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Two-Phase Flow in Flowlines

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Abstract. This paper presents the results of study of a two-phae flow in flowlines using both the Dukler case-II correlation and dimensionless group analysis of Eaton. The Eaton correlation to calculate liquid holdup was used in Dukler case-II, to predict the pressure gradient in flowlines. A computer program to predict pressure gradients was written in Basica for use on a Personal Computer. The necessary equations to calculate liquid holdup were obtained with high accuracy using a Statistical Analysis System (SAS), instead of reading the related figure. Field data from Saudi flowlines was used to evaluate this method. Results show that this improvement in the Dukler-Eaton correlations predict reasonably accurate downstream pressures in flowlines, with an average percent difference of −3.1 and a standard deviation of 9.7 percent. The results obtained were compared with the correlations for liquid hold-up of Premoli and Friedel for pressure drop and proved to be more accurate than these correlations.

Nomenclature

d	=	pipe inside diameter, in
f	=	friction factor
g_c	=	dimensionless constant (32.2 Lb_f/Lbm − sq. sec.)
H	=	local holdup
L	=	pipe length, ft
n	=	number of tests
N_d	=	pipe diameter number
N_{gv}	=	gas velocity number
N_L	=	liquid viscosity number
N_{Lv}	=	liquid velocity number
N_{Re}	=	Reynold's number
P	=	pressure, psia
P_a	=	atmospheric pressure, psia
P_c	=	calculated pressure, psia

P_m	=	measured pressure, psia
P_{pr}	=	the reduced pressure
q	=	average flow rate cuft/sec
S	=	dimensionless function
t	=	the reciprocal, pseudoreduced temperature
v	=	velocity, ft/sec
y	=	the reduced density
Z	=	compressibility factor
λ	=	volume fraction of liquid
Δ	=	difference
ρ	=	density Lb_m/cuft
μ	=	viscosity, cp
σ	=	surface tension, dynes/cm

Subscripts

g	=	gas phase
L	=	liquid phase
m	=	mixture
o	=	oil
tp	=	two-phase

1. Introduction

The simultaneous flow of liquid and gas in pipes is encountered in different petroleum engineering operations. Many correlations have been developed to predict the pressure loss in horizontal pipes for two-phase flow [1,2,3,4]. The application of these methods requires the knowledge of the fluid properties and liquid holdup for each line increment. The Dukler correlation applies an iterative procedure which assumes a value of liquid holdup, calculates the two-phase Reynold's number, and then uses the holdup correlation graph to calculate the actual holdup. This procedure should be repeated until the assumed value agrees with the calculated value of holdup within a 5 percent error. In this research, equations for Eaton's [2] liquid holdup correlation were developed, and their accuracy in the Dukler correlation was studied.

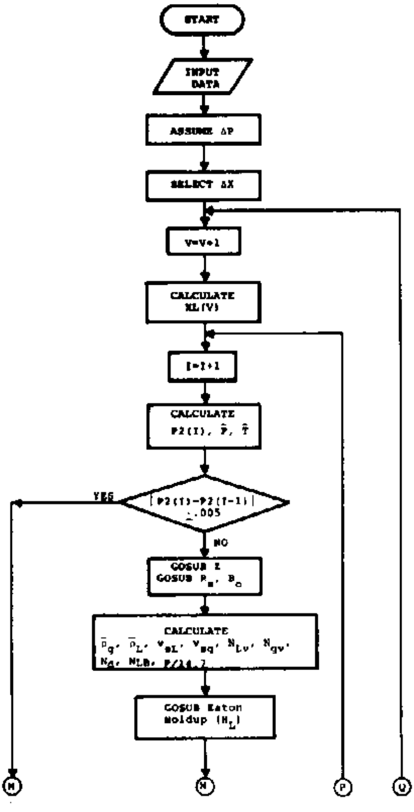
2. Program Design

Figure 1 shows the flowchart of the steps followed to calculate the pressure drop in the pipelines. The complete computer program written in Basica has two subroutines. One for calculating oil formation volume factor and gas solubility at operating pressures and temperatures by Lagrangian interpolation [5], while the other calculates the compressibility factor of gas using Hall-Yarborough equations [6,7], which were developed from the Starling Carnahan equation of state. These equations are as follows:

$$Z = \frac{0.06125\, P_{pr}\, t\ e^{-1.2(1-t)^2}}{y} \tag{1}$$

The reduced density (y) can be obtained from the solution of the equation:

$$F = -\ 0.06125\, P_{pr}\, t\, e^{-1.2(1-t)^2} + \frac{y + y^2 + y^3 + y^4}{(1 - y)^3} - (14.76t - 9.76t^2 + 4.58t^3)\, y^2 + (90.7t - 242.2t^2 + 42.4t^3)\, y^{(2.18 + 2.82t)} = 0 \tag{2}$$



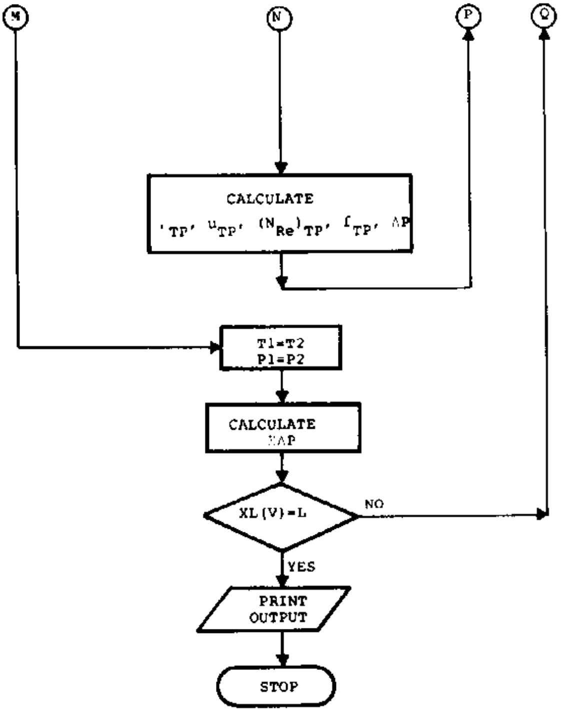


Fig. 1. Flow chart for the Dukler-Eaton correlation.

This non-linear equation can be solved for y using Newton-Raphson iterative technique.

Figure 2 shows the flow chart for calculating the gas compressibility factor. The following equations have been developed using the SAS method for the liquid holdup curve shown in Fig. 3.

For $X < 1$

$$H_L = 0.7562 + 0.6444 \log(X) + 0.1384 (\log(X))^2 \tag{3}$$

For $X \geqslant 1$

$$H_L = 0.75 + 0.3338 \log(X) - 0.1109 (\log(X))^2 \tag{4}$$

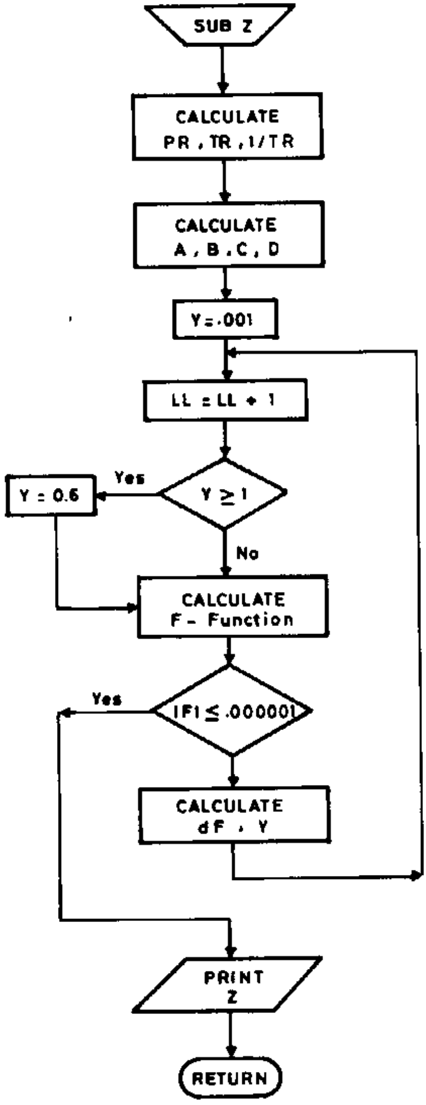
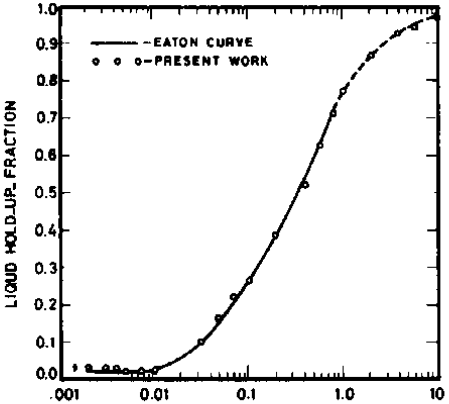


Fig. 2. Flow chart for calculating gas compressibility factor



$$\frac{N_{Lv}^{.575}}{N_{Gv}}\left(\frac{P}{P_B}\right)^{.05}\frac{1}{N_D^{.0277}}\left(\frac{N_L}{N_{LB}}\right)^{.1}$$

Fig. 3. Liquid hold-up data for 4", 6" and 8" flowlines.

where:

$$X = \frac{N_{Lv}^{0.575}}{N_{gv}\ N_d^{0.0277}}\left(\frac{P}{P_a}\right)^{0.05}\quad\left(\frac{N_L}{N_{LB}}\right)^{0.1} \tag{5}$$

and:

$$N_{Lv} = 1.938\ v_{sL}\ (\rho_L/\sigma_L)^{0.25}$$

$$N_{gv} = 1.938\ v_{sg}\ (\rho_L/\sigma_L)^{0.25}$$

$$N_d = 120.852\ d(\rho_L/\sigma_L)^{0.5}$$

$$N_{LB} = 0.00226$$

$$N_L = 0.15726\ \mu_L\ (1/\rho_L\ \sigma^3)^{0.25}$$

$$\frac{P}{P_a} = \frac{p}{14.7}$$

For two-phase flow, the friction factor was calculated from the following equation [1]:

$$f_{tp} = \frac{f_{tp}}{f_o} \times f_o \tag{6}$$

where:

$$\frac{f_{tp}}{f_o} = 1 + \left(\frac{-\ln\lambda}{S}\right) \tag{7}$$

Here:

$$S = 1.282 - 0.478\,(-\ln\lambda) + 0.444\,(-\ln\lambda)^2$$

$$- 0.094\,(-\ln\lambda)^3 + 0.00843\,(-\ln\lambda)^4 \tag{8}$$

$$f_0 = 0.00146 + 0.125\,(N_{Re})_{tp}^{-0.032} \tag{9}$$

and

$$\lambda = \frac{\bar{q}_L}{\bar{q}_L + \bar{q}_g} \tag{10}$$

Reynold's number for two-phase flow was calculated from the equation:

$$(N_{Re})_{tp} = \frac{1488\, d\, v_m\, \rho_{tp}}{\mu_{tp}} \tag{11}$$

where:

$$\rho_{tp} = \rho_L \frac{\lambda^2}{H_L} + \rho_g \frac{(1 - \lambda)^2}{(1 - H_L)} \tag{12}$$

and

$$\mu_{tp} = \mu_L\, \lambda + \mu_g (1 - \lambda) \tag{13}$$

Finally, if the acceleration term is neglected, the pressure loss due to friction is calculated from Dukler [1] equation:

$$\Delta P_{friction} = \frac{2\, f_{tp}\, L\, V_m^2\, \rho_{tp}}{12\, g_c\, d} \tag{14}$$

and,

Downstream pressure (P_2) = upstream pressure $(P_1) - \Delta P$

3. Field data

The data used for this study was obtained from reference [8], and is presented in Tables 1 and 2. The tests were conducted on 101 Saudi flowlines of 4, 6, and 8 inch diameters, with oil flow rates varying from 3533 to 16882 bbL/day and line lengths from 2241 to 19938 ft.

Table 1. Flowline data

Test No.	Well No.	Line diam. (In.)	Length of test section (Feet)	Rate STB/D	Pressure (PSIA)		Temperature (°F)			Average liquid viscosity (cp.)
					Up-Stream	Down-Stream	Up-Stream	Down-Stream	Ambient	
1	A 2	4	2,241	12,156	606	316	180	173	97	1.35
2	2	4	2,241	12,484	645	326	180	172	93	1.30
3	2	4	2,241	12,204	614	325	178	174	93	1.33
4	2	4	2,241	11,594	567	318	177	173	87	1.35
5	2	4	2,310	11,720	537	265	185	167	74	1.40
6	2	4	2,310	4,513	267	208	170	147	64	1.85
7	2	4	2,310	4,370	265	204	176	151	75	1.80
8	2	4	2,310	4,022	251	201	173	147	77	1.82
9	2	4	2,310	3,892	249	203	167	139	75	2.00
10	3	8	19,938	15,440	571	480	186	107	73	1.53
11	3	8	19,938	15,522	576	480	186	107	73	1.52
12	3	8	19,938	12,212	528	462	181	112	70	1.51
13	3	8	19,938	16,882	508	380	176	101	65	1.81
14	11	6	8,204	6,590	268	199	177	116	63	2.04
15	11	6	8,204	4,630	220	190	171	96	82	2.28
16	11	6	8,204	3,533	207	183	167	93	77	2.38
17	11	6	8,204	6,750	264	195	175	109	-	2.13

Table 2. Dukle-Eaton correlation – test analysis

Test No.	Diam In	Flow rate STB/D	P2 (measd) psi	P2 (calc) psi	%Err. P2
1	4.0260	12156	316	391.4534	–23.8776400
2	4.0260	12484	326	442.0341	–35.5932900
3	4.0260	12204	325	403.3040	–24.0935300
4	4.0260	11594	318	352.9414	–10.9878700
5	4.0260	11720	265	277.6588	– 4.7768880
6	0.0260	4513	208	196.4116	5.5713360
7	4.0260	4370	204	198.6303	2.6322160
8	4.0260	4022	201	192.4443	4.2565950
9	4.0260	3892	203	193.5710	4.6448280
10	7.8264	15440	480	473.4394	1.3667930
11	7.8264	15522	480	478.6172	0.2880987
12	7.8264	12212	462	461.9522	1.035033E-02
13	7.8264	16882	380	365.1636	3.9043310
14	6.0000	6590	199	199.6397	– 0.3214467
15	6.0000	4630	190	180.3423	5.0829840
16	6.0000	3533	183	182.6995	0.1642029
17	6.0000	6750	195	190.4737	2.3211830

The values of the fluid properties used, are as follows:

Gas-liquid ratio = 483 scf/bbl, API gravity = 32, and specific gravity = 1.2408.

Also the formation volume factor and gas solubility data are given in the computer program. Gas composition is given in Table 3. Pseudocritical pressures and temperatures were corrected [7,9] for the presence of N_2, CO_2 and H_2S gases, they are 698 psia and 467°R respectively.

Gas viscosity [9] was estimated after correction for the presence of N_2, CO_2, and H_2S gases to be 0.0127 cp and oil surface tension [10] 29 dynes/cm.

Table 3. Gas composition and its critical values (GOSP-A)

Component	Mole % Y_i	P_{ci} psia	$Y_i P_{ci}$	T_{ci} °R	$Y_i T_{ci}$
N_2	0.33	493	1.63	227	0.75
CO_2	11.03	1071	118.13	548	60.44
H_2S	3.03	1806	39.57	672	20.86
C_1	48.99	673	329.70	343	168.03
C_2	18.18	708	128.71	550	99.99
C_3	11.07	616	68.19	666	73.73
i-C_4	1.01	529	5.34	735	7.42
n-C_4	3.41	551	18.79	765	26.09
i-C_5	0.67	483	3.24	830	5.56
n-C_5	1.04	489	5.09	845	8.79
C_6	0.77	487	3.36	913	7.03
C_7	0.30	397	1.19	972	3.07
C_8	0.12	361	0.43	1024	1.23
C_9	0.05	332	0.17	1070	0.53
			Σ 723.50		Σ 483.00

4. Results

The computer output results show that the subroutines used to calculate the gas compressibility factor, gas solubility and oil formation volume factor at operating pressures and temperatures were accurate with average percent differences of 1, 2 and 0.5 respectively.

The statistical results of the calculations show that the modified Duckler-Eaton correlation yielded an average percentage error of −3.1, standard deviation of 9.7 and an average absolute percent error of 6.8.

The calculated liquid hold-up values using equations (3) and (4) were plotted on Eaton's curve in Fig. 3. The plotting shows that the curve is in a good agreement with the values. This correlation has been compared with the recent correlations using 30 test runs for Friedel [11] and Premoli [12] (see Table 4). The empirical correlation of Premoli calculates the hold-up and takes into account the mass flux effects and the average gas and liquid velocities ratio. Friedel correlation relates the two-phase frictional pressure gradient to the frictional pressure gradient for a single phase flow at the same total mass velocity, using the physical properties of the liquid phase and the two phase pressure drop multiplier. The statistical results yielded an average percentage error of −13.35% and a standard deviation of 29.55 for downstream pressures. It was also found that Premoli [12] correlation is not valid for superficial liquid velocities higher than 6 ft/sec.

The effect of the quality x (the ratio of the gas mass flow rate to the total mass flow rate) on the liquid hold-up is illustrated in Fig. 4. As the quality increases, the liquid hold-up decreases. Figure 5 shows that as the quality increases, the two phase

Table 4. Premoli-Friedel correlation - test analysis

Test No.	Diam In.	Flow rate psi	P2 (measd) psi	P2 (calc)	% Err. ΔP	% Err. P2
5	4	3700	275	312.9251	80.69173	−13.79095
6	4	941	276	282.0762	86.80335	− 2.201534
11	4	4513	208	252.5448	75.49971	−21.41578
12	4	4370	204	251.3555	77.63199	−23.21349
13	4	4022	201	239.0854	76.1707	−18.94794
14	4	3892	203	237.5457	75.09941	−17.0176
15	4	2216	195	208.5577	75.32077	− 6.952687
16	8	15440	480	535.3281	60.8001	−11.52668
17	8	15522	480	540.0536	62.5559	−12.51118
18	8	12212	462	504.352	64.1697	− 9.167097
19	8	7184	428	466.2765	79.74274	− 8.943112
20	8	2408	421	453.537	95.69681	− 7.728486
21	8	16882	380	464.5834	66.08079	−22.25879
22	8	13056	331	386.919	66.57023	−16.89396
23	8	7793	288	321.1923	73.7606	−11.52509
24	8	5315	269	295.7202	80.97038	− 9.933166
25	8	3667	269	298.7249	90.07556	−11.05015
26	6	6590	199	253.0715	78.36445	−27.1716
27	6	4630	190	211.5295	71.76507	−11.33133
28	6	3533	183	201.5935	77.473	−10.16039
29	6	2540	186	201.935	83.86861	− 8.56219
30	6	6750	195	248.2701	77.20306	−27.31801

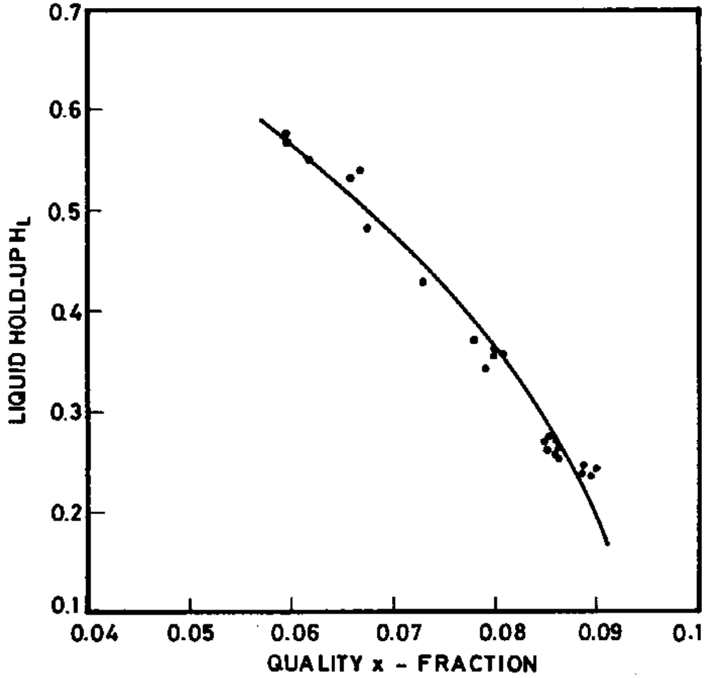


Fig. 4. Liquid hold-up as a function of quality

pressure gradient multiplier increases and consequently the percentage error in downstream pressure decreases.

Thus the statistical results were based on:

$$\text{Percent difference PD} = \frac{\text{P (measured)} - \text{(P calculated)}}{\text{P (measured)}} \times 100 \tag{15}$$

$$\text{Average percent difference APD} = \frac{\sum_{i=1}^{n} (\text{PD})_i}{n} \tag{16}$$

$$\text{Standard deviation SD} = \frac{\sum_{i=1}^{n} [(\text{PD})_i - (\text{APD})]^2}{n-1} \tag{17}$$

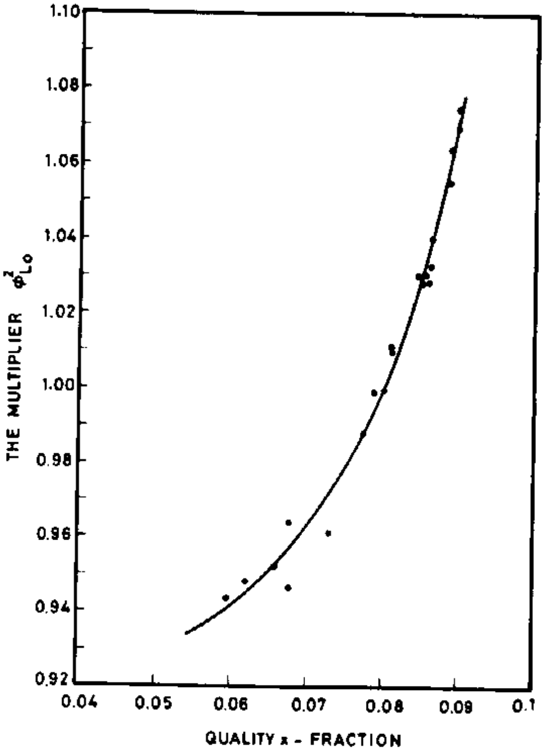


Fig. 5. Two-phase pressure multiplier ϕ^2_{Lo} as a function of quality.

5. Conclusions

Analysis of the two-phase flow field test data lead to the following conclusion:

The improved Dukler-Eaton pressure loss correlations gave good results of predicted pressure loss in 4 in., 6 in. and 8 in. diameter flow lines, proving that the liquid hold-up equations developed by the SAS method, are relatively accurate and simple.

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السريان الثنائي الأطوار في خطوط التدفق

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ملخص البحث: تم في هذا البحث دراسة نتائج استخدام كل من طريقتي التشابه لدوكلر (٢) وتحليل المجموعات اللابعدية لايتون لدراسة السريان الثنائي الأطوار. وقد استخدمت علاقة ايتون لحساب محتويات السائل بالأنبوب في علاقة دوكلر (٢) للتنبؤ بمعدل فقدان الضغط في خطوط التدفق. وقد تم تصميم برنامج لهذا الغرض على الحاسب الآلي الشخصي باستخدام لغة البيسك المتقدمة. وقد تم ايجاد المعادلات اللازمة لحساب محتوى السائل بالأنبوب بدقة باستخدام نظام التحليل الاحصائي (SAS) بدلاً من استخدام الرسوم البيانية. وقد استخدمت البيانات الخاصة ببعض خطوط التدفق السعودية لتقويم هذه الطريقة. وتبين النتائج أن الطريقة المحسنة لدوكلر ـ ايتون تتنبأ بمعدلات جيدة لحسابات الضغط في نهاية خطوط التدفق بمعدل فرق نسبي - ١ر٣٪ ومعدل انحراف ٧ر٩٪.

وقورنت نتائج هذه الطريقة بطريقتي بريمولي لحساب محتوى السائل بالأنبوب وفريدل لحساب معدل الفقدان في الضغط في الأنابيب، وأظهرت النتائج تفوق الطريقة المحسنة لدوكلر ـ ايتون في دقة حساب الضغط في نهاية خطوط التدفق بالمقارنة بطريقتي بريمولي وفريدل.