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Evaluation of the Blocking Probability of Limited Access Networks

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Abstract. We derive the probability of a customer finding no available circuit in a limited access network. We analyze a limited access network with a finite number of circuits and several classes of customers. Each class of customers has an upper bound on the number of circuits being simultaneously used by customers of that class. This upper bound may differ from a class to another. Several intelligent networks are nowadays adopting this method of access since it allows them to control the cost of communication. A common example is the Private Virtual Network which often has this capability. We provide an exact solution and a divide and conquer algorithm for determining the probabilities of finding no available circuit for each class of customers. These probabilities are essential in specifying the circuit requirements in the network.

1. Introduction

A private virtual network (PVN) is a public intelligent network capability that offers business customers the features they expect of private networks, including the ability to control their communications costs. In addition, by selectively limiting access of each individual PVN customer,the PVN architecture provides an inherent protection for the public network. To be specific, customers can potentially share all network circuits but are constrained to at most a customer-chosen limit on the number of calls in progress. One of these customers can be considered the general public whose calls in progress are limited only by the total network capacity.

A resource counter is a software device that blocks a customer's calls after that customer's number of calls in progress reaches a specified value, called a resource limit.These resource counters can emulate the finite capacity among all customer locations on a private network and are an optional feature of PVN (see [2,5]).

The network provider has a different point of view, that of assuring all customers, including the general public, to receive the grade of service corresponding to their resource limits. While resource counters give customers a way to limit their costs, they also offer the network provider the capability to protect the service of individual customers.

Although the concepts developed in this paper apply to resource limits of customers, the customers could equally well represent distinct transport services which are offered on a single integrated network.

Although the problem of calculating the blocking probability for each class of customer with resource limits is not new, there have been no efficient and exact solutions presented for realistic multi-server problems. Cooper [3] presents it only as a textbook problem whose solution can be formulated as the product form for closed network of queues.

Mitral [9] published a derivation of solution bounds whose computational complexity is polynomial in the number of customers.

For direct routing only, Bachle [1] derived a formula for the exact blocking probability but presented an efficient solution only for an approximation.

Kamoun an Kleinrock [6], among other authors, published an efficient algorithm for the exact blocking probability for a special case of a single-server problem which evaluated computer buffer capacity for several different storage schemes.

The emphasis in this paper is on the derivation of efficient and exact calculations for required statistics for engineering an entire hierarchical network with resource limits on the various parcels of offered traffic.

Although this paper derives exact and efficient formulas for engineering a network with resource limits, the traditional engineering blocking algorithms which use the Erlang-B distribution (see [4]) would require nontrivial enhancements in the Trunk Servicing and Trunk Forecasting Systems.

We focus on the engineering of a single trunk group. As usual in hierarchical network design, this approch is critical to the network decomposition which allows us to route and engineer traffic for the whole network. We derive the traffic statistics for individual customers, which would not be needed for trunk engineering but could be useful in evaluating the performance of a customer's PVN.

We derive the steady state probabilities of the system size by showing that they obey a product-form solution. These probability values are expressed in terms of a normalizing constant parameter. Then we present an efficient convolution-type algorithm to compute the exact value of this normalizing constant. We also derive additional statistics, such as the different blocking probabilities, as well as the average number of busy circuits and the steady state probabilities of the process defining the number of busy circuits.

In the following section, we present a formulation of the system as a queueing model and introduce the notations carriedout throughout the paper. In section 3, we derive analytical expressions for the steady state probability of the system size process. Section 4 presents an efficient convolution-type algorithm to derive the normalizing constant. Section 5 presents a numerical example of a limited-access system and a derivation of the queueing statistics.

2. Notations and Queueing Model

The system is modeled as a multiple server queueing system with T identical servers. Customers belong to R classes. Customers from class i, $i = 1,2,\dots, R$ arrive to the system in accordance with a Poisson process with rate λ_i. Each incoming class-i customer requires a service with exponentially distributed duration and with rate μ_i. We define the class-i offered load to be $\rho_i = \lambda_i / \mu_i$ and the total offered load to be

$$\rho = \sum_{i=1}^{R} \rho_i \tag{2.1}$$

The state of the system is described with an R-dimensional vector of integers where $\bar{n} = (n_1, n_1, \dots n_R)$ where the i^{th}, component n_i, depicts the number of class-i customers present in the system.

The system is a loss system; it has no waiting room and customers are admitted only if they are accommodated by a server. Obviously, the system cannot accommodate more than T customers simultaneously (the total number of circuits which are shared by all customer classes; T is the trunk group size). In addition, each class of customers has a resource-counter so that no more than a prespecified number of customers can gain access to the system, regardless of the availability of the servers. We assume that class i, $i = 1, 2,\dots,R$, allows no more than t_i of its customers simultaneously in the system (i.e., when the number of calls in progress for class-i customers equals t_i, additional calls by that customer class are blocked). Therefore, an arbitrary customer can be blocked, either by the resource-counter, or due to the unavailability of the servers (i.e., all T servers are busy) (see Fig. 1).

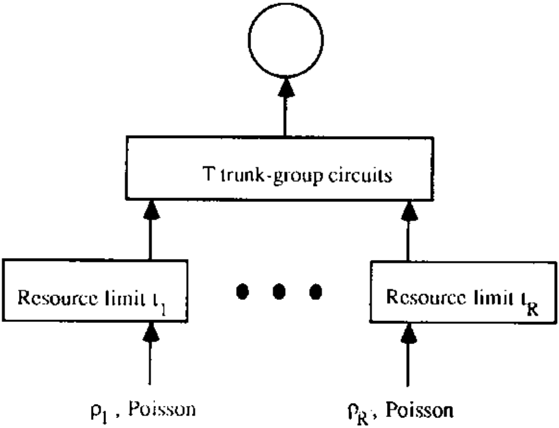

Fig. 1. A node in a limited-across virtual network with T circuits and R classes

Situations where $\sum_{i=1}^{R} t_i \leq T$ yield trivial cases in the sense that the R customer classes behave independently: we have R queueing systems, each of which is of limited capacity (system i has capacity) and only resource counter blocking may occur. The blocking probability for customer class i is given by Erlang-B loss formula (see [4]):

$$\text{P (blocking for class i)} = B\left(t_i, \rho_i\right) = \frac{\rho_i^{t_i}}{\sum_{j=0}^{t_i} \rho_i^j / j!} \tag{2.2}$$

We will therefore assume that $\sum_{i=1}^{R} t_i > T$.

Define F to be the set of all feasible values assumed by the state of the system; i.e.,

$$F = \quad \overline{n} \text{ such that } = \sum_{i=1}^{R} n_i \leq T, \text{ and } n_i \leq t_i, \ i = 1, \ldots R$$

and let

$$F_k = \quad \overline{n} \in F, \text{ such that } = \sum_{i=1}^{R} n_i = K \qquad 0 \leq K \leq T.$$

A schematic representation of the feasible space for the number of calls in progress for two customer classes having limited resource to all the trunks is given in Fig. 2.

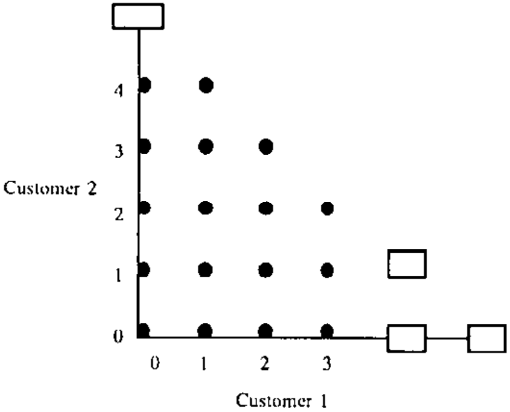

Fig. 2. Feasible states for network calls in progress T = 5, $t_1 = 3$ and $t_2 = 4$

When the resource limits for each customer are set to an unlimited value, we get the extended space shown in the same figure. For three customer-classes, the solution space is given by Fig.3. In addition, we define the following operations on n̄;

$$T_i^{+}\,\bar{n} = \left(n_1, n_2, \ldots\ldots, n_i + 1, n_i{+}1 \ldots, n_R \right)$$

$$T_i^{-}\,\bar{n} = \left(n_1, n_2, \ldots\ldots, n_i + 1, n_i{+}1 \ldots, n_R \right)$$

n̄ represents the same state of the system where n_i class-i customers are in the system for i = 1, 2, ..., R. $T_{\dagger}$ n̄ (respectively T_i n̄) represents the same state of the system with the exception that there are $n_i + 1$ (respectively $n_i - 1$) class-i customers in the system.

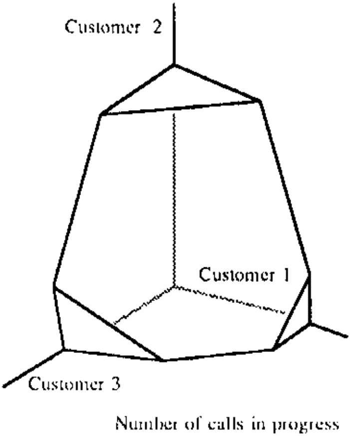

Fig. 3. Feasible states for network calls in progress, R = 3

Let p (n̄) define the steady state probability that the system is in state n̄.

Therefore, $\sum_{\bar{n} \in F}^{R} p(\bar{n}) = 1$ and $p(\bar{n}) = 0$ if $\bar{n} \notin F$.

The next section provides an analysis of the system size and derives expressions for these steady state probabilities p (n̄), as well as several other statistics of the system, mentioned above.

3. Queue Size Analysis

In this section, we show that the equilibrium joint probabilities for the number of busy circuits satisfy a product form solution. Recalling that p ($\bar{n}$) = 0 for $\bar{n} \notin F$, and noting that the underlying process is a birth and death process, the balance equations satisfied by the steady-state probabilities p ($\bar{n}$) are as follows:

$$\begin{aligned}
&\sum_{i=1}^{R}\left[(\lambda_i + n_i\mu_i)p(\bar{n})\right] = \sum_{i=1}^{R}\left[\lambda_i p\left(T_i^{-}\bar{n}\right) + (n_i+1)\mu_i p\left(T_i^{+}\bar{n}\right)\right] \quad && \bar{n} \in F, \sum_{i=1}^{R} n_i < T \\
&\sum_{i=1}^{R} n_i\mu_i p(\bar{n}) = \sum_{i=1}^{R}\lambda_i p\left(T_i^{-}\bar{n}\right) && \bar{n} \in F, \sum_{i=1}^{R} n_i = T
\end{aligned} \tag{3.1}$$

Proposition: The solution to the set of equations (3.1) has a product form as follows:

$$p(\bar{n}) = C\left[\frac{\rho_1^{n_1}}{n_1!}\,\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right] \quad \text{for} \quad \bar{n} \in F, \tag{3.2}$$

where C is a non negative real number.

Proof.

Case 1: $$0 < \sum_{i=1}^{R} n_i < T$$

Substituting the RHS of Eqs.(3.2) for p ($\bar{n}$) in Eqs. (3.1) yields:

$$C\sum_{i=1}^{R}(\lambda_i < n_i\mu_i)\left[\frac{\rho_1^{n_1}}{n_1!}\,\frac{\rho_2^{n_2}}{n_2!}\,\frac{\rho_R^{n_R}}{n_R!}\right] = C\sum_{i=1}^{R}\lambda_i\left[\frac{\rho_1^{n_1}}{n_1!}\,\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_{i-1}^{n_{i-1}}}{n_{i-1}!}\,\frac{\rho_{i-1}^{n_i-1}}{(n_i-1)!}\,\frac{\rho_{i+1}^{n_{i+1}}}{n_{i+1}!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right]$$

$$+\, C\sum_{i=1}^{R}(n_i+1)\mu_i\left[\frac{\rho_1^{n_1}}{n_1!}\,\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_{i-1}^{n_{i-1}}}{n_{i-1}!}\,\frac{\rho_{i-1}^{n_i+1}}{(n_i+1)!}\,\frac{\rho_{i+1}^{n_{i+1}}}{n_{i+1}!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right]$$

By dividing both the right and left hand sides by C, $\left[\frac{\rho_1^{n_1}}{n_1!}\,\frac{\rho_2^{n_2}}{n_2!}\,\frac{\rho_R^{n_R}}{n_R!}\right]$ Eqs. (3.1) become:

$$\sum_{i=1}^{R}(\lambda_i + n_i\mu_i) = \sum_{i=1}^{R}\lambda_i\frac{n_i}{\rho_i} + \sum_{i=1}^{R}(n_i + 1) = \mu_i\rho_i$$

Using the fact that, $\rho_i \frac{\lambda_i}{\mu_i}$, we prove the desired equality.

Case 2: $\sum_{i=1}^{R} n_i = 0$

In this case and since $p(T_i \bar{n}) = 0$, Eqs. (3.1) are:

$$C\sum_{i=1}^{R} \lambda_i p(\overline{n}) = C\sum_{i=1}^{R} (n_i + 1)\mu_i \, p\left(T_i^{+} \overline{n}\right)$$

Using the same substitutions as in case 1, we obtain:

$$C\sum_{i=1}^{R} \lambda_i = \sum_{i=1}^{R} \mu_i \, C\rho_i \ ,$$

which proves the equality.

Case 3: $\sum_{i=1}^{R} n_i = T$

In this case and after proper substitutions, Eqs. (3.1) yield

$$C\sum_{i=1}^{R} n_i \mu_i \left[\frac{\rho_1^{n_1}}{n_1!}\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right] = C\sum_{i=1}^{R} \lambda_i \left[\frac{\rho_1^{n_1}}{n_1!}\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_{i-1}^{n_{i-1}}}{n_{i-1}!}\frac{\rho_i^{n_i-1}}{(n_i-1)!}\frac{\rho_{i+1}^{n_{i+1}}}{n_{i+1}!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right]$$

Again, after dividing both the right and left hand sides by $C\left[\frac{\rho_1^{n_1}}{n_1!}\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right]$, Eqs. (3.1) become:

$$\sum_{i=1}^{R} n_i \mu_i = \sum_{i=1}^{R} \lambda_i \frac{n_i}{\rho_i} \ .$$

The above equality holds since $\rho_i \frac{\lambda_i}{\mu_i}$.

Now using the fact that, $\sum_{\overline{n}\in\Gamma} P(\overline{n}) = 1$, we can obtain the normalizing constant C by writing:

$$C = \left[\sum_{\overline{n}\in\Gamma}\left[\frac{\rho_1^{n_1}}{n_1!}\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right]\right]^{-1} \tag{3.3}$$

We readily know that

$$\sum_{\substack{0\le n_k<\infty \\ k=1,2,\ldots,R}} \left[\frac{\rho_1^{n_1}}{n_1!}\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right] e^{-\rho} = 1,$$

and that

$$\sum_{\bar{n}\in F}\left[\frac{\rho_1^{n_1}}{n_1!}\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right]e^{-\rho}\leq\sum_{\substack{0\leq n_j<\infty\\ j=1,2,\ldots,R}}\left[\frac{\rho_1^{n_1}}{n_1!}\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right]e^{-\rho}$$

Thus

$$C^{-1}e^{-\rho}\leq 1,$$

and

$$C\geq e^{-\rho}\ .$$

Depending on several parameters such as p_i and , t_i, $i = 1, \ldots, R$, and T, C may be greater or smaller than one.

In the following, we derive expressions for several statistics relevant to the performance evaluation of the system. We first denote by N be the random variable representing the number of customers in the system. N also represents the number of busy circuits and let $G_T(k)$, $k = 0,1, \ldots, R$, be the steady-state probability that k circuits of the system are busy. In other words, these variables depict the equilibrium probabilities that there are k customerspresent in the system.

$$G_T(K)=\sum_{\bar{n}\in F_k}p(\bar{n})=C\sum_{\bar{n}\in F_k}\left[\frac{\rho_1^{n_1}}{n_1!}\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right]\quad 0\leq k\leq T \tag{3.4}$$

Trunk group blocking (TG blocking) occurs when all T circuits are busy, so that

$$P(\text{TG blocking})=G_T(T)=C\sum_{\bar{n}\in F_T}\left[\frac{\rho_1^{n_1}}{n_1!}\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right] \tag{3.5}$$

A resource-counter blocking for class i_0 occurs when there are t_{i0} circuits occupied by class-i_0 customers. Therefore, the resource-counter blocking (RC blocking) probability for class i_0 is

$$P\left(\text{RC blocking } i_0\right)=C\sum_{\substack{\bar{n}\in F_T\\ n_{i_0}=t_{i_0}}}\left[\frac{\rho_1^{n_1}}{n_1!}\frac{\rho_2^{n_2}}{n_2!}\cdots\frac{\rho_R^{n_R}}{n_R!}\right]$$

The average number of busy circuits is then given by:

$$E[N]=\sum_{k=1}^{T}kG_T(k) \tag{3.6}$$

Additional moments can be similarly derived. It remains to derive the constant C. The next section presents an efficient algorithm for computing this constant C.

4. Evaluation of the Normalizing Constant

There are several ways to evaluate the constant C. However, most, including exhaustive enumeration, are not efficient. Using an approach based on convolutions, a procedure that calls itself, we develop an efficient way to evaluate the constant C and thus, all desired statistics. This procedure takes advantage of the product-form nature of the solution. This method is inspired from the ones introduced by Lam and Lien [7] and Lavenberg [8].

Considering m different classes, $i_1, i_2, \ldots, i_m$, among the R classes with the respective resource-counter constraints, $t_{i_1}, t_{i_2}, \ldots, t_{i_m}$, we can define

$$f_m^{(t_{i_1}, t_{i_2}, \ldots, t_{i_m})}(K) = \sum_{\bar{n} \in F_k} \left[\frac{\rho_{i_1}^{n_{i_1}}}{n_{i_1}!} \frac{\rho_{i_2}^{n_{i_2}}}{n_{i_2}!} \cdots \frac{\rho_{i_1}^{n_{i_m}}}{n_{i_m}!} \right] \quad \text{with } n_{i_1} + n_{i_2} + \cdots + n_{i_m} = k \tag{4.1}$$

When multiplied by the normalizing constant: $C\, f_m^{(t_{i_1}, t_{i_2}, \ldots, t_{i_m})}(k)$ is the probability that the system has exactly k busy circuits occupied solely by customers from classes $_1$, i_2, ... , i_m. Consequently, the probability that k circuits of the system are busy is given by:

$$G_T(K) = \frac{1}{C} f_R^{(t_1, t_2, \ldots, t_R)}(k) = \sum_{\bar{n} \in F_k} \left[\frac{\rho_1^{n_1}}{n_1!} \frac{\rho_2^{n_2}}{n_2!} \cdots \frac{\rho_R^{n_R}}{n_R!} \right] \quad k = 0, 1, \ldots, T, \tag{4.2}$$

so that

$$C = \left[\sum_{k=0}^{T} f_R^{(t_1, t_2, \ldots, t_R)}(k) \right]^{-1} \tag{4.3}$$

It remains to derive the expressions for $f_R^{(t_1, t_2, \ldots, t_R)}(k)$ for $k = 0, 1, \ldots, T$. We readily have:

$$f_m^{(t_{i_1}, t_{i_2}, \ldots, t_{i_m})}(0) = 1 \tag{4.4}$$

where class i_j is any of the R classes of customers. In addition, we have

$$f_1^{(t_i)}(k) = \begin{cases} \dfrac{p_i^k}{k!} & \text{if } 0 \le k \le t_i \\ 0 & \text{otherwise} \end{cases} \qquad i = 1, 2, \ldots, R \tag{4.5}$$

We also note that:

$$f_{m+1}^{(t_1, t_2, \ldots, t_{m+1})}(k) = \sum_{i=0}^{k} f_m^{(t_1, t_2, \ldots, t_m)}(i) \,.\, f_1^{(t_{m+1})}(k - i) \tag{4.6}$$

Therefore, using a Divide and Conquer approach, $f_R^{(t_1,t_2,\dots,t_R)}(k)$ for $k = 0, 1,\dots,T$, can be recursively and efficiently evaluated, as outlined in the following paragraph.

To ease the notation, we can assume, without loss of generality, that R is a power of 2. If R is not a power of 2, we can use the ceiling of $R/2^i$ and the floor of $R/2^i$ appropriately, to have an integer value. We proceed by increasing k , starting from $k = 1$ to reach $k = T$. We write

$$f_R^{(t_1,t_2,\dots,t_R)}(k) = \sum_{i=0}^{k} f_{R/2}^{(t_1,t_2,\dots,t_{R/2})}(i) \,.\, f_{R/2}^{(t_{R/2+1},\, t_{R/2+2},\dots,t_R)}(k-i)$$

and repeat the same computation, recursively, for the evaluation of $f_{R/2}^{(t_1,t_2,\dots,t_{R/2})}(i)$ This approach yields the value of the normalizing constant and $G_T(k)$ for all k, as well as

$f_{R/2^i}^{(t_{R/2^i\cdot j+1},\; t_{R/2^i\cdot j+2},\dots,t_{R/2^i\cdot(j+1)})}(k)$ for $k=1,2,\dots,T$, and $i=0,1,\dots,\log_2(R)$, in $T(T+1)R$ operations. It is also worthwhile to note that one may use the obtained results to adjust to a system with more circuits or less circuits. For instance, if we expand the network by adding a circuit, all derived results remain useful and all we have to do is to derive $f_R^{(t_1,\dots,t_R)}(T+1)$ The updated normalizing constant is then given by:

$$[C(T+1)]^{-1} = [C(T)]^{-1} + f_R^{(t_1,\dots,t_R)}(T+1)$$

Similarly, removing a circuit can be treated as follows:

$$[C(T-1)]^{-1} = [C(T)]^{-1} - f_R^{(t_1,\dots,t_R)}(T) \;.$$

Increasing or decreasing a parameter and adding a class of customers may also be efficiently treated. For instance, if we add a new class of customers $(R + 1)$, with limit t_{R+1} and offered load ρ_{T+1}, we proceed as follows:

$$f_{R+1}^{(t_1,t_2,\dots,t_{R+1})}(k) = \sum_{i=0}^{k} f_R^{(t_1,t_2,\dots,t_R)}(k-i) \,.\, f_1^{(t_{R+1})}(i)$$

$$= \sum_{i=0}^{\min(k,\,t_{R+1})} f_R^{(t_1,t_2,\dots,t_R)}(k-i)\,\frac{\rho_{R+1}^{i}}{i!}$$

This saves a lot of computation any time we wish to upgrade the system (add more trunks, or more classes).

In the following section, we present a numerical example of a limited-access system.

5. Numerical Example

In this section, we evaluate statistics of a limited-access system. We consider such a system with two different classes (see Fig.4).

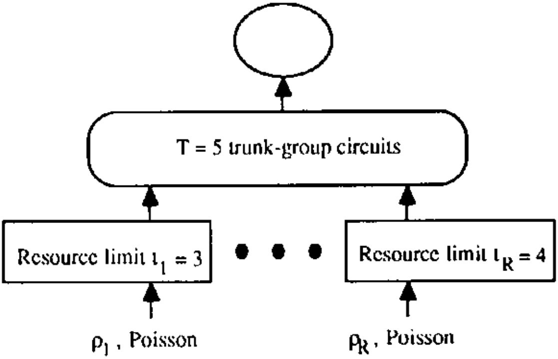

Fig. 4. The limited acess system presented in the example.

The offered load for both classes are $\rho_1 = \rho_2 = 1$. The resource-limit for class 1 and class 2 are respectively, $t_1 = 3$ and $t_2 = 4$. All these parameters are summarized inTable 1 below.

Table 1. Network profile

Virtual network profile				
t_1	t_2	T	ρ_1	ρ_2
3	4	5	1	1

The following next three tables present the intermediate steps in computing the normalizing constant C for this system.

Table 2. Basic convolution values

Convolution function		
k	$f_1^{(3)}(k)$	$f_1^{(4)}(k)$
0	1	1
1	1	1
2	½	½
3	1/6	1/6
4		1/24

The next table displays the needed pair-wise computations for and is tabulated to illustrate the feasible state space.

Table 3. Intermediate convolution values

	$f_1^{(3)}(n_1 \times f_1^{(4)}(n_2$			
n_1^* n_2	0	1	2	3
4	1/24	1/24	*	
3	1/6	1/6	1/12	**
2	½	½	¼	½
1	1	1	½	1/6
0	1	1	½	1/6

* n_1 (resp. n_2) is the number of class 1 (resp. class 2 customers in the system.

Summing along the diagonals, we get the convolution functions, we obtain the results shown by Table 4.

Table 4. Final convolution values

k	$f_2^{(3,4)}(k)$
0	1
1	2
2	2
3	4/3
4	5/8
5	5/24

Using Eq. (4.4) and summing up the the values in the last table we obtain:

$$C^{-1} = 172/24 \text{ and } C = 24/172 .$$

From Eqs. (3.6) and (4.4) we have the expected number of busy circuits given by:

$$E[N] = C \sum_{k=1}^{5} k\, f_2^{(3,4)}(k) = 1.8895$$

The overall blocking probability for an arbitrary customer (regardless of the class), either by the resource-counter or by the trunk group is given by:

$$P(\text{overall blocking}) = 1 - \frac{E[N]}{\rho_1 + \rho_2}$$

$$= 1 - \frac{1.8895}{2} = 0.0552 \ .$$

Trunk group blocking is readily obtained using the last row of Table 3 as:

$$P(\text{TG blocking}) = C \ f_2^{(3,4)} \ (5) = 0.0290 \ .$$

To obtain RC blocking for class 1 (respectively class 2), we sum up entries of the last column (respectively the first row) in Table 2 and multiply the result by C to obtain:

$$P\ (\text{RC blocking } 1) = 0.0581,$$
$$P\ (\text{RC blocking } 2) = 0.0110.$$

The overall blocking per class of customers includes TG and RC blocking and is given by:

$$P\ (\text{overall blocking } 1) = 0.0756,$$
$$P\ (\text{overall blocking } 2) = 0.0349.$$

Note that averaging over these two per class overall blocking probability values will yield the overall blocking probabilityvalue obtained above in a different fashion.

To see the effect of the normalizing constant we will consider the system with the same parameters except for $T = \infty$. In this case there is no trunk group blocking. For each individual class of customers, the RC blocking probability is equal to the overall blocking probability and is given by Erlang-B loss formula as follows:

$$P\ [\text{RC blocking}_1\ (\text{when } T = \infty)] = B(4,1) = 0.0625,$$
$$P\ [\text{RC blocking}_2\ (\text{when } T = \infty\ )] = B(4,1) = 0.0153.$$

The expected number of busy circuits is given by:

$$E\ [N\ (\text{when } T = \infty)] = 1.922.$$

The blocking probability for an arbitrary customer may be derived by averaging the individual blocking probabilities or from the expected number of circuits and is given by:

$$P\ [\text{blocking (when } T = \infty\ )] = 0.389.$$

As expected, we note that blocking decreases as we increase the number of circuits. Considering only RC blocking, we see that the T circuit system yields lower blocking probability values. This result may not be generalized and depends on the parameters of the system. Several other statistics such as the variance can also be derived.

Adding a circuit to the system (T becomes 6) is dealt with by adding the corresponding entries in the cells of Table 3 containing asterisks, summing up these entries to add the corresponding row at the end of Table 4 and proceeding accordingly to update all relevant statistics.

6. Conclusions

We have developed formulae for calculating exactly the statistics for engineering networks with customer-specific resource limits. Although the application of the formulae is straight-forward the implementation is often difficult.

We present an exact queue size analysis of a finite server system with customer-resource-limitation of access to service. Two types of blocking can occur in such a system, blocking by the resource-counter and blocking due to saturation of the system. For such an analysis, we present an algorithm to compute the equilibrium system state probabilities as well as several other statistics.

The work in this paper is a contribution to queueing theory and traffic engineering because to the best of our knowledge no other exact solution to this important problem has been published. The literature contains only exhaustive evaluation of exact solutions, efficient approximations, or solutions for particular cases.

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تقدير احتمالية الابعاد للشبكات ذات الاستعمال المحدد

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ملخص البحث. تعرض طريقة للحصول على الاحتمالية المتعلقة بعدم وجود دائرة متاحة في شبكة ذات استعمال محدود. ونحلل شبكة ذات استعمال محدود من الدوائر ومجموعة من فئات مختلفة من المستخدمين. لكل فئة من المستخدمين حد أقصى من الدوائر التي يمكن استخدامها تزامنيًا. هذا الحد الأقصى يختلف من فئة إلى أخرى. العديد من الشبكات الذكية تتبنى اليوم هذه الطريقة حيث تتمكن من التحكم في تكاليف الاتصالات. ومثال ذلك الشبكة الخاصة.

نعرض حلاً دقيقًا وخوارزم يمكن من الحصول على الاحتمالية المطلوبة. هذه الاحتمالية بالغة الأهمية في وضع مواصفات تتعلق بالدوائر داخل الشبكات.