J. King Saud Univ., Vol. 5, *Eng. Sci.* (1), pp. 43 - 53 (A.H. 1413/1993)

Hall Effects on Hydromagnetic Convective Flow in a Rotating Fluid Through Porous Medium

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(Received 17/4/1990; Accepted for Publication 30/9/1991)

Abstract. An analysis of the effects of Hall current on the combined thermal and mass diffusion through a porous medium bounded by a vertical plate is theoretically investigated when a strong magnetic field is imposed in a plane which makes an angle α with the normal to the plate. Analytical expressions for the velocity and temperature field in the boundary layer are derived. The effects of Hall current on the flow are studied for various values of α.

Nomenclature

C^+, C_w^+, C_∞^+ Species concentrations
C_p Specific heat at constant pressure
D Chemical molecular diffusivity
E Ekman number
E_c Eckert number
$\vec{E}$ Electric field
G Acceleration due to gravity
G_c Modified Grashof number
G_r Grashof number
H_0 Alpplied magnetic field
J_x^+, j_y^+, j_z^+ Components of current density
K^+ The permeability of porous media
M Magnetic parameter
m Hall parameter

P Prandtl number
$\vec{q}$ Velocity vector
S_c Schmidt number
T^+, $T_w{}^+$, T_∞^+ Temperatures
t^+ Time
u^+, v^+, w^+ Components of the velocity field $\vec{q}$
w_0 Suction velocity
x^+, y^+, z^+ Cartesian coordinates

Greek symbols

α angle
β coefficient of volume expansion
β^* coefficient of expansion with concentration
w_e cyclotron frequency
θ dimensionless temperature
θ^* dimensionless concentration
k thermal conductivity
γ kinematic viscosity
ϱ density of fluid
μ coefficient of viscosity
μ_e magnetic permeability
σ electrical conductivity
τ_2 electron collision time
τ skin-friction at the plate
τ_x skin-friction in the direction of x
τ_y skin-friction in the direction of y
τ_θ rate of heat transfer
e electric charge
n_e the number density of electron
P_e electron pressure

Introduction

In the last decade considerable progress has been made in the general theory of rota-ting fluids [1]. It is well known that in a rotating fluid near a flat plate Ekman layer exists wherein the viscous and coriolis forces are of the same order of magnitude. The effects of transverse magnetic field on such a layer was studied by Gupta [2], Raptis *et al.* [3] and Soundalgekar and Pop [4] . In an ionised gas where the density is low and/or the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiralling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both electric and magnetic fields. This phenomenon, well-known in the literature, is called the Hall effect. The study of magnetohydrodynamic viscous flows with Hall currents has important engineering applications in problems of magnetohydrodynamic generators and of Hall accelerators as well as in flight magnetohydrodynamics. Recent studies Pop [5,6], Dutta and Jana [7] on the hydromagnetic flows with Hall currents are mainly focused upon those where the magnetic field is imposed normal to the plate. Hence, the purpose of the present work is to analyse the effects of Hall current on hydromagnetic free convective flow past an infinite vertical porous plate in a rotating fluid with mass transfer through a porous medium when a strong magnetic field is imposed in a plane which makes an angle α with the normal to the plate.

Mathematical Analysis

Consider the steady magnetohydrodynamical flow in a porous medium bounded by a vertical porous plate ($z^+ = 0$) placed in a rotating fluid, when the fluid is exposed to a strong uniform magnetic field $\vec{H} = (0, H_0\sqrt{(1-\lambda^2)}, H_0\lambda)$, where $\lambda = \cos\alpha$. Since the plate is infinite in extent, all physical quantities, except pressure, are functions of z^+ only. The equation of continuity $\nabla \vec{q} = 0$ gives $w^+ = -w_0$ $(w_0 > 0)$, where $\vec{q} = (u^+, \upsilon^+, w^+)$. When the strength of magnetic field is very large, the generalized Ohm's law in the absence of the electric field [1] is of the form

$$\vec{J} + \frac{w_e \tau_e}{H_0\lambda}\vec{J} \times \vec{H} = \sigma\left(\mu e\,\vec{q} \times \vec{H} + \frac{1}{e\,n_e}\nabla . p_e\right) \qquad (1)$$

neglecting the electron pressure, thermoelectric pressure and ionslip, we have from equation (1)

$$j_{x^+} = \frac{\sigma H_0 \mu e \lambda}{1 + m^2\lambda^2}(\upsilon^+ + u^+ m\lambda)$$

$$j_{y^+} = \frac{\sigma H_0 \mu e \lambda}{1 + m^2 \lambda^2} (m\lambda \upsilon^+ - u^+) \tag{2}$$

where $m = w_e \tau_e$ is the Hall parameter.

We now consider further the case of a short-circuit problem in which the applied electric field $\vec{E} = 0$. Under these assumptions, the flow is governed by the following equations:

Momentum equations

$$-w_0 \frac{du^+}{dz^+} - 2\,\Omega\,\upsilon^+ = \gamma \frac{d^2u^+}{dz^{+2}} + g\beta\,(T^+ - T_\infty{}^+) + g\beta^*\,(C^+ - C_\infty{}^+) - \frac{\gamma}{K^+} u^+ + \frac{\mu e H^0}{\varrho} j_{y+} \tag{3}$$

$$-w_0 \frac{d\upsilon^+}{dz^+} + 2\,\Omega\,u^+ = \gamma \frac{d^2\upsilon^+}{dz^{+2}} - \frac{\gamma}{K^+} \upsilon^+ - \frac{\mu e H^0}{\varrho} j_{x+} \tag{4}$$

Energy equation

$$-w_0 \frac{dT^+}{dz^+} = \frac{k}{\varrho c_p} \frac{d^2T^+}{dz^{+2}} + \frac{\mu}{\varrho C_p} \left[\left(\frac{du^+}{dz^+} \right)^2 + \left(\frac{d\upsilon^+}{dz^+} \right)^2 \right] \tag{5}$$

Diffusion equation

$$-w_0 \frac{dC^+}{dz^+} = D \frac{d^2 C^+}{dz^{+2}} \tag{6}$$

The boundary conditions are

$$\begin{aligned} &u^+ = 0,\ \upsilon^+ = 0,\ T^+ = T_w^+,\ C^+ = C_w^+ \text{ at } z^+ = 0 \\ &u^+ \to 0,\ \upsilon^+ \to 0,\ T^+ \to T_\infty^+,\ C^+ \to C_\infty^+ \text{ as } z^+ \to \infty \end{aligned} \tag{7}$$

On introducing the following non-dimensional quantities

$$Q = \frac{u^+ + i\upsilon^+}{w_0},\ Z = \frac{w_0 z^+}{\gamma},\ \theta = \left(\frac{T^+ - T_\infty^+}{T_\infty^+ - T_\infty^+} \right)$$

$$\theta^* = \left(\frac{C^+ - C^+_\infty}{C^+_w - C^+_\infty} \right), P = \frac{\gamma \varrho C_p}{K}, Gr = \frac{\gamma g \beta (T^+_w - T^+_\infty)}{w_0^3}$$

$$Gc = \frac{\gamma g \beta (C^+_w - C^+_\infty)}{w_0^3}, E_c = \frac{w_0^2}{C_p (T^+_w - T^+_\infty)},$$

$$M^2 = \frac{\sigma \mu e^2 H_0^2 \gamma}{\varrho w_0^2}, E = \frac{\Omega \gamma}{w_0^2}, K = \frac{w_0^2 K^+}{\gamma^2}, Sc = \frac{\gamma}{D},$$

$$M_i = (2 E i + \frac{\lambda^2 M^2 (1 + i m \lambda)}{1 + \lambda^2 m^2} + \frac{1}{K})$$

in equations (3) - (6) and using equation (2), we have

$$Q'' + Q' - M_1 Q = - Gr\theta - Gc\, \theta^* \tag{8}$$

$$\theta'' + P\theta' = PE_c (Q' . \bar{Q}') \tag{9}$$

$$\theta^{*\prime\prime} + Sc\, \theta^{*\prime} = 0 \tag{10}$$

where bar denotes the complex conjugate of the corresponding quantity, primes denote the differentiation with respect to z and all the physical variables are given in the nomenclature. The corresponding boundary conditions are

$$Q = 0, \theta = 1, \theta^* = 1 \text{ at } z = 0 \quad Q \to 0, \theta \to 0, \theta^* \to 0 \text{ as } z \to \infty \tag{11}$$

To solve the equations (8)-(10) under the boundary conditions (11), we expand Q and θ in powers of E_c, the Eckert number under the assumption $E_c \ll 1$. This is justified in low speed incompressible flows. Hence we can write

$$Q = Q_0 + E_c Q_1 + 0 (E_c^2) \tag{12}$$

$$\theta = \theta_0 + Ec\, \theta_1 + 0 (E_c^2) \tag{13}$$

Substituting (12) and (13) in equations (8) and (9) and equating the like terms on both sides, we have the solution of the problem under modified boundary conditions as

$$\theta_0 = \exp(-Pz) \tag{14}$$

$$\theta^* = \exp(-Scz) \tag{15}$$

$$\theta_0 = A \exp(-hz) - A_1 \exp(-Pz) - A_2 \exp(-Scz) \tag{16}$$

$$\theta_1 = P \Big[A_3 \exp(-2\alpha z) + A_4 \exp(-2Pz)$$
$$+ A_5 \exp(-2Scz) + A_6 \exp(-(P+Sc))z$$

$$- (A_7 \cos\beta z + A_8 \sin\beta z)(\exp[-(\alpha+P)z]$$
$$- A_9 \cos\beta z + A_{10} \sin\beta z)(\exp[-(\alpha+Sc)z]$$
$$- A_{11} \exp(-Pz)\Big] \tag{17}$$

$$Q_1 = Gr\, P \Big[A_{21} \exp(-hz) - A_{12} \exp(-2\alpha z)$$
$$- A_{13} \exp(-2Pz) - A_{14} \exp(-2Scz)$$
$$- A_{15} \exp[-(P + Sc)z] + A_{16} \exp(-Pz)$$
$$+ (A_{17} \cos\beta z + A_{18} \sin\beta z)(\exp[-(\alpha+P)z])$$
$$+ (A_{19} \cos\beta z + A_{20} \sin\beta z)(\exp[-(\alpha+Sc)z])\Big] \tag{18}$$

where all the constants appearing in equations (16)-(18) are given below:

$h = \frac{1}{2}(1 + \sqrt{1+4M_1}) = \alpha + i\beta$

$M_1 = [2Ei + \frac{\lambda^2 M^2 (1+i\lambda m)}{1+\lambda^2 m^2} + \frac{1}{K}] = \alpha_1 + i\beta_1$

$A_1 = Gr/(P^2-P-M_1) = a_1 + ib_1, A_2 = Gc/(Sc^2-Sc-M_1) = a_2+ib_2$

$A = A_1 + A_2 = a + ib$

$a_3 = aa_1 + bb_1, \quad b_3 = ab_1 - a_1 b$

$a_4 = aa_2 + bb_2, \quad b_4 = ab_2 - a_2 b$

$a_5 = \alpha a_3 + \beta b_3, \quad b_5 = \beta a_3 - \alpha b_3, a_6 = \alpha a_4 + \beta b_4, b_6 = \beta a_4 - \alpha b_4,$

$a_7 = \alpha^2 + \alpha P - \beta^2, \quad b_7 = \beta(2\alpha + P)$

$a_8 = (\alpha + Sc)^2 - P(\alpha+Sc) - \beta^2, \quad b_8 = (2\alpha+2Sc-p)$

$a_9 = 2a_5 P/(a_7^2 + b_7^2), \quad b_9 = 2b_5 p/(a_7^2 + b_7^2)$

$a_{10} = 2a_6 S_c/(a_8^2 + b_8^2), \quad b_{10} = 2b_6 Sc/(a_8^2 + b_8^2)$

$a_{11} = (\alpha+p)^2 - (\alpha+p) - (\beta^2+\alpha_1), \quad b_{11} = \beta(2\alpha+2P-1)+\beta_1$

$a_{12} = (\alpha+Sc)^2 - (\alpha+Sc) - (\beta^2 + \alpha_1), \quad b_{12} = \beta(2\alpha+2S_c-1) + \beta_1$

$A_3 = (a^2+b^2)(\alpha^2+\beta^2)/(4\alpha-2p\alpha), \quad 3A_4 = \frac{1}{2} (a_1^2+b_1^2)$

$A_5 = Sc(a_2^2 + b_2^2)/(4Sc-2p), \quad A_6 = 2p(a_1 a_2 + b_1 b_2)/(p+Sc)$

$A_7 = (a_7 a_9 + b_7 b_9), \quad A_8 = (a_7 b_9 - a_9 b_7)$

$$A_9 = (a_8 a_{10} + b_8 b_{10}), \qquad A_{10} = a_8 b_{10} - a_{10} b_8$$

$$A_{11} = A_3 + A_4 + A_5 + A_6 - A_7 - A_9$$

$$a_{13} = A_7/(a_{11}^2 + b_{11}^2), \qquad b_{13} = A_8/(a_{11}^2 + b_{11}^2)$$

$$a_{14} = A_9/(a_{12}^2 + b_{12}^2), \qquad b_{14} = A_{10}/(a_{12}^2 + b_{12}^2)$$

$$A_{12} = A_3/(4\alpha\text{-}2\alpha - M_1), \qquad A_{13} = A_4/(4p^2\text{-}2p\text{-}M_1)$$

$$A_{14} = A_5/(4S_c^2\text{-}2S_c - M_1), \qquad A_{15} = A_6/(p+S_c)^2\text{-}(p+S_c)\text{-}M_1)$$

$$A_{16} = A_{11}/(p^2\text{-}p\text{-}M_1), \qquad A_{17} = a_{11}a_{13} + b_{11}b_{13}$$

$$A_{18} = a_{11}b_{13} - a_{13}b_{11}, \qquad A_{19} = a_{12}a_{14} + b_{12}b_{14}$$

$$A_{20} = a_{12}b_{14} - a_{14}b_{12}$$

$$A_{21} = A_{12} + A_{13} + A_{14} + A_{15} - A_{16} - A_{17} - A_{19}$$

Discussions of the Results

To discuss the Hall effects on the hydromagnetic free convection, we have presented the non-dimensional velocity components for P = 0.71 (air), Sc = 0.6, Gr = 5.0, Gc = 5.0, E_c = 0.01, M = 5.0, E = 0.2, and for different values of Hall parameter m, permeability parameter K and angle α in Figs 1 and 2. In selecting values of Sc, the

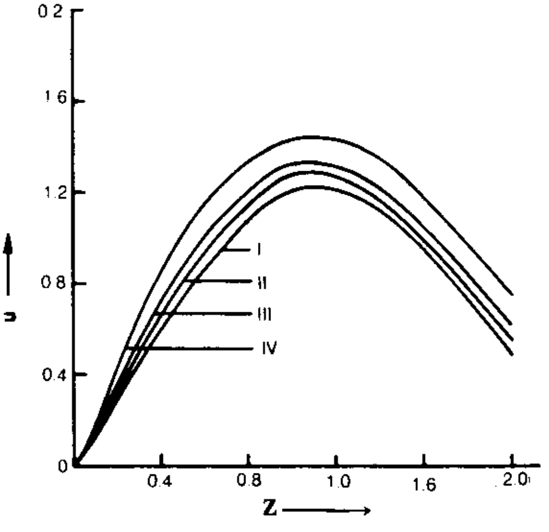

Fig. 1. Mean primary velocity profiles

Schmidt number, the diffusing chemical species of most common interest in air are considered. We have listed the values of the Schmidt number in Table 1 for different concentration species.

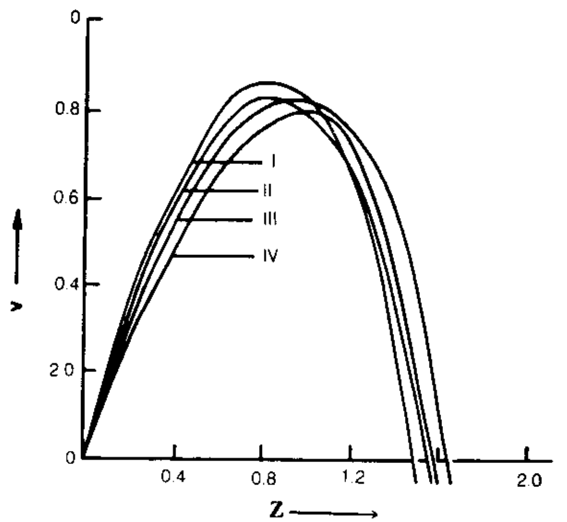

Fig. 2. **Mean secondary velocity profiles**

Table 1. **Thermodynamic and transport properties at 25° and latm.**

Species	Sc	P
H_2	0.22	0.71
H_e	0.32	0.71
H_2o	0.60	0.71
NH_3	0.78	0.71

The non-dimensional skin-friction τ on the plate z=0 is given by

$$\tau = \tau_x + i\tau y = \left.\frac{dQ}{dz}\right|_{z=0}$$

$$= PA_1 + Sc\, A_2 - hA + PGr\, Ec\, [\, 2\alpha\, A_{12} + 2PA_{13}$$

$$+ 2\,Sc\,A_{14} + (P+Sc)\,A_{15} - PA_{16} - (\alpha+P)\,A_{17}\,A_{18}$$
$$- (Sc + \alpha)\,A_{19} + A_{20} - h\,A_{21}\,]$$

and the rate of heat transfer τ_θ from the plate is

$$\tau_\theta = -\left.\frac{d\theta}{dz}\right|_{z=0}$$
$$= P + PEc\left[2\alpha A_3 + 2PA_4 + 2\,Sc\,A_5 + (P+Sc)\,A_6 - (\alpha+P)\,A_7 + A_8 - (\alpha+Sc)\,A_9 - A_{10} - PA_{11}\right]$$

The numerical values of τ_x , τ_y and τ_θ are calculated for different values of m and α and are entered in Tables 2 and 3 respectively.

Table 2. Values of τ_x and τ_y

	τ_x			τ_y		
m/α	0°	30°	45°	0°	30°	45°
0.5	4.064	4.075	4.092	-1.020	-1.007	-0.925
1.0	4.312	4.336	4.361	-1.835	-1.516	-0.205
1.5	4.648	4.654	4.659	-2.516	-2.013	-1.935

Table 3. Values of τ_θ.

m/α	0°	30°	45°
0.5	0.6590	0.6602	0.6634
1.0	0.6212	0.6238	0.6241
1.5	0.5489	0.5927	0.5539

From Fig. 1 and Fig. 2 it is seen that with the increase in Hall parameter m, and permeability parameter K (from curves I & II and I & IV), the primary velocity increases and decreases with the increase of angle α (from curves III & IV) whereas the secondary velocity profiles give reverse type of behavior with the increase of Hall parameter m, permeability parameter K and angle α.

From Table 2 it is observed that τ_x (the skin-friction) increases with the increase in Hall parameter m and angle α whereas τ_y decreases with the increase in m and increases with the increase in angle α. From Table 3 it is seen that the rate of heat transfer τ_θ increases with the increase of angle α whereas it decreases with the increase of the Hall parameter m.

Acknowledgement. The author is grateful to the referees for their valuable suggestions.

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تأثيرات «هول» على سريان الحمل الهيدرومغناطيسي في سائل دوّار خلال وسط مسامي

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ملخص البحث. تم التحليل النظري لتأثيرات تيار هول على انتشار الكتلة والحرارة المشترك خلال وسط مسامي محدود بلوح رأسي عندما يؤثر مجال مغناطيسي قوي في مستوى يميل بزاوية α مع العمود على اللوح وتم اشتقاق معادلات للسرعة وللمجال الحراري في الطبقة الحدودية وبذلك أمكن دراسة تأثيرات تيار «هول» على السريان لقيم مختلفة للزاوية α .