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MECHANICAL ENGINEERING

Correlations for Predicting Average Global Solar Radiation Incident on a Horizontal Surface in Amman, Jordan

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Abstract. The present work develops several empirical formulas for estimating the average hourly, daily and monthly global solar radiation incident on a horizontal plane in Amman, Jordan. The developed formulas correspond to sinusoidal correlations which express the values of the total solar radiation available in terms of the hour, day and month of the year. The determined coefficients of the sinusoidal correlations are expressed in polynomial form in terms of the hour and month of the year to allow the use of a single formula for determining the hourly and monthly variations of solar radiation in Jordan. The present correlations are based on hourly measurements of solar radiation over the period (1983-1987) and are considered suitable for solar energy collection applications. The computed solar radiation values, using the present correlations, were compared with measured data. The results obtained indicate that the overall mean error ranges from 0.2 to 9.5%.

Introduction

Solar energy conversion into a useful form of energy in a given locality depends on the knowledge of the soalr radiation available in that locality. Hourly and daily average values of irradiations incident on a thermal solar system are considered important parameters in predicting and simulating the performance of such a system. Daily and monthly average values of total insolation are considered adequate for predicting the thermal energy output of flat plate collectors [1,2]. These average values could be obtained either by direct measurements or from applying appropriate prediction correlations. As available measurements of solar radiation are very limited in Jordan, the use of prediction formulas becomes necessary to assess the availability of solar energy in the country. Linear regression correlations of the Angstrom-type have been used by the author [3,4] to predict the availability of monthly global radiation in various sites of Jordan. The correlations require the knowledge of the bright sunshine duration.

The general expression of sinusoidal function for predicting global radiation is:

$$G(t) = A + B \sin [(2\pi/T)t - F] \quad (1)$$

where G(t) = average global radiation at any given time t.
A = mean value of global radiation.
B = amplitude of the sinusoidal function which specifies the amount of the global radiation oscillation about its average value.
T = period in units of time.
F = phase angle which parameterizes the extent to which the sinusoidal function is shifted horizontally.

In the above correlation, t is the time of global radiation variation and stands for the hour(h) of the day, the day number (n) of the year or the month(m) of the year. The ratio $2\pi/T$ represents the angular frequency which characterizes how often the global radiation cycle occurs.

The prediction of global solar radiation using the general expression of sinusoidal function has the advantage that it does not require the knowledge of the bright sunshine hours as well as the calculation of the extraterrestrial radiation and length of the day. All it requires is the determination of the time period (hour of the day, day of the month or month of the year) during which the global radiation is needed to be predicted.

The purpose of the present work is to provide one sinusoidal correlation for estimating each of the hourly, daily and monthly values of the average, maximum and minimum global radiation incident on a horizontal surface in Amman, Jordan (Latitude: 32°N, Longitude: 36°E, Elevation:980 m). The present correlations are based on hourly measurements of solar radiation performed by the Royal Scientific Society (RSS) over the period (1983-1987).

The instruments used to measure the hourly global solar radiation data are Kipp and Zennon Pyranometer and Friedrichs weather station micro processor. The accuracy of the recording instrument is better than 0.1%. The accuracy of the Pyranometer used is better than 1%. The accuracy was checked at the end of the measuring period of 1985 and was found to be better than ±3% [5].

Results and Discussion

Correlations of the Average Hourly Global radiation

The average global radiation incident on a horizontal surface during a given hour has been obtained for each day of the year by averaging the corresponding hourly measured data available for the period (1983-1987). The averaged data for all hours from 06:00 to 17:00 hours Local Standard Time, LST, are plotted against the day of the year as shown in Fig. 1. The averaged data for each hour presented in Fig. 1, have been fitted to a sinusoidal function of the form:

$$G_h(n) = A_h + B_h \sin[(2\pi/365)n - F_h] \quad (2)$$

where $G_h(n)$ is the average global radiation as a function of the day number, n(n= 1,2,...,365) for a given hour (h) of the day (h= 06, 07,...,17 hours LST).

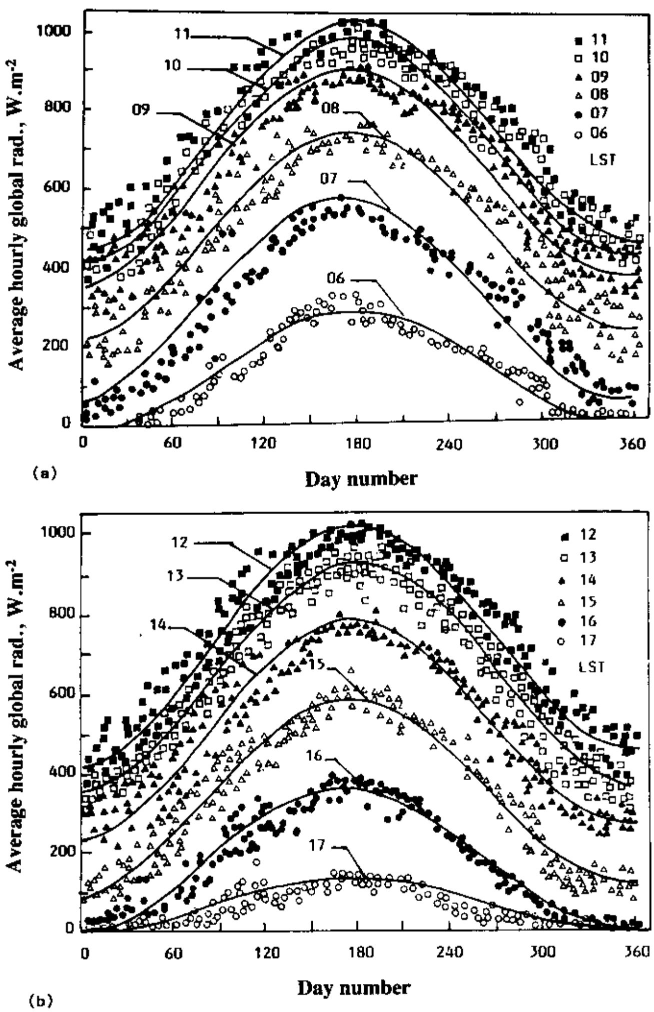

Fig. 1. Annual variation of the average hourly global radiation for all hours of the day.
(a) From 06:00 to 11:00 hours LST.
(b) from 12:00 to 17:00 hours LST.
Solid curves correspond to correlation (2).

A computer program was written to compute the values of the constants A_h, B_h and F_h of equation (2) using least squares technique. The obtained values of these constants are listed in Table 1. The sinusoidal correlations corresponding to each hour are also plotted in Fig. 1.

Table 1. Hourly values of the constants A_h, B_h and F_h of correlation (2) and the mean percentage error for each hour.

Time hours LST	A_h $W.m^{-2}$	B_h $W.m^{-2}$	F_h degrees	% error
06	128.3	145.0	77.4	–46.7
07	308.0	243.9	80.0	9.2
08	464.7	247.0	82.3	5.5
09	601.7	256.5	82.4	3.5
10	684.7	265.3	84.1	3.5
11	720.7	269.0	87.2	3.5
12	697.1	280.7	88.2	3.2
13	621.2	274.3	89.6	4.5
14	491.6	254.1	86.3	4.9
15	329.6	230.4	82.9	8.7
16	169.9	176.5	81.4	2.0
17	48.4	66.1	79.7	50.4

The average global radiation values for each hour of the day were computed using correlation (2) and the corresponding values of the constants A_h, B_h and F_h are listed in Table 1. The predicted data, ($G_{pred.}$) were compared with the measured values ($G_{meas.}$) for each hour. (365 readings). The residual ($G_{pred.}$ - $G_{meas.}$) as well as the percentage error, defined as ($G_{pred.}$ - $G_{meas.}$)/ $G_{meas.}$ can be used to check the accuracy of the used correlation. The overall mean error of correlation (2) for every hour, defined as: $(1/365) \sum_{d=1}^{365} | [G_{pred.}(d)-G_{meas.}]/G_{meas.}(d) |$ was computed to verify the correlation. The results of computation are listed in Table 1. It can be seen from Table 1 that the overall mean error is less than 6% for most sunshine hours of the day. It is noted that, the overall mean errors at 06:00 and 17:00 hours LST are considered large (about 50%) due to the fact that the irradiation data of the first and last hours are not complete. In addition, the radiation intensity is low at these hours which results in large errors. Thus, the present model fails to predict the global radiation at these hours with high accuracy. However, this large error occurs near sunrise and sunset hours and is of little practical singificance since most of the radiation incident during the day is confined within about or four-fifths of the day around solar noon.

Table 1 indicate that, the magnitude of the mean value of global radiation (A_h) increases from 128.3 $W.m^{-2}$ at 06:00 hours to reach a maximum value of 720.7 $W.m^{-2}$

at 11.00 hours. The mean value then starts to decrease until a minimum value of 48.4 $W.m^{-2}$ is obtained at 17:00 hours. The integrated value of the constants A_h represents the annual mean daily global radiation reaching the horizontal surface. This value amounts to 5.266 $KWh.m^{-2}$ per day as compared to the corresponding measured value of 5.302 $KWh.m^{-2}$ per day. The resulting error is only 0.7%. The behavior of the amplitude (B_h) and the phase angle (F_h) is similar to that of A_h. The amplitude increases from a value of 145.0 $W.m^{-2}$ at 06:00 hours to a maximum value of 280.7 $W.m^{-2}$ at 12:00 hours, then decreases to reach its minimum value at 17:00 hours. Similarly, the phase angle starts to increase from its value at 06:00 hours to reach a maximum value of 89.6 at 13:00 hours then decreases to 79.7 at 17:00 hours. The shift in the phase angle indicates a change in the location of the maximum value of global radiation intensity.

The hourly values of the constants A_h, B_h and F_h listed in Table 1 can be related to the independent variable h by a single formula of polynomial form. This will allow the use of a single equation for representing the hourly variations of global solar radiation. This resulting equations are as follows:

$$A_h = 2702.525 + 697.089h - 42.031\,h^2 + 0.613h^3 \qquad (3)$$

$$B_h = 29.683 + 5.771\,h + 4.797h^2 - 0.291h^3 \qquad (4)$$

$$F_h = 68.889 + 0.100h + 0.369h^2 - 0.019h^3 \qquad (5)$$

The coefficients of equations (3)-(5) have been computed using the least squares technique. The regression coeffecients (r) for equations (3)-(5) are 0.99, 0.96 and 0.94 respectively. The day number at which the hourly global radiation reaches its maximum value ($n_{h,max}$) can be obtained by differentiating equation (2) with respect to the day number and equating the result to zero. The expression obtained is:

$$n_{h,max} = 365(0.25 + 0.5\,F_h/\pi) \qquad (6)$$

Equation (6) reveals the role of the phase angle in determing the day number at which the maximum value of the hourly global radiation occurs for each hour. The maximum values of the hourly global radiation ($G_{h,max}$) can be determined by substituting equation (6) into equation (2). The relation obtained is:

$$G_{h,max} = A_h + B_h \qquad (7)$$

The above result is expected as equation (2) reaches its maximum value when $\sin[(2\pi/365)n - F_h]$ is equal to unity. The hourly values of $n_{h,max}$ and $G_{h,max}$ were determined, using equation (6) and (7), respectively, and are shown in Table 2. It can be seen from Table 2 that the smallest and greatest phase shifts occur at the day numbers of 170 for 06:00 hours and 182 for 13:00 hours, respectively. This result indicates that the maximum shift is 12 days. Table 2 also indicates that $G_{h,max}$ reached an upper value of about 990 $W.m^{-2}$ and occured at 11:00 hours while the lower value is reached at 17:00 hours and amounts to 115 $W.m^{-2}$.

Table 2. Hourly values of $n_{h,max}$ and $G_{h,max}$ of equations (6) and (7)

Time hours LST	$n_{h,max}$	$G_{h,max}$ $W.m^{-2}$
06	170	273.3
07	172	551.9
08	175	711.7
09	175	858.2
10	177	950.0
11	180	989.7
12	181	977.8
13	182	895.5
14	179	745.7
15	175	560.0
16	174	346.4
17	172	114.5

Correlations for the Total Monthly Hourly Global Radiation

The averaged global radiation data of each hour from 06:00 to 17:00 hours LST for all days of the month were integrated to yeild the total monthly hourly values of global radiation. The obtained values were plotted against the hour of the day for each month of the year as shown in Fig. 2. The variation of the monthly hourly data for each of the 12 months have been fitted to a sinusoidal function of the following form, using the least squars method:

$$G_m(h) = A_m + B_m \sin[(2\pi/11.5)\,h - F_m] \qquad (8)$$

where $G_m(h)$ is the total global radiation for a given month (m) as a function of the hour. In the above correlation, h=1 stands for 06:00 hours, h=2 stands for 07:00 hours, and h=12 stands for 17:00 hours LST. The values of the constants A_m, B_m and F_m of equation (8) were determined for each month and are given in Table 3. Using the monthly values of the constants A_m, B_m and F_m and equation (8), the total monthly hourly global radiation data were calculated for each of the 12 months. These values are plotted against the hour of the day as shown in Fig. 2. The predicted monthly hourly global radiation for each month ($G_{pred.}$) were compared with the corresponding measured data (G_{meas}). The overall mean error of correlation (8), defined as:

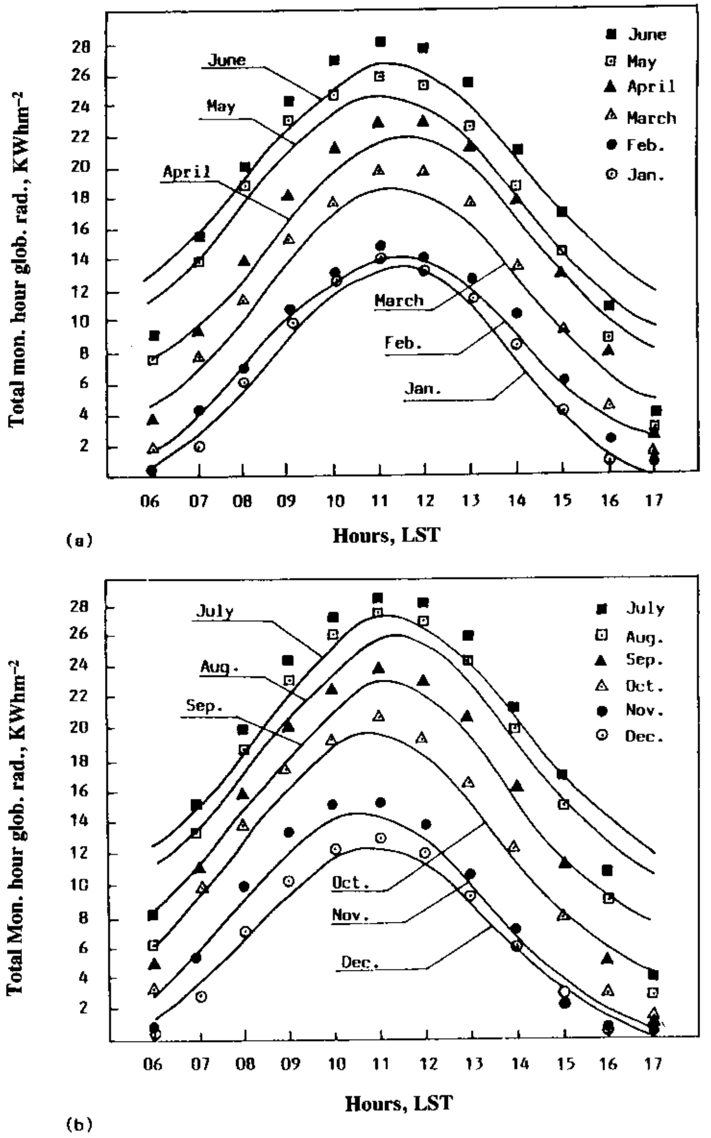

Fig. 2. Hourly variation of the average total monthly global radiation for all months of the year.
(a) From January to June.
(b) From July to December.
Solid curves correspond to correlation (5).

$(1/2)\sum_{i=1}^{12} | [G_{pred.}(i)-G_{meas.}(i)]/G_{meas.} |$, was calculated for every month to check the accuracy of the used model. The results are also presented in Table 3. As may be seen from Table 3, the overall mean error is less than 7% for every month which indicate good agreement between predicted values and measured data. Table 3 also indicates that the average value of the monthly mean hourly global radiation (A_m) increases from a value of 7 KW per m^2 for the month of January to a maximum value of 20 KW per m^2 for the month of July. The magnitude of the constant A_m then decreases to a minimum value of 6.8 KW per m^2 for the month of December. It is also observed that the minimum value of the phase angle (F_m) is 86° and occurs for the month of November while the maximum value of F_m is 115°.

Table 3. Monthly values of the constants A_m, B_m and F_m of correlation (8) and the mean percentage error for each month

Month	A_m $KW.m^{-2}$	B_m $KW.m^{-2}$	F_m degrees	% error
January	7.055	6.913	104.0	4.4
February	7.800	5.557	115.2	5.3
March	11.641	6.798	109.8	3.7
April	14.855	7.018	115.2	−5.9
May	17.704	6.824	99.7	−0.3
June	19.675	6.757	105.1	−0.4
July	20.013	7.234	108.5	−0.7
August	18.494	7.700	105.7	2.3
September	15.323	7.684	101.1	−3.6
October	12.542	7.159	92.9	−1.3
November	8.160	6.446	86.0	3.7
December	6.846	5.600	97.0	6.9

In order to permit the use of a single formula for expressing the monthly variation of the global solar radiation, the constants A_m, B_m and F_m of Table 3 are expressed in polynomial form in terms of the month of the year m as follows:-

$$A_m = 0.678 + 5.008m - 0.298m^2 - 0.007m^3 \quad (9)$$

$$B_m = 7.144 - 0.709m + 0.188m^2 - 0.012m^3 \quad (10)$$

$$F_m = 101.1 + 6.7m - 1.3m^2 + 0.1m^3 \quad (11)$$

The regression coefficients of the above relations are 0.96, 0.85 and 0.81 respectively.

Correlations for the Total Yearly Hourly Global Radiation

The values of the total monthly global radiation for each hour from 06:00 to 17:00 hours LST have been integrated over all months of the year to obtain the total yearly hourly global radiation ($G_{h,ann.}$). The data obtained were plotted against the hour of the day as shown in Fig. 3. Using the least squares technique, the sinusoidal equation:

$$G_{h,ann} = 160.107 + 80.740 \sin[(2\pi/11.5)h - 1.807] \quad (12)$$

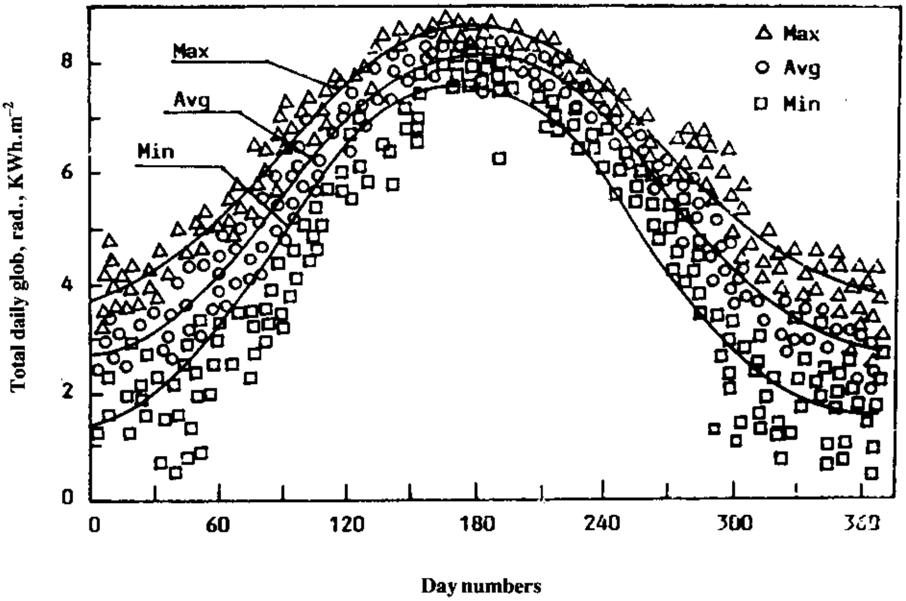

Fig. 3. Hourly variation of the total annual global radiation. Solid curve corresponds to correlation (6).

was obtained to fit the annual global radiation data for each hour(h). Equation (12) indicates that, the mean value of the yearly hourly global radiation is 160.107 $KW.m^{-2}$ per year. The total global radiation incident over the whole of the year is 160.107×12 hours which equals to 1921.284 $KW.m^{-2}$ per year which indicate good availability of solar radiation. The predicted values of the total annual hourly global radiation were calculated for each hour using equation (12) and the obtained values are also plotted in Fig. 3. The predicted and measured values of $G_{h,ann}$ were compared and the percentage error for each hour was calculated. The overall mean error was found to be 3.2% which indicate good agreement between predicted and measured values.

Correlations for the Total Daily Global Radiation

The measured data of daily global radiation over the period (1983-1987) were processed to determine the average, maximum and minimum daily values for all days of the year. The annual variations of the average global radiation ($G_{d.avg.}$) incident over the whole of the day, as well as the maximum ($G_{d,max.}$) and minimum ($G_{d,min.}$) global radiation are shown in Fig. 4. The daily data have been successfully fitted to sinusoidal functions expressed as follows:

$$G_{d,avg} = 5.327 + 2.681 \sin[(2\pi/365)n - 1.550] \quad (13)$$

$$G_{d,max} = 6.074 + 2.376 \sin[(2\pi/365)n - 1.513] \quad (14)$$

$$G_{d,min} = 4.430 + 3.071 \sin[(2\pi/365)n - 1.426] \quad (15)$$

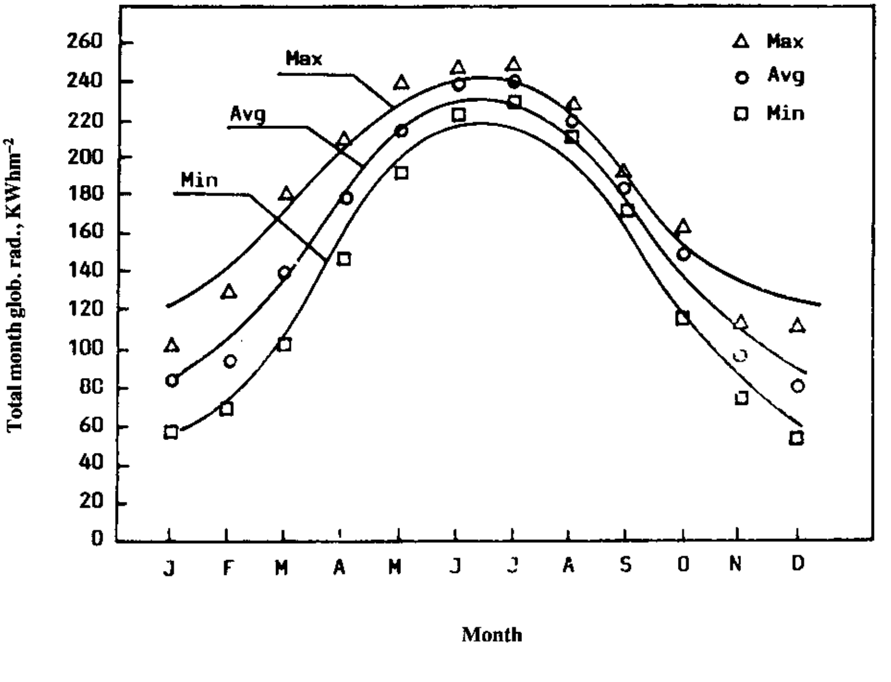

**Fig. 4. Annual variation of the average, maximum and minimum global radiation incident over the whole of the day.
Solid curves corresponds to correlations (7 - 9).**

where n is the day number of the year (*i.e.* n= 1,2, ...365). The mean values, the amplitudes and the phase shifts of the above relations were calculated using the least squares method. Correlation (13)-(15) are also plotted in Fig. 4. The average, maximum and minimum daily global radiation, computed using correlations (13)-(15), were compared with that of the corresponding measured values. It was found that, the mean error introduced by using formulas (13)-(15) are less than 2.6, 2.5 and 9.5%, respectively. The above formulas indicate that the mean values of the yearly variations of $G_{d,avg.}$, $G_{d,max}$ and $G_{d,min}$ are 5.327, 6.014 and 4.430 KWh^{-2} per day and are obtained at n equals to 90, 88 and 83, respectively.

Correlations for the total Monthly Global Radiation

The average, maximum and minimum global radiation incident over the whole of the month $G_{m,avg}$, $G_{m,max}$ and $G_{m,min}$ respectively have been calculated by integrating the daily radiation data of $G_{d,avg}$, $G_{d,max}$ and $G_{d,min}$, respectively for each month of the year. Fig. 5 shows a plot of the measured values of $G_{m,avg}$, $G_{m,max}$ and $G_{m,min}$ against the month of the year. The monthly variations of the data presented in Fig. 5 were fitted to the following sinusoidal correlations, using the least squares technique.

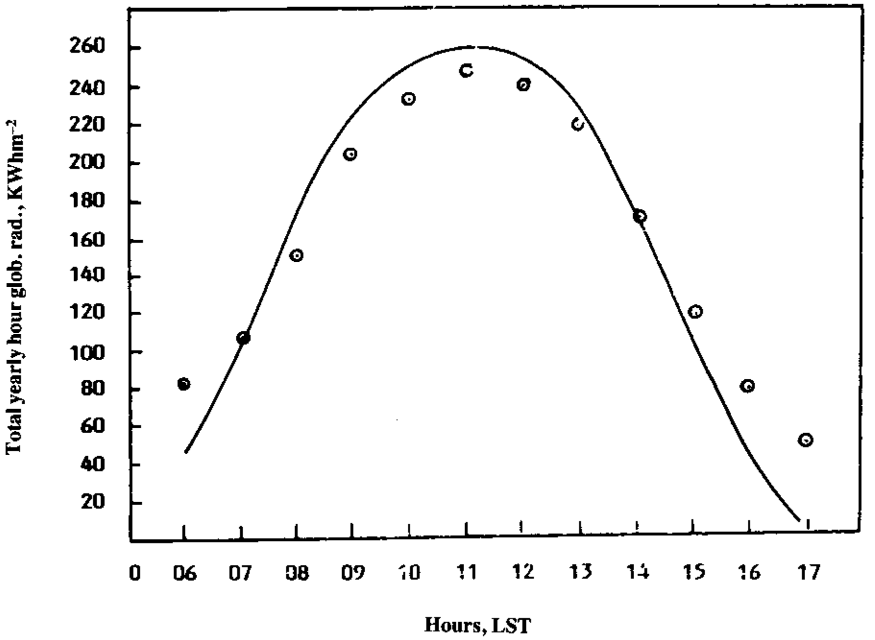

Fig. 5. Annual variation of the average, maximum and minimum global radiation incident over the whole of the month.
Solid curves corresponds to correlations (10 - 12).

$$G_{m,avg} = 161.555 + 71.803 \sin[(2\pi/11.5)m - 2.024] \quad (16)$$

$$G_{m,max} = 182.458 + 60.464 \sin[(2\pi/11.5)m - 1.860] \quad (17)$$

$$G_{m,min} = 135.921 + 81.738 \sin[(2\pi/11.5)m - 2.158] \quad (18)$$

where m is the month of the year (m= 1,2, ..., 12) with m=1 corresponds to January, m=2 corresponds to February and m=12 corresponds to December. It can be observed from formulas (16)-(18) that, the mean values of the annual variations of $G_{m,avg}$, $G_{m,max}$ and $G_{m,min}$ are 161.555, 182.458 and 135.921 $KWh.m^{-2}$ per month, respectively while the amplitude of the variations are 71.803, 60.464 and 81.738 $KWh.m^{-2}$ per month, respectively. Correlation (16) indicates that, the average total global radiation incident over the year is 161.555 × 12 and equals to 1938.660 $KWh.m^{-2}$ per year as compared to the annual value of 1921.284 $KWh.m^{-2}$ per year calculated based on integrating the hourly radiation data. This discrepancy is less than 1% and is due to considering the day length from sunrise to sunset as 12 hours.

Conclusion

The present work provides several empirical formulas of the form:

$$G(t) = A + B \sin[(2\pi/T)t - F]$$

which best represent the distribution and behavior of the measured hourly, daily and monthly global radiation data. The developed formulas allow the computation of the average, maximum and minimum global radiation intensity incident on a horizontal surface in Amman, Jordan over the whole of hour, day and month of the year for solar energy collection applications. The accuracy of the derived correlations were tested against the measured data over the period (1983-1987). The agreement between predicted values and measured data was found to be satisfactory.

The obtained values of the constants A, B and F of the present work are different from those calculated for Athens and Greece [6] (Latitude: 38° and Longitude: 24°). A comparison of the present constants obtained for the average hourly global radiation correlation with the corresponding values calculated for Athens shows a difference in the range of 10-60% for A, 5-20% for B and 1-6% for F, depending on the hour of the day. It can be observed that, the largest difference occurs for the constant A. This is due to the fact that the constants, mainly the constant A depend on the latitude of the location and on the turbidity of the atmosphere.

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علاقات تقدير الإشعاع الشمسي الكلي الواقع على سطح أفقي في مدينة عمان ـ الأردن

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ملخص البحث. تبحث هذه الورقة في استنباط عدة علاقات لحساب المعدّل اليومي والشهري لكمية الإشعـاع الشمسي الكـلي وكذلك الإشعاع الشمسي الواقع في الساعة على سطح أفقي في مدينة عمان بالأردن. وتبين الورقة أن المعادلات المستنتجة توازي معادلات جيبية تعبر عن معدل تغيّر كمية الإشعاع الشمسي الكلي بدلالة الساعة واليوم والشهر. ويظهر البحث أن العلاقات الحالية قد بُنيت على قياسات لمعدل الإشعاع الشمسي بالساعة خلال الفترة الممتدة من عام **١٩٨٣** إلى عام **١٩٨٧**. وقد تم التعبير عن ثوابت المعادلات الجيبية المستنبطة بواسطة معادلة واحدة وذلك بهدف استعمال معادلة واحدة لتقدير قيمة التغيير اليومي والشهري للإشعاع الشمسي الكلي.

وتعتبر العلاقات الحالية مناسبة للتطبيقات المتعلقة بتجميع الطاقة الشمسية حيث تم في هذا البحث مقـارنـة قيم الإشعاع الشمسي المقاسة بمقدار الإشعاع المحسوب بواسطة المعادلات المستنتجة في هذه الورقة. وتشير النتائج التي تم الحصول عليها إلى أن هناك توافقا جيدا بين القيم المقاسة والمقادير المحسوبة حيث تراوحت نسبة الخطأ الكلي بين ٣,٠ ـ ٥,٩٪.