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The Specification of the Functional Form of the Labor Demand Equation

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Abstract. A flexible labor demand function is presented. In this function there is no prior assumption about the technology where the data decides on the appropriate functional form. Thus, the labor demand function avoids any estimation bias which could result from imposing the wrong restrictions on the technology. The estimation results suggest that the labor demand is non-linear function of input prices and the size of output. The demand for labor is significantly elastic with respect to the wage level, the price of capital, and the output level. A test of imposing prior assumptions about the technology shows that restricting the labor demand function to follow Cobb-Douglas, Generalized Leontief, or other specified functions gives different and sometimes insignificant elasticities.

Introduction

In estimating the labor demand function there is no theoretical presumption regarding the appropriate function of the labor demand. This paper is concerned with precisely the question of choosing empirically the appropriate functional form of the labor demand.

A Flexible Labor Demand Function

The specification of the labor demand function is controversial. The question raised is what labor demand function should be used. Whether to use the Cobb-Douglas [1] form which is attractable because it eases the calculation and the estimation of the labor elasticities. Or to use the Generalized Leontief function [2]. Yet some people have assumed the labor demand function has the translog form [3] which gives the labor share variables rather than the labor demand variables. And, finally, some people have used the Constant Elasticity of Substitution Function, CES [4]. Each of these forms imposes some restrictions on the labor demand behavior. For example the Cobb-Douglas and the CES assume a constant elasticity of substitution between the labor and the other production factors.

In this paper, a more general labor demand function is provided. Where there are no prior restrictions to be imposed on the labor behavior. In this case the data determine the shape of the labor demand function. The function that is proposed here is the Box-Cox labor demand function.

The proposed form is a general form of the Generalized Leontief labor demand function which produces the Cobb-Douglas, Generalized leontief, and other forms of the labor demand functions. This form is more flexible because it does not impose any restrictions on the labor relations and on the technology; i.e., it does not impose restrictions on the elasticity of substitution or on the rate of technical substitution.

Here the labor demand function is:

$$\frac{L^{\lambda} - 1}{\lambda} = a_1 \frac{Y^{\lambda} - 1}{\lambda} = a_2 \frac{X^{\lambda} - 1}{\lambda} \tag{1}$$

where

$$X = W^{-.5} R^{.5} Y$$

L: The labor demand.
W: The wage rate.
R: The interest rate.
Y: The output level.

This general equation has been proposed by Box and Cox [5].

Here

$$L^{\lambda} = \frac{L^{\lambda} - 1}{\lambda} \quad \forall \lambda \neq 0$$
$$= \ln L \quad \forall \lambda = 0 \tag{2}$$

Proof:

$$\lim_{\lambda \to 0} L^{\lambda} = \lim_{\lambda \to 0} \frac{\left(\frac{d}{d\lambda}\right)\left(L^{\lambda} - 1\right)}{\left(\frac{d}{d\lambda}\right)(\lambda)} = \lim_{\lambda \to 0} \left(L^{\lambda} \ln L\right) = \ln L$$

If we define the demand for capital to be:

$$\frac{\left(k^{\lambda} - 1\right)}{\lambda} = b_1 \frac{\left(Y^{\lambda} - 1\right)}{\lambda} + b_2 \frac{\left(Z^{\lambda} - 1\right)}{\lambda} \qquad (3)$$

where

$$Z = W^{.5} R^{-.5} Y$$

and define the cost functioin as

$$C = WL + RK \qquad (4)$$

then the cost function is:

$$C = W\left[1 + a_1\left(Y^{\lambda} - 1\right) + a_2\left(X^{\lambda} - 1\right)\right]^{\frac{1}{\lambda}} + R\left[1 + b_1\left(Y^{\lambda} - 1\right) + b_2\left(Z^{\lambda} - 1\right)\right]^{\frac{1}{\lambda}} \qquad (5)$$

Proof

Rewrite equation 1 as

$$\frac{L^{\lambda}}{\lambda} = \frac{1}{\lambda} + a_1 \frac{\left(Y^{\lambda} - 1\right)}{\lambda} + a_2 \frac{\left(X^{\lambda} - 1\right)}{\lambda}$$

If we multiply by λ

$$L^{\lambda} = 1 + a_1\left(Y^{\lambda} - 1\right) + a_2\left(X^{\lambda} - 1\right)$$

$$L = \left[1 + a_1\left(Y^{\lambda} - 1\right) + a_2\left(X^{\lambda} - 1\right)\right]^{\frac{1}{\lambda}} \tag{6}$$

Similarly:

$$K = \left[1 + b_1\left(Y^{\lambda} - 1\right) + b_2\left(Z^{\lambda} - 1\right)\right]^{\frac{1}{\lambda}} \tag{7}$$

substituting equations 6 and 7 for K and L in equation 4 gives:

$$C = W\left[1 + a_1\left(Y^{\lambda} - 1\right) + a_2\left(X^{\lambda} - 1\right)\right]^{\frac{1}{\lambda}} + R\left[1 + b_1\left(Y^{\lambda} - 1\right) + b_2\left(Z^{\lambda} - 1\right)\right]^{\frac{1}{\lambda}}$$

Here the Box-Cox procedure involves the specification of a general power function that contains the Generalized Leontief and Cobb-Douglas as special cases. For $\lambda = 1$, equation 1 becomes:

$$L = a_1 Y + a_2 X \tag{8}$$

which is the Generalized Leontief function. However, for $\lambda = 0$, equation 1 is:

$$LnL = a_1 LnY + a_2 \ln x \tag{9}$$

$$LnL = c_1 LnY + c_2 \ln R + c_3 LnW$$

which is equivalent to:

$$LnL = c_1 LnY + c_2 (LnW - LnR) \tag{10}$$

where:

$$c_1 = a_1 + a_2,\ c_2 = .5a_2,\ c_3 = -\ .5a_2$$

Proof:

$$\text{Ln L} = a_1 \text{ Ln Y} + a_2 \text{ Ln } (W^{-.5} R^{.5} Y)$$

$$\text{Ln L} = a_1 \text{ Ln Y} + a_2 (- .5 \text{ Ln W} + .5 \text{ Ln R} + \text{Ln Y})$$

$$\text{Ln L} = a_1 \text{ Ln Y} - .5\, a_2 \text{ Ln W} + .5\, a_2 \text{ Ln R} + a_2 \text{ Ln Y}$$

$$\text{Ln L} = (a_1 + a_2) \text{ Ln Y} - .5\, a_2 \text{ Ln W} + .5\, a_2 \text{ Ln R}$$

$$\text{Ln L} = c_1 \text{ Ln Y} + c_2 \text{ Ln R} + c_3 \text{ Ln W}$$

$$\text{Ln L} = c_1 \text{ Ln Y} + c_2 (\text{Ln W} - \text{Ln R})$$

The major point behind the use of this flexible form is that we avoid possible mis-specification of the functional form by choosing a possible wrong structure which may lead to inconsistent and biased estimates. Here we choose the appropriate functional form biased on an empirical investigation where no prior reasoning dictates the correct functional form. Estimating equation (1) makes the estimation technique itself choose the transformation which best fits the data.

Estimation and Results

In order to estimate the flexible labor demand function, equation (1), the non-linear least square estimation is applied where the following procedures are performed:

The first step is to simplify the estimation problem by dividing the variables by their geometric mean, which makes convergence of estimation easier. The λ value that maximizes the likelihood function (i.e., minimizes the error sum of squares) is chosen by systematic grid search, searching from $\lambda = -3$ to $\lambda = 3$ in steps of 0.1. $\lambda = 0.8$ gives the greatest likelihood functioin value. Repeating the search from $\lambda = 0.7$ to $\lambda = 0.9$ in steps of 0.01 gives $\lambda = 0.84$. Finally, searching from $\lambda = 0.83$ to $\lambda = 0.85$ in steps of 0.001 gives $\lambda = 0.844$ to be the value that minimizes the error sum of square.

The second step is to estimate the parameters of the original model (λ, a_0, a_1, a_2, σ^2). The last step of this procedure is to find the covariance matrix of parameter estimates from the original data. The covariance matrix is:

$$\sigma^2 \begin{vmatrix} X'X \; - \; X'U\lambda \\ -U'_\lambda X \, U'U_{\lambda\lambda} \; + \; U'_\lambda U_\lambda - 2T^{-1}\sigma^2 \, \Sigma \ln L \end{vmatrix}^{-1}$$

where U is the error term of the equation. and

$$U_\lambda = \frac{\partial U}{\partial \lambda} = \frac{\left(L^{0.844} \ln L\right)}{0.844} - \frac{\left(L^{0.844} - 1\right)}{(0.844)^2} - \frac{\left(Y^{0.844} - \mathrm{Ln}Y\right)}{0.844}$$

$$+ \frac{\left(Y^{0.844} - 1\right)}{(0.844)^2} - \frac{\left(X^{0.844} \ln X\right)}{0.844} + \frac{\left(X^{0.844} - 1\right)}{(0.844)^2}$$

$$U_{\lambda\lambda} = \frac{-2U_\lambda}{0.844} + \frac{\left[L^{0.844} (\ln L)^2\right]}{0.844} - \frac{\left[Y^{0.844} (\ln Y)^2\right]}{0.844} - \frac{\left[X^{0.844} (\ln X)^2\right]}{0.844}$$

T = the number of observations.

Results

The result of the estimation is:

$$L = -0.72 + 0.05\,Y - 0.02\,X \qquad R^2 = 0.948$$
$$(4.8) \quad (7.14) \quad (-\,6.06)$$

where the T-statistics are in parentheses.

The wage elasticity ω is calculated to be:

$$\omega = \frac{dL}{dX} \, \frac{dX}{dW} \, \frac{W}{L}$$

(1) The data are based on U.S. manufacturing during the period from 1929-1968. The purpose of this study is to show a specific econometrics technique of estimating a labor demand function. As a result, the period of the data should not affect the outcomes of this paper.

The estimated wage elasticity is:

$$\omega = \frac{(-0.2)\left(0.5\ W^{-0.5}\ R^{0.5}\ Y\right)}{L}$$

$$\omega = -0.40$$

The output elasticity of labor demand μ is:

$$\mu = 0.05\left(\frac{Y}{L}\right) = 1.965$$

If we impose some restrictions on our estimates to see how prior assumptions about the functional form give a different estimate. For example, if we restrict the value of λ = 1, the Generalized Leontief, the estimated equation is:

$$L = 0.40 + 0.32\,Y + 0.006\,X \qquad R^2 = 0.867$$
$$(5.35)\ (3.28)\qquad (-0.82)$$

The estimated wage elasticity is:

$$\omega = -0.12$$

However, the coefficient on X is not significant which suggests that the wage elasticity is not significantly different from zero.

Similarly, if we impose a prior assumption that the technology follows Cobb-Douglas, λ = 0, then we get an inelastic wage. Here the estimation result is:

$$LnL = -2.31 + 0.65\,LnY - 0.15\,(LnW - LnR) \qquad R^2 = 0.86$$
$$(-12.17)\quad (13.45)\quad (-0.895)$$

Conclusion

This paper has presented a flexible labor demand function. The presented function avoids a possible estimation bias which results from imposing the wrong restrictions on the production technology. The estimation results show that restricting the labor demand to follow either Cobb-Douglas or the Generalized Leontief form gives an insignificant wage elasticity. However, the flexible labor demand function shows significant labor demand elasticity.

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تحديد العلاقة الدالية للطلب على العمل

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ملخص البحث. يهدف هذا البحث إلى دراسة دالة الطلب على العمل مع الأخذ بعين الاعتبار عدم وضع أية قيود أو افتراضات مسبقة على تركيبة الدالة. دالة العمل المقترحة تعطي للبيانات الإحصائية الحرية في اختيار الدالة المناسبة دون تدخل من الباحث.

النتائج المتحصل عليها تؤكد أن دالة الطلب على العمل هي دالة غير خطية لا تتفق مع أي من الأشكال التي سبق طرحها مثل دالة كوب ـ دوقلاس أ ودالة ليونيتيف. بعد إجراء اختبار للدوال السابقة اتضح أن بعضها يعطي نتائج غير صحيحة خصوصًا فيما يتعلق بالمرونات السعرية للطلب على العمل.