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AGRICULTURAL ENGINEERING

Friction Coefficient for Center Pivot Systems

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Abstract. An important objective of any center pivot irrigation system is to achieve a uniform distribution of water delivered through the sprinklers. In this study friction coefficients are developed for designing center pivot irrigation systems using two of the most common methods used for estimating friction head loss in sprinkler irrigation pipelines. A center pivot system operating under field conditions was used to verify the analysis. In the analysis, the change of sprinkler discharges along the lateral affects the Reynolds number and hence the state of flow along the lateral has been taken into account. The calculation of friction coefficients and friction head losses from the two equations are compared with the field data.

Introduction

Engineers are continually searching for simpler hydraulic calculation methods that do not need high accuracy. In a general pipe section, for example, it is of practical interest to estimate the locations of energy line and the hydraulic grade line. There are numerous methods for computing friction head loss in pipe lines. Some investigators [1;2] use the Hazen-Williams equation and other similar equations that account for the pipe material in estimating the friction head loss. While other researchers [3;4] consider the flow to be laminar or turbulent, hence the friction coefficient and the head loss are generally dependent on both the flow regime and the roughness of the pipe material. In the design of center pivot irrigation systems selection of appropriate pipe diameters, and calculation of the friction loss along the lateral are very important. A center pivot sprinkler irrigation system consists of a sprinkler line rotating about a single pivot point. The water is introduced at the pivot point and flows outward through the pivot lateral supplying each individual sprinkler heads. There is an interrelationship between the individual sprinkler discharges and sprinkler spacings along the lateral.

To accurately compute friction head loss in the lateral line, it is logical to start at the last outlet on the line and work back to the supply line, computing the discharge and friction loss between two successive outlets. This tedious process has been simplified for non-rotating sprinkler laterals by a procedure developed by Christiansen [5]. He developed an adjustment factor (called a friction coefficient) to correct the friction loss calculated using a general pipe friction loss equation that assumes all of the water is carried to the end of the line. This friction coefficient depends on the number of evenly spaced outlets along the lateral, the location of the first outlet relative to the lateral inlet and the material of which the lateral is made. This friction coefficient was derived by Christiansen [5] under the assumption that the sprinkler discharge is approximately the same from each of the sprinklers evenly spaced along the lateral. This assumption also holds true for linear-move systems when allowable variation in flow rate along lateral is not very great [6]. In Christiansen's analysis, the case of a pipe with uniform flow, the problem was oversimplified by assuming the friction coefficient to be constant along the pipe [7]. This assumption considers the flow to be wholly rough and depends upon the pipe roughness only which is not the case in practice [8]. Other investigators consider the design friction coefficient to be the average value at the inlet and outlet of the pipe [9].

There are many methods for computing friction head loss in pipe lines. Two of the most common and convenient methods applicable to the flow of water through irrigation sprinkler systems are the Hazen-Williams equation and the Darcy- Weisbach equation. Each equation uses a friction coefficient to account for decrease in flow rate along the lateral to estimate the friction head loss and the pressure distribution. Each friction coefficient must be so selected that each equation correctly yields the friction head loss.

In the case of center pivot sprinkler irrigation system design, the use of the friction coefficient derived by Christiansen [5] is not applicable, because discharges of the sprinklers located on the center pivot lateral vary (increase) considerably as the radial distance from the pivot increases. The sprinkler irrigation industry generally uses the Hazen-Williams equation for calculating the friction loss in pipes. This equation uses a constant factor for each pipe material that does not vary with Reynolds number. Chu and Moe [10] determined a friction coefficient for a center pivot system using the Scobey equation by calculating the average of the values at the pivot inlet and at the lateral end. Reddy and Apolayo [6] also derived a friction coefficient for center pivot systems, but they used a short pivot lateral with only 50 sprinklers. The objective of the research was to derive friction coefficients for Hazen-Williams and Darcy- Weisbach equations for design of center pivot systems with long laterals. The analysis takes into consideration the variable sprinkler discharges, the flow regimes and relative roughness along the lateral.

Friction coefficients

The change in total head from one sprinkler position on a center pivot line to another is a combination of the friction loss in the main line, and the branching flow losses at each sprinkler. The head loss due to friction in any particular section of a center pivot lateral between sprinklers may be calculated from the Hazen-Williams equation or Darcy-Weisbach equation. The general friction coefficient for each equation will be determined to be used to estimate actual total friction head loss along the center pivot lateral, when the pipe is assumed to carry the total flow for the full length of the lateral pipe.

The Hazen-Williams equation can be used to calculate friction head loss in any individual section along the pivot lateral. This equation is written as:

$$Hfi = k\ Li\ (Qi/C)^{1.852}\ .\ D^{-4.87} \quad (1)$$

in which Hfi = friction head loss in section i of the lateral (m); k = a constant, 1.22 $\times 10^{10}$; Li = length of section i of the lateral (m); Qi = flow rate in section i (L/s); C = Hazen-Williams coefficient which is based on the pipe material; D = inside diameter of center pivot lateral (mm) at section i. The total friction head loss that occurs along the lateral is computed by:

$$Hf = \sum_{i=1}^{N} Hfi \quad (2)$$

in which Hf = total friction head loss in the lateral; N = number of sprinkler outlets on the lateral.

If the center pivot irrigation system consists of a lateral of constant diameter and constant spacing between sprinklers (Li=L/N), and also the sprinkler discharge is proportional to its radial distance from the center pivot then Eq. 1 and 2 could be confined to give:

$$Hf = k\ Li.\ C^{-1.852}.\ D^{-4.87} \sum_{i=1}^{N} Qi^{1.852} \quad (3)$$

For convenience Eq. 3 can be put in the form:

$$Hf = B \sum_{i=1}^{N} Qi^{1.852} \qquad (4)$$

in which

$$B = K\,Li\,C^{-1.852}\,D^{-4.87} \qquad (5)$$

Therefore Eq. 4 could be written as:

$$Hf = B(Q^{1.852} + Q2^{1.852} + Q3^{1.852} + \ldots.. + QN^{1.852}) \qquad (6)$$

in which Q1 = Q, and Q is the discharge at the lateral inlet. Taking the individual sprinkler discharges into consideration (Fig. 1) then Eq. 6 can be written in the form:

$$Hf = B\,[\,Q^{1.852} + \sum_{i=1}^{N} (Q - \sum_{j=1}^{i-1} qj)^{1.852}\,] \qquad (7)$$

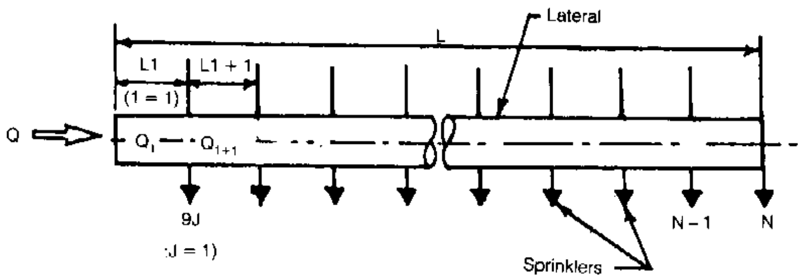

Fig. 1. Definition sketch of a center-pivot irrigation lateral.

in which qj is the flow rate from jth sprinkler on the lateral (Fig. 1). The discharge from any sprinkler in the lateral at any distance from the pivot with constant spacing between sprinklers is computed by (10):

$$qj = (2\,Q\,rj\,Ss)\,/\,L^2 \qquad (8)$$

in which rj = the distance between the jth sprinkler and the pivot point; and Ss = Sprinkler spacing which is constant. Since Q and L are constants, and also:

$$rj = J\ Ss \tag{9}$$

$$Ss = L/N \tag{10}$$

Substituting Eqs. 9 and 10 into Eq. 8 will give:

$$qj = (2Qj)/N^2 \tag{11}$$

Substitution of Eq. 11 into Eq. 7 and rearranging:

$$Hf = B\ Q^{1.852} [\ 1 + \sum_{i=2}^{N} (1-2/N\ N^2 \sum_{J=1}^{i-1} J\ )^{1.852}\ ] \tag{12}$$

For convenience Eq. 12 is rearranged and put in the form:

$$Hf = K.\ L.\ D^{-4.87}\ \ (Q/C)^{1.852}\ .\ Fc(N) \tag{13}$$

in which:

$$Fc(N) = 1/N + \sum_{i=2}^{N} 1/N\ [\ 1-2/N^2 \sum_{j=1}^{i-1} j\ ]^{1.852} \tag{14}$$

in which Fc(N) is therefore the general friction coefficient for the design center pivot irrigation systems, and is a function of only the number of equally spaced sprinklers on the lateral.

The Darcy-Weisbach formula can also be used for calculating energy losses in pipes [11, p. 73]. The friction coefficient in Darcy-Weisbach equation is a function of the relative roughness and Reynolds number, and the absolute roughness height is a constant for a given surface and the friction coefficient is dimensionally consistent. The Hazen-Williams uses a constant friction coeffieicnt equation for each pipe material. Because when the Darcy-Weisbach equation is properly used with the correct roughness height for the pipe material, variation in pipe diameter does not introduce any significant error, as it does with other commonly used equations [11]. The Darcy-Weisbach equation:

$$Hfi = f.\ Li/D\ .\ Vi^2/2g \tag{15}$$

in which Hfi is the friction head loss in pipe section i; f is the friction coefficient in pipe section i; Vi is the average velocity in section i; and g is the acceleration due to gravity. For circular pipe Eq. 15 may be put in the form:

$$Hfi = (\ 8f.\ Li.\ Qi^2)\ /\ \pi^2\ D^5\ g \tag{16}$$

The Darcy-Weisbach equation calculates the friction head loss from the pipe inlet to the section under consideration. Since each zone on the Moody diagram (Fig. 2) has a corresponding friction coefficient formula, it is convenient to analyze each case separately. The friction coefficient in the Darcy-Weisbach equation is a function of the relative roughness of the pipe material and the Reynold's number (Rn). The Reynold's number is defined by the following equation

$$Rn = Vi\ D/\gamma \tag{17}$$

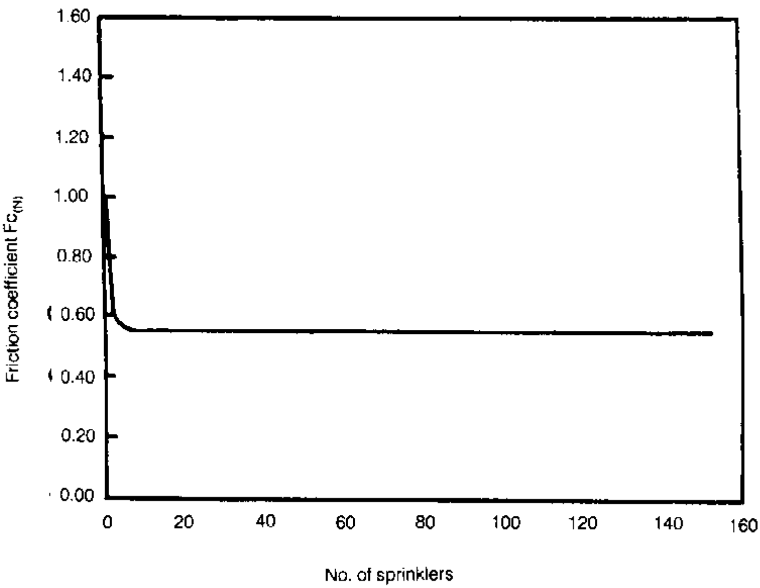

Fig. 2. Friction head loss in the lateral at a center pivot system computed by the two equations.

in which γ is the kinematic viscosity of the water.

The flow rate is usually high in a center pivot lateral, and hence, the flow is generally turbulent. The Darcy-Weisbach friction coefficient could be evaluated by the following equation [12]:

$$f = \frac{1.325}{[\ln (E/3.7D + 5.74/Rn^{0.9})]^2} \tag{18}$$

in which E is the absolute roughness of the pipe. The preceeding formula yields an error of less than 1% in the range $10^{-6}<E/D<10^{-2}$ and $5\times10^3 < Rn < 10^8$. This equation covers a significant portion of the turbulent zone on Moody diagram and it has been used in many engineering applications [13].

Results and Discussions

The center pivot system used in this analysis has a steel lateral pipe, the length was 402 m with diameter of 168.22 mm. There were a total of 154 spray-type nozzles connected to drop tubes hanging from the lateral at a uniform spacing of 2.55 m. The flow rate at the pivot was 54.7 l/sec. The sprinkler discharge was measured from each individual nozzle by the volumetric measurements. These measurements were made to check the design equations. It is of practical importance to calculate the friction coefficients and friction head loss from measured data.

The values of the general friction coefficient ,Fc(N), were calcualted using Eq. 14, as a function of the number of uniformly spaced sprinklers on the center pivot lateral. The calculated values obtained are presented in Fig. 3. The shape of the curve in Fig. 3 shows that the change of friction coefficient values, Fc(N), with respect to the number of outlets is relatively large for a small number of sprinklers (less than 6), and the change in the calculated Fc-value is very small when the number is greater than 10. The Fc-value became constant (Fig. 3) when there were more than 30 sprinklers. It can also be noted from Fig. 3 that when the number of sprinklers is from 31 to 154 the Fc-value is 0.548. Since this is the practical range of number of sprinklers for most center pivot designs, this Fc-value could be used in Hazen-Williams equation (Eq. 13) for calculating total friction head loss in a center pivot lateral without causing any significant error.

The values of friction coefficient, f, for each pipe section were calculated using Eq. 18, as a function of the relative roughness and Reynolds number. The calculated

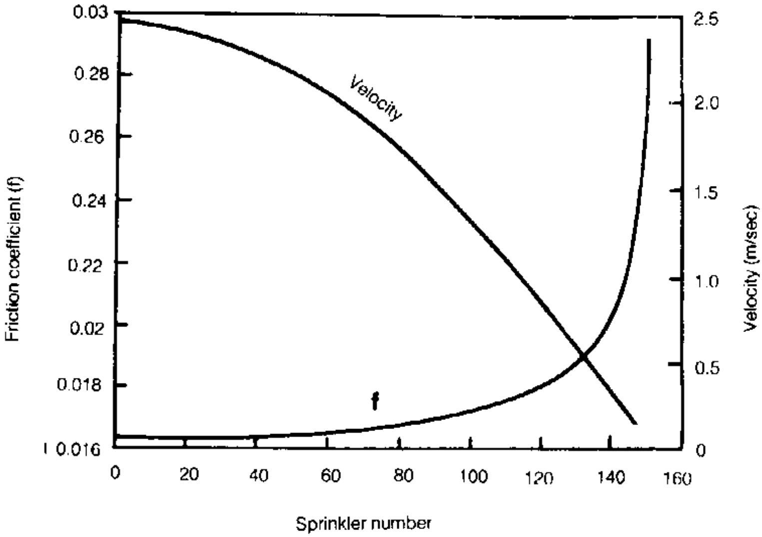

Fig. 3. Relationship between friction coefficient ($Fc_{(N)}$) and number of sprinklers on center pivot lateral.

f values and the change in water velocity along the pivot lateral are shown in Fig. 4 which indicates that f values are increasing along the lateral. This increase is very small up to sprinkler number 100, and then increased rapidly as shown in Fig. 4. Also, the water velocity is decreasing along the pivot lateral (Fig. 4). The average f value of 0.017 could be used in this experiment and other similar conditions to calculate the friction head loss for each section along the lateral without causing any significant problem. The friction coefficient approaches a constant value for large Reynold's numbers [14, p. 347]. The Reynold's number ranged from 424907 near the pivot to 10268 at the end of the lateral. This means that the flow is turbulent throughout the pivot lateral.

Equation 13 was used to calculate the total friction loss at the end of the lateral line, which Eq. 16 was used to calculate friction head loss at each section along the lateral. However, it is also important to know the friction head loss rate along the line (Fig. 2). As the total discharge decreases with respect to the length of the line, the energy gradient will not be a straight line but is a curve. The friction head loss computed by Eq. 13 was compared with the summation (using Eq. 2) of friction head loss computed by Eq. 16 for individual sections as shown in Fig. 2.

Therefore, the constant value of Fc obtained from Fig. 3 is only function of number of sprinklers, and hence, can safely be used as friciton coefficient in Hazen-Williams equation to compute friciton head loss for all discharges, pipe material and diameter, and number of sprinklers above 10. But in Eq. 18 friction coefficient "f" for Darcy-Weisbach equation is dependent on Reynold's number and relative roughness. Therefore the constant value when computing friciton head loss ($f = 0.017$) is only valid for the only discharge, material and diameter used in this study, and also for number of sprinklers not exceeding about 100. For any other discharges, material or diameter the f value should be calculated using Eq. 18 and average value of f determined by drawing figure similar to Fig. 4.

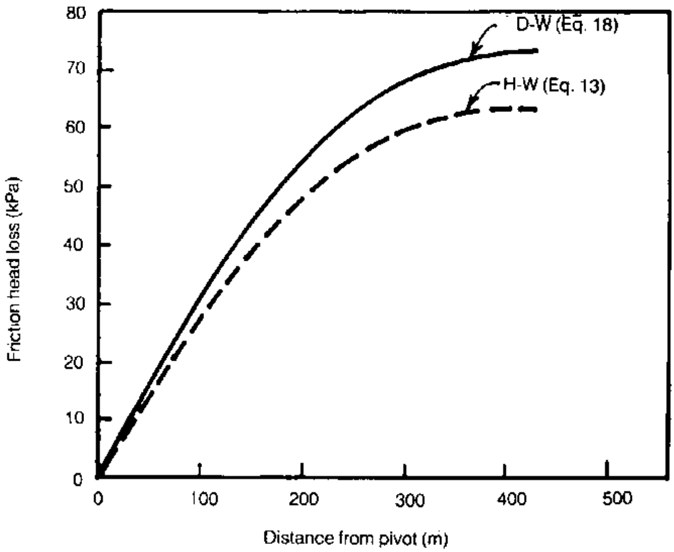

Fig. 4. Relationship between friction coefficient (f), flow velocity and sprinkler number on center pivot system.

Conclusion

The study derived two friction coefficients for two of the most common head loss equations used in sprinkler irrigation laterals to calculate the friction head loss in a center pivot irrigation system. The change of individual sprinkler discharge and the effect of variation of Reynold's number and accordingly the state of flow along the lateral is considered and discussed. The results of the calculation of friction head losses along the lateral are compared.

This derived friction coefficient for center pivot systems will assist to determine more accurately the pressure distribution along the lateral, and select the appropriate pipe size for the irrigation systems.

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معامل الاحتكاك لنظام الري المحوري

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(استُلم في ٢/١/١٩٩٣م؛ قُبل للنشر في ٢٢/٦/١٩٩٣م)

ملخص البحث. الهدف ار[illegible] من نظام الري المحوري أن يتم توزيع المياه من الرشاشات بكفاءة جيّدة. في هذه الدراسة تمّ إيجاد معامل احتكاك لمعادلتين تستخدم بكثرة عند تقدير فواقد الاحتكاك في أنابيب الري عند تصميم نظم الري المحوري. وقد تم التحقق من النتائج باستخدام نظام ري محوري يعمل تحت الظروف الحقليّة. وأشارت النتائج إلى وجود علاقة بين تصرف الرشاشات على طول خط الرش المحوري ورقم رينولد ونظام السريان. وتم مقارنة نتائج المعادلتين ومناقشتها.