric programming with a constraint that limits the probability of an unforeseen tool failure. Savsar and Kilic (1991) compared three types of tool replacement policies in the multi stage machining systems using simulation. Recently, Savsar (2006) also considered the effect of different maintenance policies on productivity of flexible manufacturing cells.

The level of cutting quality is another major endogenous issue in machining systems, which has received limited attention. It has been found that a workpiece surface quality significantly affects its proper functioning as the condition of the surface is related to the likelihood of failure produced by fatigue, creep and stress corrosion (Jang 1992). The absence of proper consideration of quality cost when determining optimal machining conditions causes inappropriate decisions. Quality of machined workpieces can be measured by the level of surface roughness and dimensional accuracy. These quality

issues are directly affected by the selection of process parameters (cutting speed and feed rate). Therefore, quality must be treated as a cost element and be included in the objective function of machining economic model. The capability of a cutting tool deteriorates over time as result of tool wear which affect quality of the machined workpiece. It is well known that the rate of deterioration also depends on the selection of process parameters.

In this paper, a simulation optimization model is developed to solve single-pass machining problem in turning operations with practical constraints when tool life is stochastic. In addition, to the operating and tool factors, the developed model incorporates tool failure and tool maintenance (tool replacement) elements due to the stochastic nature of tool life. The quality cost is included in the developed models based on the deviation from target dimension. The objective is the minimization of total production cost or time per workpiece including maintenance costs (tool failure and tool replacement costs) and quality costs (deviations from target dimension). The different machining economics problems constraints are taken into consideration in the developed model. These constraints are related to the process boundaries and limitations. The rest of the paper is organized as follows: Section 2, reviews simulation optimization approaches, Section 3, presents the formulation of the machining economic problem, Section 4 provides the developed simulation optimization model for the stochastic machining economic problem, Section 5 is devoted for the simulation experiments and model solutions. Section 6 presents a sensitivity analysis and finally, Section 7 concludes the paper and provides direction for future research.

### Simulation optimization

For several decades, simulation has been used as descriptive tool by the researchers in the modeling and analysis of complex real systems (Tekin and Sabuncuoglu 2004). Incorporating optimization features into simulation systems opens a door for simulation in terms of new application areas and research possibilities. With the ability of performing optimization, simulation can also become an operational tool to solve various short-term decision making problems as well as strategic and tactical problems (Tekin and Sabuncuoglu 2004).

Simulation optimization provides a structured approach to find optimal input parameter values, where optimal is measured by a function of the output

variables associated with a simulation model (Swisher et al. 2004).

Several researchers have presented excellent surveys on the topic of simulation optimization see for example Swisher et al. (2004); Tekin and Sabuncuoglu (2004); Boesel et al (2003); Fu (1994); Law et al. (2002); Fu et al. (2005); Kelton and Barton (2003); Fu (2002); Swisher et al. (2003); Carson and Maria (1997); Azadivar (1999) and Andradottir (1998) who surveyed simulation optimization literature since 1988. They covered several new advances in this subject including multiple comparisons with the best and metaheuristics such as tabu search and simulated annealing.

The commercial simulation software development has recognized the value of simulation optimization modules and add-ons to their simulation packages (Swisher et al. 2004). Fu et al. (2005) listed a number of simulation software packages which included optimization routines, as shown in Table 1.

**Table 1. Commercial implementation of evolutionary approach to simulation optimization**

Optimization Package (Simulation Platform)	Vendor	Primary Search Strategies
AutoStat (AutoMod)	AutoSimulation, Inc.	Evolutionary, Genetic Algorithm.
OptQuest (Arena, Crystal, Ball, et al.)	Optimization Technologies, Inc.	Scatter Search and tabu search, neural networks
OPITIZ (SIMUL8)	Visual Thinking International Ltd.	Neural networks
SimRunner (ProModel)	PROMODEL Corp.	Evolutionary
Optimizer (WITNESS)	Lanner Group, Inc.	Simulated annealing tabu search

### Arena and OptQuest

In this research we use Arena software package which has built in optimization systems, OptQuest. OptQuest allows the model builder to bound the search space for the input parameters to set a maximum search length and to return the parameters combinations that meet the model criteria (Rocwell Automation 2004). OptQuest for Arena now has a tree-structured user interface that displays the optimization model components (controls, responses, constraints, objectives, suggested solutions, and options). It depends on algorithm that incorporates combination strategies based on scatter search, and tabu search, along with neural networks for screening out candidates likely to be poor.

OptQuest enhances the analysis capabilities of Arena by searching for optimal solutions within the simulation models. Many simulation models are embedded in the broader context of a decision problem, where the ultimate goal is to determine the best values for a set of controls. This usually involves running a simulation for an initial set of decision variables, analyzing the results, changing one or more variables, re-running the simulation, and repeating this process until a satisfactory solution is obtained. This process can be very tedious and time consuming even for small problems and it is often not clear how to adjust the controls from one simulation to the next. OptQuest overcomes this limitation by automatically searching for optimal solutions within Arena simulation models. The optimization problem is described in OptQuest that searches for the values of controls that maximize or minimize a predefined objective taking into account the defined problem constraints.

Recent developments in the area of optimization have allowed for the creation of intelligent search methods capable of finding optimal or near optimal solutions to complex problems involving elements of uncertainty. Often, optimal solutions can be found among large sets of possible solutions even when exploring only a small fraction of them. OptQuest is the result of implementing these search technologies in combination with simulation models built for Arena. Once the optimization problem is described (by means of selecting controls, the objective, and possibly imposing constraints), Arena is called every time a different set of control values needs to be evaluated. The optimization method used by OptQuest evaluates the responses from the current simulation run, analyzes and integrates these with responses from previous simulation runs, and determines a new set of values for the controls, which are then evaluated by running the Arena model. This is an iterative process that successfully generates new sets of values for the controls, not all of them improving, but which, over time, provides a highly efficient trajectory to the best solutions. The process continues until some termination criterion is satisfied usually stopping after a number of simulations or when the OptQuest determines the objective value has stopped improving. Its ultimate goal is to find the solution that optimizes (maximizes or minimizes) the value of the model's objective. Once OptQuest exits, the controls in the Arena model are returned to their original default values. The Arena model is completely unaffected by OptQuest. More information about OptQuest optimizer can be found in

Rockwell Automation (2004). Figure 1 illustrates the linking between Arena and OptQuest.

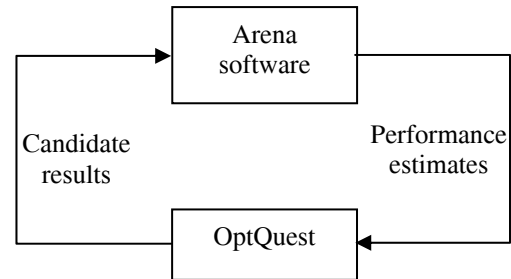


Fig. 1. Simulation optimization concept in Arena software.

### Model formulation

The following notation is used in this paper:

$a$ ,  $b$ , and  $\lambda$  Taylor tool life constants.

$C_D$  cost of out of specification workpiece (\$/Workpiece)

$C_f$  failure cost of the tool (\$)

$C_m$  operator/machine rate (\$/min)

$C_r$  cost of new tool (\$/tool edge)

$d$  depth of cut (mm).

$D$  Initial diameter of the workpiece (mm).

$d_{max}$  maximum allowable depth of cut (mm).

$d_{min}$  minimum allowable depth of cut (mm).

$E(C_{pr})$  The expected total production cost (\$/workpiece).

$E(D)$  The expected diameter of the machined component at end of tool life cycle (mm).

$E(P_u)$  The expected number of workpieces out of specification.

$E(t)$  The expected tool life (min)

$E(T_{pr})$  The expected total production time (min/workpiece).

$E(W)$  The expected tool wear of at the end of tool life cycle (mm).

$f$  feed rate (mmpr)

$f(t)$  probability density function of tool life normal distribution.

$F(T)$  tool failure function.

$F_{max}$  maximum allowable feed rate (mmpr).

$F_{max}$  maximum force.

$F_{min}$  minimum allowable feed rate (mmpr).

$L$  Length of the workpiece (mm).

$PW_{max}$  maximum power.

$R$  tool nose radius (mm).

$T$  tool replacement interval (min).

$t_c$  tool change time (min).

$t_h$  loading/unloading time (min).

$t_m$  machining time (min).

- $t_u$  time consumed in producing out of specification components (min).
- $t_z$  remaining tool life as result of early tool replacement (min).
- USL upper specification limit of the workpiece diameter (mm).
- V cutting speed (mpm)
- $V_{max}$  maximum allowable cutting speed (mpm).
- $V_{min}$  minimum allowable cutting speed (mpm).
- $\delta$  initial tool flank wear (mm)
- $\mu$  normal distribution mean.
- $\theta$  tool flank wear (mm)
- $\sigma$  tool life standard deviation.
- $\vartheta_0, \psi_0, \theta_1$  and  $\theta_2$  cutting force and power constants.

Additional notation will be introduced later as needed.

The problem under study in this paper is a single pass machining economic problem with practical constrained and stochastic tool lives. The problem also includes the maintenance and quality costs. The objective is to determine the optimal machining conditions including cutting speed, feed rate, and the tool replacement interval in order to minimize the total expected production cost or total production time per workpiece.

The total expected production cost per workpiece ( $E(C_{pr})$ ) = Loading/Unloading cost ( $A_o$ ) + machining cost ( $A_1$ ) + tool replacement cost ( $A_2$ ) + tool failure cost ( $A_3$ ) + quality cost ( $A_4$ ).

The total expected production time per workpiece ( $E(C_{pr})$ ) = Loading/Unloading time ( $B_o$ ) + machining time ( $B_1$ ) + tool replacement time ( $B_2$ ) + tool replacement time with failure ( $B_3$ ) + time lost in producing out of specifications parts ( $B_4$ )

$$\text{Therefore, } E(C_{pr}) = A_o + A_1 + A_2 + A_3 + A_4. \quad (1)$$

The terms of the total expected cost  $E(C_{pr})$  per expected tool life  $E(t)$  are described as follows:

- loading/unloading cost:  $A_o = C_m t_h$  (2)

- machining cost (AL-Ahmari 2001):  $A_1 = C_m t_m$ , where  $t_m = \frac{\pi DL}{1000Vf}$  (3)

- tool replacement cost per cycle (tool change cost + expected tool acquisition cost) (Al-Ahmari and Aziz 2006):

$$A_2 = (C_m t_c + C_r (1 - F(T)) t_m) / E(t) \quad (4)$$

- expected tool failure cost (Al-Ahmari and Aziz 2006; Hui et al (2001)):  $A_3 = C_f t_m F(T) / E(t)$  (5)

- expected quality cost is calculated using the average number of parts out of specifications ( $E(P_u)$ ) (Al-Ahmari 2006):

$$A_4 = C_D E(P_u) t_m / E(t) \quad (6)$$

Using equations (2)-(6), The objective functions in equation (1) can be rewritten as:

Min:

$$E(C_{pr}) = C_m t_h + C_m t_m + \frac{t_m}{E(t)} [C_m t_c + C_r (1 - F(T)) + C_f F(T) + C_D E(P_u)] \quad (7)$$

Subject to:

Limits of cutting speed:

$$V_{min} \leq V \leq V_{max} \quad (8)$$

Limits of feed rate:

$$f_{min} \leq f \leq f_{max} \quad (9)$$

Limits of depth of cut:

$$d_{min} \leq d \leq d_{max} \quad (10)$$

Cutting force constraint:

$$\vartheta_0 f^{\theta_1} d^{\theta_2} \leq F_{max} \quad (11)$$

Power consumption constraint:

$$\Psi_0 V f^{\theta_1} d^{\theta_2} \leq PW_{max} \quad (12)$$

Surface finish constraint:

$$0.0321 f^2 / r \leq R_{max} \quad (13)$$

The decision variables are V, f and T.

The description of the above constraints are well known and documented in the literature such as Al-Ahmari (2001) and Hui et al. (2001).

In the same manner the expected total production time per workpiece  $E(t_{pr})$  is computed as:

$$E(t_{pr}) = B_o + B_1 + B_2 + B_3 + B_4 \quad (14)$$

The details of these terms are described below:

- loading/unloading time:

$$B_o = t_h \quad (15)$$

- machining time (AL-Ahmari 2001):  $B_1 = t_m$ , where

$$t_m = \frac{\pi DL}{1000Vf} \quad (16)$$

- tool replacement time (with no failure):

$$B_2 = t_c(1-F(T))t_m/E(t) \quad (17)$$

- tool replacement time with failure:

$$B_3 = t_f t_m F(T)/E(t) \quad (18)$$

- time spent in producing out of specification workpieces (Al-Ahmari 2004),  $t_u$ :

$$B_4 = t_u \quad (19)$$

Using equations (15)-(18), The objective functions in equation (1) can be rewritten as:

$$E(t_{pr}) = t_h + t_m + \frac{t_m}{E(t)} \left[ t_c(1-F(T)) + t_f F(T) + t_u \right] \quad (20)$$

Subject to the constraints (8)-(13).

Optimization of the cutting conditions with stochastic tool life is much more difficult to achieve than optimization of the cutting conditions for the deterministic tool life due to considerations of the stochastic nature of other factors of the manufacturing process, such as maintenance and quality factors. Therefore, simulation optimization approach is the best way to solve this problem.

### Simulation optimization modeling

We consider that tool life follows the normal distribution with mean  $\mu$  and variance  $\sigma^2$ . That is,  $X \sim N(\mu, \sigma^2)$ . The mean  $\mu$  represent the physical tool life ( $\lambda/(v^a f^b)$ ).

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2} \frac{(t-\mu)^2}{\sigma^2}}, \quad t \geq 0. \quad (21)$$

$$E(C_{pr}) = C_m t_h + C_m t_m +$$

$$\begin{cases} \frac{t_m}{T} \left[ C_m t_c + C_r \left( 1 + \frac{1}{E(t) - T} \right) \right], & \text{if } \begin{cases} T \leq E(t) \\ D + W(T) \leq USL \end{cases} \\ \frac{t_m}{T} \left[ C_m t_c + C_r \left( 1 + \frac{1}{E(t) - T} \right) + C_D \frac{T - USL - D - \delta}{\theta t_m} \right], & \text{if } \begin{cases} T \leq E(t) \\ D + W(T) > USL \end{cases} \\ \frac{t_m}{E(t)} \left[ C_m t_c + C_r (1 - F(T)) + C_f F(T) \right], & \text{if } \begin{cases} T > E(t) \\ D + W(E(t)) \leq USL \end{cases} \\ \frac{t_m}{E(t)} \left[ C_m t_c + C_r (1 - F(T)) + C_f F(T) + C_D \frac{E(t) - USL - D - \delta}{\theta t_m} \right], & \text{if } \begin{cases} T > E(t) \\ D + W(E(t)) > USL \end{cases} \end{cases} \quad (27)$$

If  $\Phi(\cdot)$  is the cumulative distribution function of the standard normal. The cumulative failure distribution function  $F(t)$  is given as follows:

$$F(T) = \Phi\left(\frac{T - \mu}{\sigma}\right) \quad (22)$$

The expected tool life length is as follows:

$$E(t) = t \sim N(\mu, \sigma^2) \quad (23)$$

The total amount of linear tool wear at time  $t$  resulting from cutting process is obtained as follows:

$$W(t) = \theta t + \delta \quad (24)$$

Where the random variables  $\theta$  and  $\delta$  are flank wear and the initial tool flank wear, respectively. Then, the expected number of parts out of specifications ( $E(P_u)$ ) in equation (6) becomes:

$$E(P_u) = (E(t) - t_u)/t_m \quad (25)$$

Where,  $t_u$  is the production time needed to produce out of specification workpieces limit (Hui et al. 2001).

$$t_u = (USL - D - \delta)/\theta \quad (26)$$

The above equations are used in the objective functions illustrated in equations (7) and (18), taking into account the constraints (8)-(13). In simulation model the objective functions have four different cases, as shown in Eqs. (27) and (28).

In the above objective functions there four different cases that may be happen in practice:

1. Case 1: If tool replacement time is less than or equal to expected tool life,  $T \leq E(t)$ , and, the expected produced diameter of the finished workpiece is within specification limits,  $D + W(T) \leq USL$ , then there is amount of tool life is not

and

$$E(t_{pr}) = t_h + t_m + \begin{cases} \frac{t_m t_c}{T}, & \text{if } \begin{cases} T \leq E(t) \\ D+W(T) \leq USL \end{cases} \\ \frac{t_m}{T} [t_c + T - (USL - D - \delta) / \theta], & \text{if } \begin{cases} T \leq E(t) \\ D+W(T) > USL \end{cases} \\ \frac{t_m}{E(t)} [t_c + t_f F(T)], & \text{if } \begin{cases} T > E(t) \\ D+W(E(t)) \leq USL \end{cases} \\ \frac{t_m}{E(t)} [t_c + t_f F(T) + E(t) - (USL - D - \delta) / \theta], & \text{if } \begin{cases} T > E(t) \\ D+W(E(t)) > USL \end{cases} \end{cases} \quad (28)$$

utilized,  $t_z = E(t) - T$  and there are number of workpieces equal to  $(E(t) - T) / t_m$  are lost as result of un-utilizing  $t_z$ . The cost of this case is computed based on the portion of un-utilized tool life. The time function is similar to the classical machining economic time. This case is represented by the first choice in objective functions (27) and (28), respectively..

2. Case 2: If tool replacement interval less than or equal to expected tool life,  $T \leq E(t)$  and the produced workpieces are out of specification limit due to tool wear,  $D+W(T) > USL$ . In this case, tool life is not violated but the required dimensional tolerance of the produced part is exceeded. Therefore, the expected number of out of specification workpieces,  $E(Pu) = (T - USL - D - \delta) / \theta t_m$ . The quality cost is included in the cost function (second choice in equation (27)) and the time consumed in producing out of specification workpieces is considered in time function (second choice in Eq. (28)).
3. Case 3: if tool replacement interval is greater than expected tool life,  $T > E(t)$  and the produced workpieces are within specification limit,  $D+W(T) \leq USL$ , then tool failure is considered based on the cumulative density function of tool life. In this case, the cost and time associated with tool failure is included in the third choice of objective functions (27) and (28), respectively..
4. Case 4: if tool replacement interval is greater than expected tool life If  $T > E(t)$  and produced workpieces are out of specification limit,  $D+W(T) > USL$ . In this case then tool failure cost and time are included in the objective functions (forth choice). Also, the cost of produced workpieces out of specification limit and their associated processing times are included in cost and time functions (27) and (28), respectively.

The objective is to minimize (27) or (28) based on their above four cases, subject to the constraints (8)-(13). We use these formulations to build the simulation model. The flowchart of the developed simulation model is illustrated in Fig. 1.

OptQuest for Arena is run to develop the optimization model that is linked to the developed simulation model illustrated in Figure 1. OptQuest has a tree-structured user interface that displays the optimization model components (controls, responses, constraints, objectives, suggested solutions, and options) as shown in Fig. 2.

To the run the optimization model the following steps are implemented by OptQuest automatically:

1. Select controls for the optimization model: the controls of our problem are the decision variables: V, f, and T. The lower bound, suggested values, and upper bounds of the variables are given.
2. Select responses to show the resulting values or outputs from the Arena simulation. The responses selected for our problem are the average values of the minimum production cost and minimum production time per workpiece ( $E(C_{pr})$  and  $E(t_{pr})$ ).
3. Select the constraints to limit the search to solutions that satisfy these restrictions shown in equations (8)-(13).
4. Define the objective, ( $E(C_{pr})$  or  $E(t_{pr})$ ).
5. Set other optimization options. The simulation length is 1000 minutes and number of replications is (3 to 6). We accept the default setting for the rest of parameters.
6. Run the optimization model to display the progress of the search and to plot the best objective value for each simulation, as illustrated in Figure 3. At the completion of the optimization, the best solution is displayed. Figure 3 is generated automatically by OptQuest and updated after each simulation run to illustrate the progress of searching the best solution.

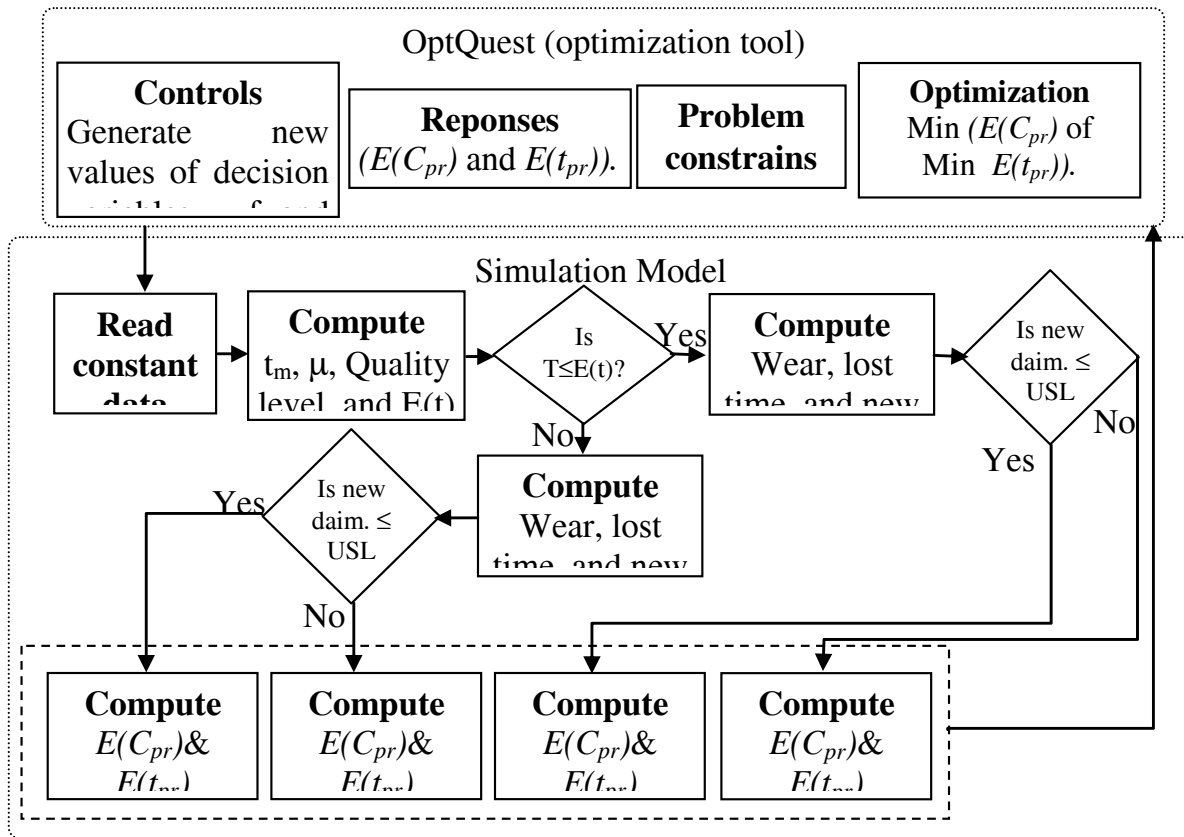


Fig. 1. Flowchart of the developed simulation optimization model for machining economic problem.

File Edit View Add Run Help

Controls User Specified

Controls User Specified

User Specified Summary								
Included	Control /	Element Type	Type	Low Bound	Suggested Value	High Bound	Step	Description
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<input type="checkbox"/>	cpr	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	cr	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	delta	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	diameter	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	extratime	Variable	Continuous	0	0	0	N/A	
<input checked="" type="checkbox"/>	Feed	Variable	Continuous	0.9	0.9	0.9	N/A	
<input type="checkbox"/>	finalcpr	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	finalcpr	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	lambda	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	length	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	losttime	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	maxtquality	Variable	Continuous	0	0	0	N/A	
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<input type="checkbox"/>	newdiameter	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	partsofcontrol	Variable	Continuous	0	0	0	N/A	
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<input type="checkbox"/>	sigma	Variable	Continuous	0	0	0	N/A	
<input checked="" type="checkbox"/>	speed	Variable	Continuous	1	60	500	N/A	
<input type="checkbox"/>	tc	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	th	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	theta	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	tm	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	toolife	Variable	Continuous	0	0	0	N/A	
<input type="checkbox"/>	toolskiameter	Variable	Continuous	0	0	0	N/A	

Add Control From Array

Select All Clear All Modify

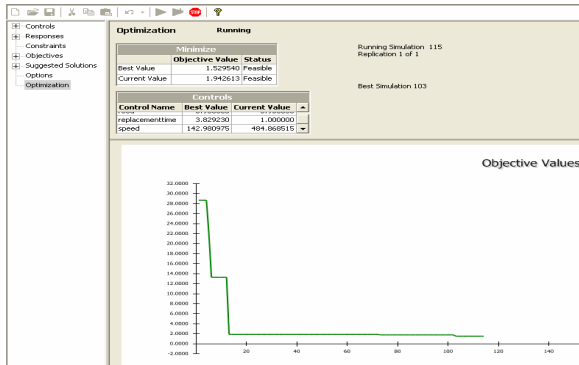


Fig. 3. Search for optimal solution in OptQuest.

### Simulation experiments

Simulation model is developed using Arena 9.0 and verified and validated using deterministic data. Replication length is 1000 min to make sure that all model reaches its study state. The simulation model is linked to OptQuest to find the optimal values of the decision variables that minimize the expected total cost,  $E(C_{pr})$  or expected total time,  $E(t_{pr})$  per workpiece. The selected controls in OptQuest are cutting speed ( $V$ ), feed rate ( $f$ ), tool replacement interval ( $T$ ) with their boundaries and the responses are  $E(C_{pr})$  and  $E(t_{pr})$ . The rest of constraints are presented in the Constraint section in OptQuest. In the objective section,  $E(C_{pr})$  or  $E(t_{pr})$  are selected to find optimal values of controls that minimize the expected total cost or time per workpiece. Stop option is set to 2000 runs and tolerance value (error) is 0.0001. Machining data used in simulation experiments are illustrated in Table 2.

Table 2. Data of the simulation model

$a$	1.2
$b$	0.1
$C_d$	20 \$/Workpiece
$C_f$	20 \$/Workpiece
$C_m$	0.5 \$/min
$C_r$	4.5 \$/edge of tool
$\delta$	0.0001 mm
$D$	50 mm
$\lambda$	9,100.00
$L$	200 mm
$\sigma$	3.5
$t_c$	2 min
$t_h$	1.0 min
$\theta$	0.001 mm/min
$t_e$	2 min
$t_f$	6 min
$USL$	50.02 mm
$\theta_1$	0.8
$\theta_2$	0.9
$\Psi_0$	4500

The developed simulation/optimization model is run to minimize  $E(C_{pr})$  and  $E(t_{pr})$ . The optimal results for both criteria are illustrated in Table 3.

It is clear from the above table that, the expected final workpiece diameter and tool wear are larger when using the minimum production costs criterion, than using minimum time criterion. Tool replacement time and the expected tool life are shorter when using the minimum production time criteria as result of increasing the value of cutting speed. The optimum value of minimum time cutting speed is greater than optimum value of minimum cost cutting speed.

Table 3. Optimal results for both criteria

Objective function	Min $E(C_{pr})$	Min $E(t_{pr})$
$E(C_{pr})$ \$/piece	1.337	1.723
$E(t_{pr})$ min/piece	2.271	2.188
$E(D)$ mm	50.010	50.006
$E(W)$ mm	0.010	0.006
$E(P_u)$	0.000	0.000
$T$ min	9.430	6.120
$V$ mpm	155.920	284.710
$f$ mmpr	0.900	0.900
$E(t)$ min	21.416	10.373

### Sensitivity analysis

In this section we consider the effect of changing model inputs on the optimal values of the decision variables and objective function, as illustrated in Table 4. The analysis indicates that if cost of tool,  $C_r$ , increases, the expected total production cost  $E(C_{pr})$  increases and production time,  $E(t_{pr})$  decreases. The expected diameter of the finished part and the expected number of parts out of control,  $E(P_u)$  increase as tool cost increases as result of trading off between tool cost and quality cost. Also, tool replacement interval,  $T$ , and cutting speed,  $V$ , also increase. The expected tool life,  $E(t)$  decrease due to increasing cutting speed,  $V$ .

The results show that there is no significant effect on changing the value of failure cost and quality cost because the optimal values of decision variables are still in the optimum range. It should be noted that if the tool change time increases, then  $E(C_{pr})$ ,  $E(T_{pr})$ ,  $E(D)$ ,  $E(W)$ ,  $T$ , and  $E(t)$  increase, and  $V$  decreases. Changing value of tool life standard deviation,  $\sigma$ , also has effects on the optimal values of the problem. The values of  $E(C_{pr})$ ,  $E(T_{pr})$ ,  $E(D)$ ,  $E(W)$ ,  $T$ , and  $E(t)$  increase, and  $V$  decreases as  $\sigma$  increases. There is no clear effect of changing values of  $USL$ . The results also illustrates that if wear rate,  $\theta$ , increases, the

		$E(C_{pr})$ \$/wps.	$E(T_{pr})$ Min/wps.	$E(D)$ mm	$E(W)$ mm	$E(P_u)$	$T$ min	$V$ mpm	$f$ numpr	$E(t)$ mm
$C_r$	4.5	1.337	2.271	50.010	0.010	0.00	9.43	155.92	0.90	21.42
	7	1.446	2.288	50.011	0.011	0.00	11.17	143.00	0.90	23.77
	15	1.560	2.242	50.012	0.012	5.11	40.44	255.18	0.90	11.83
	25	1.691	2.245	50.013	0.013	7.63	72.20	247.16	0.90	12.29
$C_f$	20	1.337	2.271	50.010	0.010	0.00	9.43	155.92	0.90	21.42
	40	1.336	2.266	50.010	0.010	0.00	9.51	158.82	0.90	20.95
	60	1.336	2.266	50.010	0.010	0.00	9.53	158.57	0.90	20.99
	80	1.336	2.266	50.010	0.010	0.00	9.53	158.55	0.90	20.99
$C_d$	5	1.336	2.266	50.010	0.010	0.00	9.53	158.58	0.90	20.98
	20	1.337	2.271	50.010	0.010	0.00	9.43	155.92	0.90	21.42
	35	1.336	2.266	50.010	0.010	0.00	9.52	158.61	0.90	20.98
$t_c$	2	1.337	2.271	50.010	0.010	0.00	9.43	155.92	0.90	21.42
	5	1.369	2.339	50.011	0.011	0.00	10.90	150.15	0.90	22.41
	7	1.390	2.382	50.012	0.012	0.00	11.62	146.22	0.90	23.14
	9	1.410	2.424	50.013	0.013	0.00	12.50	141.45	0.90	24.08
$\sigma$	3.5	1.337	2.271	50.010	0.010	0.00	9.43	155.92	0.90	21.42
	5	2.687	2.293	50.011	0.011	0.00	10.58	141.82	0.90	23.97
	7	1.805	2.359	50.013	0.013	0.00	12.89	112.50	0.90	31.65
$USL$	50.02	1.337	2.271	50.010	0.010	0.00	9.43	155.92	0.90	21.42
	50.05	1.336	2.266	50.010	0.010	0.00	9.52	158.63	0.90	20.98
	50.1	1.336	2.266	50.010	0.010	0.00	9.52	158.60	0.90	20.98
$\theta$	0.001	1.337	2.271	50.010	0.010	0.00	9.43	155.92	0.90	21.42
	0.005	1.379	2.228	50.020	0.020	0.00	3.98	232.10	0.90	13.26
	0.009	1.490	2.257	50.020	0.020	0.00	2.21	262.35	0.90	11.44
	0.01	1.512	2.237	50.020	0.020	0.00	1.96	310.36	0.90	9.39

values of  $E(C_{pr})$ ,  $E(D)$ ,  $E(W)$  and  $V$  increase, but values of  $E(T_{pr})$ ,  $T$ , and  $E(t)$  decrease.

### Conclusions

Optimization of the cutting conditions with stochastic tool life is much more difficult to achieve than optimization of the cutting conditions for the deterministic tool life. This requires considerations of the stochastic nature of other factors of the manufacturing process, such as maintenance and quality factors. This paper presents an application of concept of simulation optimization in complex machining systems problems. Machining systems are highly stochastic systems due to the nature of variability of tool life which has significant effects on production cost, time and quality. Solving such problems analytically is somewhat difficult. Therefore, it has been found in this research that the new trend of simulation optimization is capable of solving such stochastic machining system problems with the help of integrated simulation software that links discrete-event simulation tools with optimization tools such as Arena 9.

Using of simulation optimization approach in solving machining economic problems provides high flexibility in considering the different scenarios of the problem and high capability in representing the stochastic nature of tool lives using the convenient simulation probability functions. In the model developed, different responses of model variables can be considered by selecting the appropriate objective function in the optimization tool. This would simplify comparisons of the machining economic criteria against each other.

This paper shows that, the selection of machining conditions have effects not only on tool life, but also on the cutting quality of the tool during its life and tool maintenance. Further research can be done on the integration of machining economic problem with other planning problems in manufacturing systems utilizing the new concept of simulation optimization environment.

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## حل مسألة اقتصاديات التشغيل العشوائية باستخدام طريقة أمثلة المحاكاة

عبدالرحمن بن مشيب الأحمري

قسم الهندسة الصناعية، كلية الهندسة،

جامعة الملك سعود، ص ب ٨٠٠ الرياض ١١٤٢١

المملكة العربية السعودية

(قدم في ٢٠٠٧/١٢/٠٢ م؛ وقبل للنشر في ٢٠٠٨/٠٩/١٤ م)

الكلمات المفتاحية: اقتصاديات التصنيع، أمثلة المحاكاة، أعمار أدوات القطع العشوائية.

**ملخص البحث.** ظهر في السنوات الأخيرة وبشكل سريع طرق أمثلة المحاكاة. وفي هذا البحث تم استخدام المفهوم الحديث لأمثلة المحاكاة في حل مسألة اقتصاديات التصنيع عندما يكون عمر أداة القطع عشوائي. يحدد النموذج الذي تم تطويره في هذا البحث القيم المثلى لمتغيرات عملية التصنيع وكذلك زمن تغيير أداة القطع مع الأخذ في الاعتبار القيود الأخرى المتعلقة بالعملية عندما يكون الهدف تقليل وقت وكلفة الإنتاج لكل وحدة منتجة مع دراسة التكاليف الأخرى المصاحبة (تكاليف الصيانة والجودة). تعتبر النماذج العشوائية أكثر ملاءمة في المجال التطبيقي لهذه المسائل نظراً للطبيعة العشوائية لأعمار أدوات القطع. تم اختبار وعرض الطريقة المقترحة باستخدام مسائل رقمية. كما تم تحليل الحساسية لدراسة تأثير مدخلان النموذج على قيم النواتج المختلفة.