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Linear Arrays Pattern Synthesis with Multiple Broad Nulls Using Partial Amplitude Control

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Abstract. The purpose of this paper is to present a simple procedure for partial amplitude control to suppress multiple wide band interferences in linear arrays. The partial element excitations are determined to meet a specified array pattern using the simplex method of linear programming to solve an overdetermined system of linear equations. The procedure achieves the minimum number of controlled edge elements for the prescribed sidelobe level (SLL), main beam width (MBW), and multiple wide band interference suppression in arbitrary directions. The performance of multiple interference suppression using this technique is independent of the number of array elements.

Introduction

Interference suppression can be achieved by generating a null in the direction of the interference by controlling the phases and/or the amplitudes of the received signals at the elements of the array [1-3]. When the direction of arrival of the interference is not known exactly or when the interference source is varying with time, then the pattern has to be nulled over an extended sector. So far, a fully adaptive array is used to achieve wide band nulls where every element of the array is individually controlled adaptively [4,5]. In practice, corrolators and attenuators must be added to the antenna system which requires an expensive hardware for large arrays. When the number of external jammers are much smaller than the number of degrees of freedom, then a partially adaptive array is preferred. It is shown that a partially adaptive array can be achieved by controlling the elements of the array at a subarray level, by controlling certain auxiliary elements, or by searching for the best phase and/or amplitude settings according to a certain criteria. Controlling the signal characteristics at the subarray level reduces the amount of extra hardware but generally will cause distortion in the far field pattern [6]. On the other hand, null formation by searching for the best coefficient setting avoids the expensive hardware but takes considerable time to form the nulls [4,5,7]. A simple algorithm for canceling specific sidelobe using edge elements alone is presented in El-Azhary *et al* [8].

In this paper, a partial amplitude control technique based on the minimax approximation is applied to suppress wide sectors in the prescribed directions. This technique determines the minimum number of perturbed edge elements and the corresponding current ratio excitations for the prescribed SLL, MBW, and wide band sectors suppression in arbitrary directions. The obtained number of perturbed edge elements and the corresponding current distribution that produces such a pattern are considered as being optimum in the minimax sense. The algorithm is based upon the simplex method of linear programming and uses a step-wise approach which successively increases the degree of approximation until the desired accuracy is achieved. The edge elements are chosen to suppress the wide band interferences because they are ideal for canceling specific sidelobes [8]. Furthermore, the number of the array elements does not affect the performance of this method.

Problem Formulation

Consider a linear array of (2N) isotropic equispaced elements with interelement spacing of d_0. When the current excitations are symmetrically even ($a_i = a_{2N-i+1}$)and because the element positions are symmetrically odd around the center of the array, then the array factor can be written as

$$F_o(u) = 2\sum_{i=1}^{N} a_i \cos(d_i ku) \tag{1}$$

where

$$d_i = d_o(i - \frac{2N+1}{2}) \tag{2}$$

a_i is the initial current excitation of the ith element, d_i is the element position with respect to the center of the array, u = sin (θ) , (θ is the scanning angle from broadside), and k is the wavenumber (2π/λ). In an interfering environment, J broad nulls are required in the pattern to suppress J wide band interferences at angular location u_j, j = 1,2,....., J. An array with conventional feed network has a frequency dependent antenna pattern such that a jth interference source at a fixed direction u_j appears to cover an angular pattern sector

$$\Delta u_j = B.u_j \tag{3}$$

where B is the relative frequency bandwidth. In practice only few elements are used to suppress the interferences when the number of interferences are much smaller than the number of array elements (2N). It is known that the far-field lobes of the edge elements of a uniformly excited array are nearly equal in width to the sidelobes of the array itself. Therefore, the edge elements are ideal for cancellation of specific sidelobes of the pattern. This fact supports the concept of partial adaptation using the edge elements to suppress wide sectors. Assume that the first and the last M elements are used to suppress the interference by perturbing the corresponding amplitudes of their current excitations. Therefore, the new current excitations can be written as

$$b_i = a_i + \xi_i \quad i = 1,2,\ldots,M \tag{4}$$

Where ξ_i are the current perturbations of the first and the last M elements. The current perturbations are constrained to be symmetrically even with respect to the center of the array. Then, the perturbed pattern can be written as

$$F(u) = 2\sum_{j=1}^{M} b_i \cos(d_i ku) + 2\sum_{i=M+1}^{N} a_i \cos(d_i ku) \tag{5a}$$

or

$$F(u) = F_0(u) + 2\sum_{i=1}^{M} \xi_i \cos(d_i ku) \tag{5b}$$

So it is required to minimize M and to determine the current perturbations for given SLL, MBW, and suppressed wide band sectors at prescribed locations. To calculate these current perturbations, let the desired function be defined as the initial antenna array in the main beam region and zero elsewhere, i.e.,

$$D(u) = \begin{cases} F_0(u) & 0 \le u \le u_o \\ 0 & \text{elsewhere} \end{cases} \tag{6}$$

where u_0 is the location of the first null of the initial pattern. Now, the perturbed pattern F(u) can approximate the desired function D(u). However, to maintain the level of the perturbed pattern for the whole jth sector within small quantity, δ^*_{2j} and at the same time maintain the main beam characteristics as the initial pattern within certain tolerance, δ^*_1, the perturbed pattern F(u) should satisfy the following equation

$$F(u) = \begin{cases} F_o(u) \pm \delta_1^* & 0 \le u \le u_o \\ 0 \pm \delta_{2j}^* & u = u_j \pm 0.5\Delta u_j, \quad j = 1,2,\ldots,J \\ 0 \pm \delta_3^* & \text{elsewhere} \end{cases} \tag{7}$$

where δ^*_3 is the tolerance in the sidelobe region. Then the following relation must hold

$$\delta_{2j}^* << \delta_3^* \quad j = 1,2,\ldots,J \tag{8}$$

Let the weighted deviation error E(u) be defined as

$$E(u) = W(u)[D(u) - F(u)] \tag{9}$$

where W(u) is a weighting function that enables the designer to choose the relative size of the error in different angular locations.

Minimum Edge Elements Approach

The minimum edge elements approximation problem with sector suppression can now be formulated as follows; find the minimum number of perturbed edge elements M and the corresponding set of coefficients $\{\xi_i\}$, (i = 1, 2, ..., M), that minimizes the maximum absolute value of E(u) over the region $u \in \{0,1\}$. Using the notation $\|E(u)\|_\infty$ to denote this minimum value [L_∞ norm of E(u)], the approximation problem may be stated mathematically as

$$\|E(u)\|_\infty = \min_{\{\xi_i\}} \left[\max_{u \in \{0,1\}} |E(u)| \right] \tag{10}$$

Let the weighted function W(u) be defined as

$$W(u) = \begin{cases} 1 & 0 \le u \le u_o \\ \dfrac{\delta_1^*}{\delta_{2j}^*} & u = u_j \pm 0.5\Delta u_j, j = 1,2,\ldots,J \\ \dfrac{\delta_1^*}{\delta_3^*} & \text{elsewhere} \end{cases} \tag{11}$$

According to the above discussion, the error E(u) should satisfy the inequality

$$|E(u)| \le \delta_1^* \tag{12}$$

In order to achieve the perturbed pattern with exact desired tolerances, the normalized coefficients must be used. Therefore, substituting Eq. (5b) into (7) and normalizing the result yields

$$\frac{F_o(u) + 2\sum_{i=1}^{M} \xi_i \cos(d_i ku)}{2\sum_{i=1}^{N} a_i + 2\sum_{i=1}^{N} \xi_i} = \begin{cases} \dfrac{F_o(u)}{2\sum_{i=1}^{N} a_i} \pm \delta_i & 0 \le u \le u_o \\ 0 \pm \delta_{2j} & u = u_j \pm 0.5\Delta u_j, j = 1,2,\ldots,J \\ 0 \pm \delta_3 & \text{elsewhere} \end{cases} \tag{13}$$

where δ_1, δ_{2j}, and δ_3 are the corresponding exact desired tolerances. Let the normalizing factor be defined as

$$\xi_o = \frac{1}{2\sum_{i=1}^{N} a_i + 2\sum_{i=1}^{N} \xi_i} \tag{14}$$

then the left hand side of eq. (13), which is the normalized perturbed pattern, can be written as

$$F_n(u) = \xi_o F_o(u) + \sum_{i=1}^{M} \xi_i^* \cos(d_i k u) \tag{15}$$

where

$$\xi_i^* = 2\xi_o \xi_i \tag{16}$$

and $F_n(u)$ is the normalized perturbed pattern. Substituting Eq. (15) in (13), then the following equation is obtained

$$\xi_o F_o(u) + \sum_{n=1}^{M} \xi_i^* \cos(d_i k u) = \begin{cases} F_{on}(u) \pm \delta_i & 0 \le u \le u_o \\ 0 \pm \delta_{2j} & u = u_j \pm \frac{1}{2}\Delta u_j, j = 1,2,\ldots,J \\ 0 \pm \delta_j & \end{cases} \tag{17}$$

where $F_{on}(u)$ is the normalized initial pattern. Consequently, the absolute value of the normalized weighted deviation error is given by

$$|E_n(u)| = W(u)|D_n(u) - F_n(u)| \le \delta_1 \tag{18}$$

In practice, to minimize the deviation error, Eq. (18) should be forced at a sufficient number of points, P, in the interval U {0,1}. Therefore, by forcing Equation (18) at P sample points $\{u_i, 1 = 1, 2, \ldots, P\}$ with P > M, the problem is reduced to an overdetermined system of linear equations, i.e.,

$$|E_n(u_i)| = W(u_i)|D_n(u_i) - F_n(u_i)| \le \delta_1 \quad \text{for } i = 1,2,\ldots,P \tag{19}$$

The normalized perturbed function Fn(u), which satisfies Eq. (19) in the minimax sense of Eq. (10) can be solved using the simplex method of linear programming. Therefore, the following dual linear programming of a maximization problem is formulated as maximize

$$\sum_{i=1}^{P} W(u_i) D(u_j)(S_i - t_i) \tag{20}$$

subject to

$$\sum_{i=1}^{P}(s_i - t_i)W(u_i)\phi_n(u_i) = 0 \tag{21}$$

$$\sum_{i=1}^{P}(S_i + t_i) \le 1 \tag{22}$$

$$S_i > 0, \quad t_i > 0 \tag{23}$$

where

$$\phi_n(u_i) = \begin{cases} F_o(u) & \text{for } n = 0 \\ \cos(d_n k u_i) & \text{for } n = 1,2,\ldots,M_1 \end{cases} \tag{24}$$

and M_1 is the expected maximum number of allowable controlled elements to be set by the designer. The parameters S_i and t_i are slack variables and the set of coefficients $\{\xi_o, \xi_1^* \xi_2^*, \ldots, \xi_M^*\}$ is evaluated as the marginal cost in the simplex tableau through steps 2 and 3 of the following optimization process:

Step 1. If an estimate of the minimum number of controlled edge elements, M_0, can be provided by the designer, set

$$n = M_o \tag{25a}$$

otherwise, set

$$n = 0 \tag{25b}$$

Step 2. Construct the initial simplex tableau to solve Eq. (20) subject to Eqs. (21-25).

Step 3. Utilize the efficient three stage algorithm that was developed by Barrodale and Phillips and the routine CHEB [9] for solving Eq. (20).

Step 4. If Eq. (19) is satisfied, go to step 7.

Step 5. If $(n+1)>M_1$, modify the required specifications and go to step 1.

Step 6. Add the following constraint to Eq. (21) to modify the number of perturbed edge elements from n to n+1

$$\sum_{i=1}^{P}(S_i - t_i)W(u_i)\cos(d_{n+1}ku_i) = 0 \tag{26}$$

then go to step 3.

Step 7. Calculate ξ_i and b_i according to Eqs. (4) and (16), where M=n.

A computer program has been developed to perform the above algorithm and some of the obtained results are given in the following section.

Results and Discussion

In this section, computer simulation results are presented to demonstrate the validity of this technique to suppress multiple wide band sectors in the sidelobe region using partial amplitude control. The simulations are based on a uniformly excited initial pattern with 100 equispaced linear array elements with interelement spacing of $\lambda/2$. The sample points, P, are chosen as 400 points in the main beam and sidelobe regions.

Figure 1a shows the initial pattern and Fig. 1b shows the perturbed pattern with three suppressed sectors imposed in the sidelobe region around the centers $u_j = 0.326$, 0.51, and 0.83 with relative bandwidths of 5%, 20%, and 10% respectively. All the sector depths, δ_{2j}, are reduced to -70 dB level while the SLL, δ_3, is set to 13 dB and the main beam tolerance, δ_1, is set to 0.01. Table 1 gives the computed current ratios which produce the perturbed pattern of Fig. 1b where only the first and the last twelve elements, which are determined by the algorithm, are perturbed to suppress the prescribed three sectors. Note that the current excitations have even symmetry which results in an even symmetry pattern. Therefore, the number of attenuators required is effectively halved.

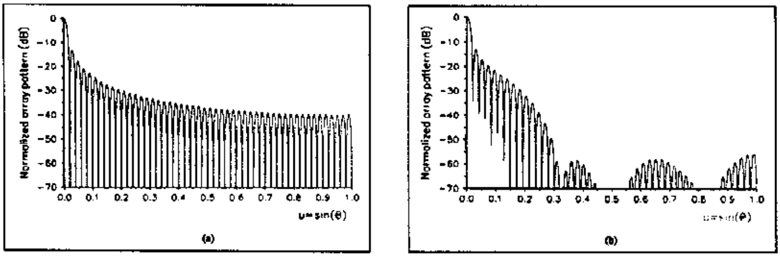

Fig.1. a- Uniformly excited initial array pattern with 2N=100 and $d_0 = \lambda/2$.
b- Simulated perturbed antenna array pattern with three wideband sectors imposed at u_j = 0.326, 0.51 and 0.83 for B=5%, 20% and 10% respectively.

Table 1. Computed edge element current ratios $\{b_n\}$ for Fig. 1

Elem no.	b_n	Elem no.	b_n
1,100	0.11191	7,94	1.25877
2,99	0.25452	8,93	1.25593
3,98	0.47973	9,92	1.20446
4,97	0.73357	10,91	1.12402
5,96	0.98020	11,90	1.05835
6,95	1.13453	12,89	1.00859
13,88	1.0		

Now, the performance of sector suppression with partial amplitude control technique using the minimum edge elements approximation algorithm is studied numerically. Fig. 2 shows the number of the first and last controlled elements, M, versus the sector depth when the center of the suppressed sector is imposed at $u_j = 0.45$ with 13 dB SLL and 0.01 main beam tolerance. It can be seen that as the sector depth increases the order M generally increases. Also, the achieved order M increases as the relative bandwidth of the jammer increases. Figure 3 shows the order M versus SLL of the perturbed pattern with fixed sector depth of 60 dB when the center of the imposed sector is at $u_j = 0.45$. As shown in the figure, the obtained order increases for larger SLL and larger relative bandwidth. Table 2 gives the order M needed to suppress one sector using different array sizes, 2N. The sector depth, centered at the peak of the sidelobe around $u_j = 0.45$, is suppressed to -80 dB while the SLL is set to 13 dB. Furthermore, the main beam tolerance is kept relatively constant for the different array sizes since the main beam width increases as the array size decreases. It is apparent that the order does not depend on the array size since a partial control is used (M<<N). Consequently, M degrees of freedom are used to suppress the same prescribed wide sector and (N-M) degrees of freedom are used to approximate the initial pattern. Finally, Fig. 4 shows the effect of varying the angular location of the suppressed sector on the number of controlled edge elements. From the figure, the number of controlled edge elements increases significantly in the vicinity of the main beam.

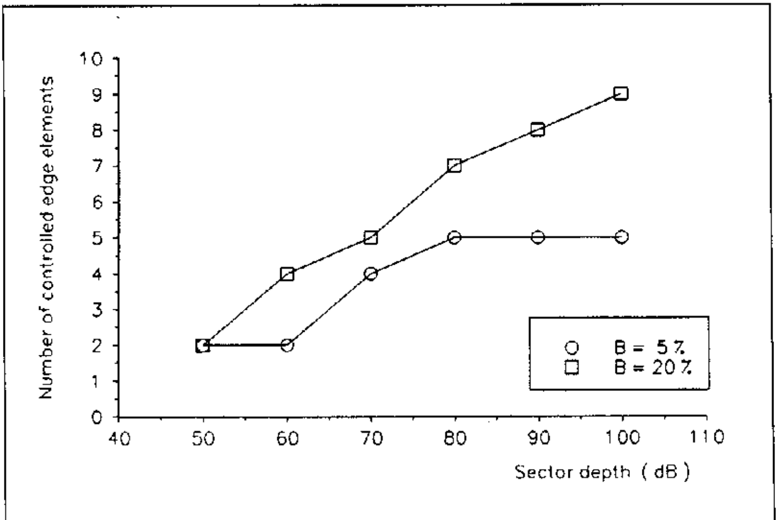

Fig.2 . Number of controlled edge elements versus sector depth for the perturbed pattern when the center of the suppressed sector is imposed at $u_j = 0.45$ with SLL=13 dB and $\delta_1 = 0.01$.

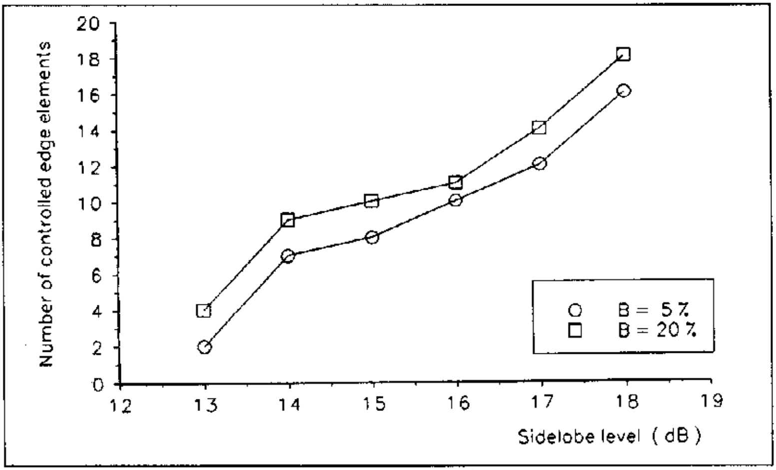

Fig.3 . Number of controlled edge elements versus SLL for the perturbed pattern when the center of the suppressed sector is imposed at $u_j = 0.45$ with sector depth=60 dB and $d_1 = 0.01$.

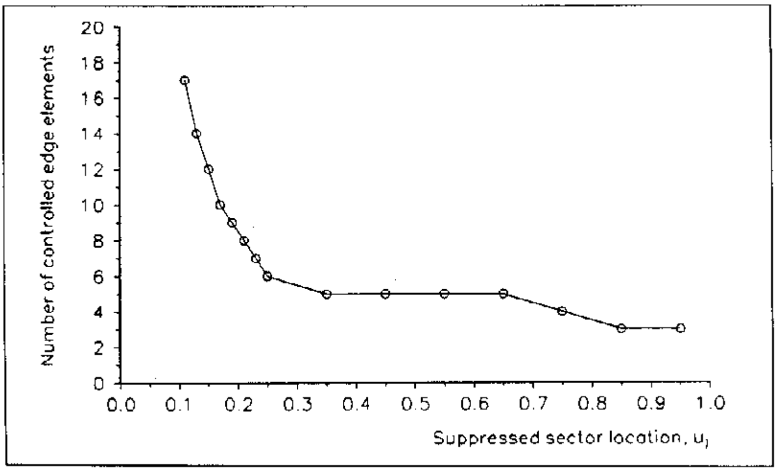

Fig.4 . Number of controlled edge elements versus angular location of the suppressed sector when B=10%, sector depth =60 dB, SLL=13 dB and $\delta_1 = 0.01$.

Table 2. Computed number of controlled edge elements, M, for different array sizes, 2N

2N	δ_1	M(*) B=5%	M(*) B=20%
200	0.0055	4	7
150	0.0073	4	7
100	0.0110	4	7
80	0.0137	4	7
60	0.0183	4	7

(*) The suppressed sector depth centered at the peak of the sidelobe around $u_j = 0.45$ is reduced to -80 dB and the SLL is set to 13 dB

Conclusion

This paper presented a simple procedure for multiple wide band sector suppression by controlling element current ratios in partially adaptive arrays. The above results demonstrate the capability of this method to achieve multiple wide band sector suppression with minimum number of controlled edge elements by maintaining the levels of the whole sectors below specified values while keeping the SLL and the MBW within required specifications. In effect, the pattern parameters can be controlled directly while achieving the desired levels of the prescribed sectors depths. Also, the performance of sector suppression using this technique is independent of the number of array elements. Furthermore, the main beam characteristics of the perturbed pattern is almost unchanged from the initial pattern.

This technique is based upon the simplex method of linear programming to solve an overdetermined system of linear equations, and uses a step-wise approach which successively increases the degree of approximation until the desired accuracy is achieved. The number of computations is greatly reduced since a partial amplitude control of edge elements is used (M<<N). Furthermore, if an estimate of the minimum number of controlled elements can be provided, then the number of iterations can be reduced. The obtained number of edge elements and the corresponding current distribution that produces such a pattern will be considered as being optimum in the minimax sense.

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تركيب نمط بتصفيرات عريضة ومتعدّدة لمصفوفات هوائيّة خطّية بتحكم جزئي لسعة التيار

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ملخص البحث . يقدّم هذا البحث طريقة لخفض مستوى التشويش العريض النطاق والمتعدّد بالتحكّم الجزئي لسعة تيّار عناصر المصفوفة . تحسب سعة تيّار العناصر الطرفيّة لمطابقة نمط معيّن باستخدام البرمجة الخطّية لحل معادلات فوق محدّدة. تمكّن هذه الطريقة التحكّم بأقل عدد من العناصر الطرفيّة للمصفوفة والحصول على مواصفات معيّنة للإشعاع الرئيس والإشعاعات الجانبية مع خفض مستوى التشويش المتعدّد. إنّ أداء هذه الطريقة لخفض مستوى التشويش المتعدّد مستقل عن عدد عناصر المصفوفة.