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Propulsion of Two Ciliated Micro-Organisms Along Their Line of Centres

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Abstract. A fluid mechanical model for the propulsion mechanism of two ciliated micro-organisms moving along their line of centres is studied. The overall effect of cilia is replaced by velocities prescribed as functions of the polar angle θ on spherical control surfaces. The corrections in the free stream velocity due to the presence of a second organism are obtained. The stream line patterns are analysed for particular cases.

1. Introduction

In their natural habitates bacteria or micro-organisms generally organise themselves in colonies. Therefore the study of multiple organism systems is more useful than the study of a single organism situation. Latter conditions can represent the limiting case at low dispersed phase. In dispersions interactions between organisms can be of primary importance. The magnitude of the interaction among the organisms is, in general, governed by the following variables **a)** their shapes and sizes, **b)** the distance between them, **c)** their orientations with respect to each other, **d)** their individual orientations relative to the direction of external forces like gravity, and **e)** their velocities and spins relative to the fluid at infinity.

The complexity of ciliary movement restricts most of the theoretical models for ciliary locomotion developed by several authors [1,2,3], to simpler geometries like planar and cylindrical surfaces of infinite extent or a finite spherical surface. Keller and Wu [4] have improved the spherical model of Blake [1] by considering a prolate spheroidal control surface, upon which velocities were prescribed to represent ciliary propulsion. These models have made important contributions to the understanding of several aspects of the problem concerned with steady or quasi-steady state.

In this paper an attempt is made to study the propulsion mechanics relating to the motion of two organisms along their line of centres (Fig. 1). This study is an extension of the model of Blake [2] to two body systems. It is supposed that the organisms are separated by a large distance but sufficiently close to interact hydrodynamically. The effect of the beating cilia is replaced by surface velocities pre-

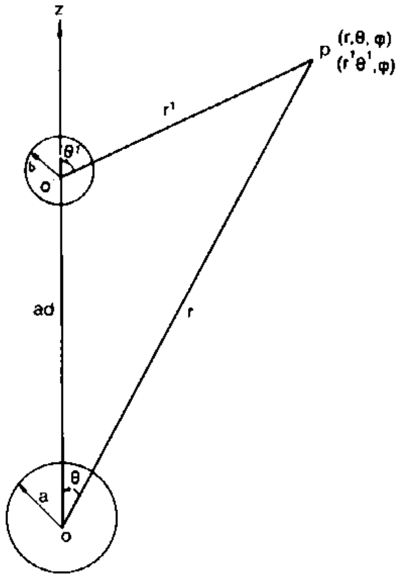

Fig. 1. Geometric configuration of two bodies moving along their line of centres

scribed as functions of the polar angle θ on porous spheres. The net flux of the fluid into the cilia layer is assumed to be zero. Expressions for the free stream velocities are obtained by considering the fluid mechanical forces experienced by the organisms as zero [5]. The effects in the flow field due to the presence of a second organisms are estimated up to $O(l/e^3)$ where e is the distance between the sphere centres. The streamline patterns for particular cases are analysed.

In addition to knowing the methods used lead to suitable approximate solutions of the quasi-steady equations of motion, it is desirable to know how well the predicted effects will be realised physically. Experimental work has lagged behind the theory; this is because until very recently the techniques available were not sufficiently accurate. Experimental confirmation of the phenomena observed for organism is much less conclusive. It is hoped that the present investigation will serve to lay the ground work for further improvement of the model that will enhance its versatility and accuracy in application to the phenomena of ciliary propulsion.

2. Formulation of the Problem

The radii (a,b) of the spherical organisms are assumed to take any positive values and the velocities of the organisms in the general case are different. The organisms move with instantaneous velocities U_oU_a and U_oU_b in an otherwise unbounded medium which is at rest at infinity and possess no spin. The organisms are assumed to be very close to neutral buoyancy. The polar axis of the spherical polar system of coordinates is taken along the line of centres of the organism with the origin at either of the centres. The problem considered here is inherently unsteady. However, one may neglect the unsteady terms compared with the viscous terms under the situations discussed in the following two paragraphs.

In order to assess the relative magnitudes of the time dependent terms ($\partial q/\partial t$) and the inertial terms ($\bar{q}.\nabla\bar{q}$) compared with the viscous terms in the Navier-Stokes equations the following dimensionless quantities are introduced.

$$\bar{q}^* = \frac{\bar{q}}{U_o}, \quad t^* = \frac{tU_o}{e} \qquad \text{and} \qquad \nabla^* = L\nabla \tag{1}$$

Thus

$$\frac{\bar{q}.\nabla\bar{q}}{\gamma\nabla^2\bar{q}} = \left(\frac{U_oL}{\gamma}\right)\frac{\nabla^*\bar{q}^*}{\nabla^{*2}\bar{q}^*} \tag{2a}$$

and

$$\frac{\partial\bar{q}/\delta t}{\gamma\nabla^2\bar{q}} = \left(\frac{U_oL^2}{\gamma e}\right)\frac{\partial\bar{q}^*/\delta t^*}{\nabla^{*2}\bar{q}^*} \tag{2b}$$

where L is a length scale over which the viscous terms change, U_0 is a characteristic velocity, e = ad is the distance between the centres of organisms, (Fig. 1), and is large enough to neglect $1/e^n$ for $n \geqslant 4$.

The fluid movement due to the propulsion of a micro-organism is observed to be confined in the neighbourhood of its surface of a thickness around the equator equal to the radius of the body [6,7]. Therefore, the viscous effects cannot spread beyond a region of radius $r = O(e)$ in the presence of the second organism when measured from either centre. When $e/r = O(1)$ and $L = r$ the flow will be steady since the local Reynolds number $R_1 = U_or/\gamma << 1$. When $(e/r) << 1$ and $L = e$ the flow can be considered as steady for the Reynolds number $R_e = U_oe/\gamma$ is small. Consequently, quasi-steady flow prevails in the whole of the fluid if $R_e << e/r$, which is equivalent to the condition $U_oL^2/e << 1$. Cooley and O'Neill [8] have shown that this condition is in accordance with the experimental data of Mackay and Mason [9] for solid spheres. Therefore one may neglect the unsteady and inertial terms provided

$$\left(\frac{U_o L^2}{\gamma e}\right) << 1 \text{ and also } \frac{\partial \bar{q}^*/\partial t^*}{\nabla^{*2}\bar{q}^*}, \text{ and } \frac{\nabla^* \bar{q}^*}{\nabla^{*2}\bar{q}^*} \quad \text{are of order unity.}$$

The radius, stream function and pressure are expressed non-dimensionally by;

$$r = \frac{r^*}{a}, \psi = \frac{\psi^*}{U_o a^2}, p = \frac{a\,p^*}{(U_o \varrho \gamma)} \tag{3}$$

Here * denotes the physical variables, ϱ is the density of fluid and ψ is the stream function. Introducing the Stokes stream function and using (3), the form-invariant linearized equation of motion (under the coordinate transformations given in equation (8) later) for an axi-symmetric flow is governed by the equation

$$D^4\psi = R_2\{r^{-2}\frac{\partial(\psi.D^2\psi)}{\partial(r,\mu)} + 2r^{-2}D^2\psi L\psi] \tag{4a}$$

and

where

$$D^2 = \frac{\partial^2}{\partial r^2} + \frac{1-\mu^2}{r^2}\frac{\partial^2}{\partial \mu^2}, \quad L = \frac{\mu}{1-\mu^2}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \mu} \tag{4b}$$

and $\mu = \cos\theta$

A Stokes approximation to the equation (4a) is

$$D^4\psi = 0 \tag{4c}$$

The equation (4c) is solved under the prescribed boundary conditions

$$-\frac{\partial\psi}{\partial\mu} = \sum_{n=1}^{\infty} A_n P_n(\mu) \tag{5a}$$

$$-\frac{\partial\psi}{\partial r} = \sum_{n=1}^{\infty} B_n P_n^1(\mu) \tag{5b}$$

on $r = 1$,

$$\left.\begin{aligned} -\frac{\partial\psi}{\partial\mu'} &= \beta^2 \sum_{m=1}^{\infty} C_m P_m(\mu') \qquad (6a) \\ -\frac{\partial\psi}{\partial r'} &= \beta \sum_{m=1}^{\infty} D_m P_1^1(\mu') P_m^1(\mu') \qquad (6a) \end{aligned}\right\} \quad \text{on } r' = \beta\,,$$

and

$$\psi \approx \begin{cases} -\frac{1}{2} U_a r^2 (1-\mu^2) \quad \text{as} \quad r \longrightarrow \infty & (7a) \\ -\frac{1}{2} U_b r'^2 (1-\mu'^2) \quad \text{as} \quad r' \longrightarrow \infty & (7b) \end{cases}$$

Here $\beta = b/a$, $\mu' = \cos\theta'$, U_a and U_b are the nondimesionalized velocities with which corresponding organisms move along their line of centres. The primed coordinate is centred at the centre of the organism of radius 'b' (Fig. 1).

3. Solutions

Let (r,θ,ϕ) and $(r\eta,\theta',\phi')$ denote the spherical polar coordinates of the same point in space with respect to O and O' respectively. Since the equation (4c) is form-invariant with respect to the coordinate transformations

$$\left.\begin{aligned} r\sin\theta &= r'\sin\theta', \\ r\cos\theta &= r'\cos\theta' \\ \phi &= \phi' \end{aligned}\right\} \qquad (8)$$

an appropriate solution of (4c) in presence of both the organisms moving along their line of centres is either

$$\psi = -\frac{1}{2} U_a r^2 (1-\mu^2) + \sum_{n=1}^{\infty} [E_n r^{-n} + F_n r^{-n+2}] P_1^1(\mu) P_n^1(\mu)$$

$$+ \sum_{m=1}^{\infty} [G_m r'^{-m} + H_m r'^{-m+2}] P_1^1(\mu') P_m^1(\mu') \qquad (9a)$$

or

$$\psi = -\frac{1}{2} U_b r'^2 (1-\mu'^2) + \sum_{n=1}^{\infty} [E_n r^{-n} + F_n r^{-n+2}] P_1^1(\mu) P_n^1(\mu)$$

$$+ \sum_{m=1}^{\infty} [G_m r'^{-m} + H_m r'^{-m+2}] P_1^1(\mu') P_m^1(\mu') \tag{9b}$$

with respect to the centre O or O′ respectively, where E_n, F_n, G_m and H_m are constants. The following equations of the transformation are true when $r, r' < d$ [10].

$$\frac{P_n^1(\mu')}{r'^{n+1}} = \frac{(-1)^{n+1}}{d^{n+1}} \sum_{j=1}^{\infty} \frac{(n)_{j+1}}{(j+1)!} \left(\frac{r}{d}\right)^j P_j^1(\mu), \tag{10a}$$

$$\frac{P_n^1(\mu)}{r^{n+1}} = \frac{1}{d^{n+1}} \sum_{j=1}^{\infty} (-1)^{j+1} \frac{(n)_{j+1}}{(j+1)!} \left(\frac{r'}{d}\right)^j P_j^1(\mu'), \tag{10b}$$

$$\frac{P_n^1(\mu')}{r'^{n-1}} = \frac{(-1)^{n-1}}{d^{n-1}} \sum_{j=1}^{\infty} \left[\frac{(2n-1)j-(n-2)}{(n+j)(2j-1)} - \frac{(2n-1)}{(2j+3)} \left(\frac{r}{d}\right)^2\right] \frac{(n)_{j+1}}{(j+1)!} \left(\frac{r}{d}\right)^j P_j^1(\mu), \tag{11a}$$

$$\frac{P_n^1(\mu)}{r^{n-1}} = \frac{1}{d^{n-1}} \sum_{j=1}^{\infty} (-1)^{j-1} \left[\frac{(2n-1)j-(n-2)}{(n+j)(2j-1)} - \frac{(2n-1)}{(2j+3)} \left(\frac{r'}{d}\right)^2\right] \frac{(n)_{j+1}}{(j+1)!} \left(\frac{r'}{d}\right)^j P_j^1(\mu'), \tag{11b}$$

In view of the relations (10a,b) and (11a,b) the equations (9a,b) can be expressed in terms of (r,θ), (r',θ') and using the boundary conditions (5a,b) and (6a,b), and get infinite sets of equations involving the constants E_n' F_n, G_m, and H_m for n,m = 1,2,3,..., Retaining terms up to $O(1/d^3)$, the above constants are calculated as

$$E_1 = \frac{\beta}{4}[\frac{3U_b}{2d}(1-\frac{3}{5d^2}+\frac{9\beta}{4d^2}-\frac{\beta^2}{3d^2})-\frac{U_a}{\beta}(1+\frac{9\beta}{4d^2})+A_1(\frac{1}{\beta}-\frac{3}{4d^2})$$

$$-B_1(\frac{2}{\beta}+\frac{3}{2d^2})+\frac{C_1}{2}(\frac{1}{d}-\frac{3}{5d^3}+\frac{9\beta}{4d^3}+\frac{\beta^2}{d^3})$$

$$+D_1(\frac{1}{d}-\frac{3}{5d^3}+\frac{9\beta}{4d^3}-\frac{\beta^2}{d^3})-\frac{\beta}{d^2}(C_2+3D_2)$$

$$-\frac{3}{2d^3}(A_2+3B_2)+\frac{3\beta^2}{2d^3}(C_3+4D_3)] \quad (12a)$$

$$F_1 = \frac{\beta}{4}[\frac{3U_b}{2d}(-3+\frac{1}{d^2}-\frac{27\beta}{4d^2}+\frac{\beta^2}{d^2})+(3U_a+A_1+2B_1)(\frac{1}{\beta}+\frac{9}{4d^2})$$

$$+\frac{C_1}{2d}(-3+\frac{1}{d^2}-\frac{27\beta}{4d^2}-\frac{3\beta^2}{d^2})+D_1(-\frac{3}{d}+\frac{1}{d^3}-\frac{27\beta}{4d^3}+\frac{3b^2}{d^3})$$

$$+\frac{3\beta}{d^2}(C_2+3D_2)+\frac{9}{2d^3}(A_2+3B_2)-\frac{9\beta^2}{2d^3}(C_3+4D_3)] \quad (12b)$$

$$G_1 = \frac{\beta^4}{4}[\frac{U_a}{2d}(\frac{3}{\beta}-\frac{1}{\beta d^2}+\frac{27}{4d^2}-\frac{9\beta}{5d^2})-\frac{U_b}{\beta}(1+\frac{9\beta}{4d^2})+\frac{A_1}{2d}(\frac{1}{\beta}+$$

$$\frac{1}{\beta d^2}+\frac{9}{4d^2}-\frac{3\beta}{5d^2})+\beta_1(\frac{1}{\beta d}-\frac{1}{\beta d^3}+\frac{9}{4d^3}-\frac{3\beta}{5d^3})$$

$$+C_1(\frac{1}{\beta}-\frac{3}{4d^2})-D_1(\frac{2}{\beta}+\frac{3}{2d^2})+\frac{1}{\beta d^2}(A_2+3B_2)$$

$$+\frac{3\beta}{2d^3}(C_2+3D_2)+\frac{3}{2\beta d^3}(A_3+4B_3)] \quad (12c)$$

$$H_1 = \frac{\beta}{4}[\frac{3U_a}{2d}(-3 + \frac{\beta^2}{d^2} + \frac{1}{d^2} - \frac{27\beta}{4d^2}) + \frac{A_1}{2d}(-3 + \frac{\beta^2}{d^2} - \frac{3}{d^2} - \frac{27\beta}{4d^2})$$

$$+ B_1(-\frac{3}{d} + \frac{\beta^2}{d^3} + \frac{3}{d^3} - \frac{27\beta}{4d^3}) - \frac{3}{d^2}(A_2+3B_2) - \frac{9\beta}{2d^3}(A_3+4B_3)$$

$$- \frac{9\beta^2}{2d^3}(C_2+3D_2)] + (3U_b+C_1+2D_1)(1 + \frac{9\beta}{4d^2}) \qquad (12d)$$

$$E_2 = -\frac{1}{2}(B_2 + \frac{\beta^2}{d^3}C_2 + \frac{3\beta^2}{d^3}D_2 - \frac{H_1}{d^2}) \qquad (13a)$$

$$F_2 = \frac{1}{6}(A_2 + 3B_2 + \frac{5\beta^2}{d^3}C_2 + \frac{15\beta^2}{d^3}D_2 - \frac{5}{d^2}H_1) \qquad (13b)$$

$$G_2 = -\frac{\beta^5}{2}(\frac{A_2}{d^3} + 3B_2 + \frac{D_2}{\beta} + \frac{F_1}{d^2}) \qquad (13c)$$

$$H_2 = \frac{\beta^3}{6}(\frac{5}{d^3}A_2 + \frac{15}{d^3}B_2 + \frac{C_2}{\beta} + \frac{3D_2}{\beta} + \frac{5}{d^2}F_1) \qquad (13d)$$

$$E_3 = -\frac{1}{2}(\frac{A_3}{12} + B_3 - \frac{H_1}{d^3}) \qquad (14a)$$

$$F_3 = \frac{1}{2}(\frac{A_3}{4} + B_3 - \frac{7H_1}{5d}) \qquad (14b)$$

$$G_3 = -\frac{\beta^5}{2}(\frac{C_3}{12} + D_3 - \frac{\beta^2}{d^3}F_1) \qquad (14c)$$

$$H_3 = \frac{\beta^3}{2}(\frac{C_3}{4} + D_3 - \frac{7\beta^2}{5d^3}F_1) \qquad (14d)$$

and for $n \geqslant 4$

$$E_n = -[(n-2)A_n + n(n+1)B_n]/[2n(n+1)] \tag{15a}$$

$$F_n = [A_n + (n+1)B_n]/[2(n+1)] \tag{15b}$$

$$G_n = -\beta^{n+2}[n(n+1)D_n + (n-2)C_n]/[2n(n+1)] \tag{15c}$$

$$H_n = \beta^n[(n+1)D_n + C_n]/[2(n+1)] \tag{15d}$$

The expression for ψ can be obtained in terms of (r,θ) as

$$\begin{aligned}\psi(r,\mu) = [& -\frac{1}{2}U_a r^2 + \frac{E_1}{r} + F_1 r + G_1\frac{r^2}{d^3} + H_1\left(\frac{r^2}{d} - \frac{r^4}{5d^3}\right) \\ & - \frac{\beta^2 r^2}{2d^2}(C_2+3D_2) + \frac{3\beta^2 r^2}{4d^3}(C_3+4D_3)]\,P_1^1(\mu)\,P_1^1(\mu) \\ & + [\frac{H_1}{d^2}\left(\frac{r^3}{3} - \frac{5}{6} + \frac{1}{2r^2}\right) + \frac{1}{2}\left(\frac{A_2}{3} + B_2 + \frac{5\beta^2}{3d^3}C_2 + \frac{5\beta^2}{d^3}D_2\right) \\ & - \frac{1}{2r^2}\left(B_2 + \frac{\beta^2}{d^3}C_2 + \frac{3\beta^2}{d^3}D_2\right) - \frac{r^3\beta^2}{3d^3}(C_2+3D_2)]\,P_1^1(\mu)\,P_2^1(\mu) \\ & + [\frac{H_1}{d^3}\left(\frac{r^4}{5} - \frac{7}{10r} + \frac{1}{2r^3}\right) + \frac{1}{8r}(A_3+4B_3) - \frac{1}{24r^3}(A_3+ \\ & 12B_3)]\,P_1^1(\mu)\,P_3^1(\mu) + \sum_{n=4}^{\infty}\ [-\frac{1}{2r^n}\left(B_n + \frac{(n-2)A_n}{n(n+1)}\right) \\ & + \frac{1}{2r^{n-2}}\left(B_n + \frac{A_n}{n+1}\right)]\,P_1^1(\mu)\,P_n^1(\mu)\end{aligned} \tag{16}$$

The stream function in terms of (r',θ') can similarly be obtained. The pressure expressions in the neighbourhood of the spheres $(r,r' < d)$ are calculated from the momentum equations. The nondimensionalized drag force D, on the organism of radius 'b' is obtained as

$$D_b\left(= -\frac{\text{drag}}{(6\pi\varrho\gamma U_o a)}\right) = \frac{4}{3}\ \frac{H_1}{\beta^2} \tag{17}$$

where H_1 is given in (12d)

Similarly the drag D_a on the organism of radius a is obtained by writing $\beta = 1$ and replacing H_1 by F_1 in the equation (17). The speeds of free-propulsion of two organisms are obtained from the equations $D_a = 0$, $D_b = 0$, respectively as

$$V_b = -\frac{1}{3}[(C_1+2D_1) - \frac{3}{d^2}(A_2 + 3B_2) - \frac{2}{d^3}(A_1 - B_1)$$

$$-\frac{9}{2d^3}(A_3 + 4B_3)] \tag{18a}$$

$$V_a = -\frac{1}{3}[(A_1+2B_1) + \frac{3\beta^2}{d^2}(C_2+3D_2) - \frac{3\beta^3}{d^3}(C_1 - D_1)$$

$$-\frac{9\beta^3}{2d^3}(C_3+4D_3)] \tag{18b}$$

4. Discussion

From the equations (18a,b) it is clear that the organisms move in the same direction with zero relative velocity or the organisms approach to or recade from each other with the same speed one gets

$$(C_1+2D_1) = \pm (A_1+2B_1), \tag{19}$$

$$(A_2+3B_2) = \mp \beta^2 (C_2+3D_2), \tag{20}$$

and

$$[(A_1 - B_1) + \frac{9}{4}(A_3+4B_3)] = \pm \beta^3[(C_1-D_1) + \frac{9}{4}(C_3+4D_3)] \tag{21}$$

The free stream velocity with which the organisms move in such a situation is

$$V_0 = \mp \frac{1}{3} \Big| [(A_1+2B_1) - \frac{3}{d^2}(A_2+3B_2) - \frac{2}{d^3}[(A_1-B_1)$$

$$+\frac{9}{4}(A_3+4B_3)]]\Bigg| \tag{22}$$

It is clear from (18 a,b) that the interaction caused by the presence of the second organism reduces (or increases) the speed of the free propulsion of a single organism (radius 'a') by a factor of $O(\beta^2/d^2)$ if (A_1+2B_1) and (C_2+3D_2) are of opposite sign (or of same sign).

If $A_n = B = 0$, $n \geq 1$ the organism with radius 'a' is inert and experiencing drag $D_a = -(4/3)F_1$. The free stream velocity V_b of the organism with radius 'b' is then obtained from the equation $H_1 = 0$ as

$$V_1 = -\frac{1}{3}[(C_1 + 2D_1) + \frac{3U_a}{2d}(-3 + \frac{1}{d^2} + \frac{\beta^2}{d^2})$$

$$-\frac{9\beta^2}{2d^3}(C_2 + 3D_2)] \tag{23}$$

Here U_a is the velocity of the inert sphere. If

$$U_a = \frac{3\beta^2(C_2 + 3D_2)}{(1 + \beta^2 - 3d^2)} \tag{24}$$

then the free stream velocity V_1 is the same as that of the case of a single organism in an unbounded fluid [7].

The corresponding results of two inert spheres moving with velocities U_a and U_b along their line of centres can be obtained from the present analysis by putting $A_n=B_n=C_n=D_n = O$ for $n = 1,2,3,\ldots$. The drag formulae in (17) are then reduced to

$$D_a = U_b[-\frac{3\beta}{2d} + \frac{\beta}{2d^3}(1 - \frac{27\beta}{4} + \beta^2)]$$

$$+ U_a(1 + \frac{9\beta}{4d^2}) + O(\frac{1}{d^4}) \tag{25a}$$

$$D_b = U_a[-\frac{3}{2d\beta} + \frac{1}{2d^3}(\frac{1}{\beta} - \frac{27}{4} + \beta)]$$

$$+ U_b [(\frac{1}{\beta} + \frac{9}{4d^2})] + O(\frac{1}{d^4}) \tag{25b}$$

The above two results agree with the results of Happel and Brenner [11] if terms up to the $O(1/d^3)$ are retained.

The results of a single organism [2] can be deduced from the present analysis as $d \rightarrow \infty$.

The stream function with respect to the laboratory frame is

$$\psi' = \frac{1}{2} U_a r^2 \sin^2\theta + \psi \tag{26}$$

where ψ is given by (16). The stream lines $\psi' \times 10^5$ = constant are drawn in a meridian plane in Figs (2a,b,c) as they would appear to an observer in front of whom the sphere is passing at that moment. The constants A_n, B_n, C_n, D_n are taken to be zero for $n \geqslant 2$. These stream lines due to the three dimensional doublet originate from the fore-region of the body surface and close on its aft region except for the trivial one along the z-axis.

In case of a freely swimming organism for which A_1, C_1, B_1 and D_1 are prescribed such that D=0, is considered. Figure 2 project the stream pattern for a self-propelling spheres of equal radius $\beta = b/a = 1.0$ with **a)** $A_1 = C_1 = 0.01$, $B_1 = D_1 = -1.505$, and **b)** $A_1 = -C_1 = 0.0$, $B_1 = -D_1 = -1.5$. The stream lines for a self-propelling non identical spheres $\beta = 1.5$ with $A_1 = C_1 = 0.0$, $B_1 = D_1 = -1.5$ is shown in Fig 2(c). Comparison of the Figs 2(a)and 2(b) indicates that if $A_1 = C_1 \neq 0$ (here $A_1 = C_1 > 0$) the disturbance spreads at a faster rate (thereby affecting a larger volume of fluid) than when $A_1 = C_1 = 0$. However, in case of Fig. 2(c) the disturbance spreads at still more faster rate than that of Figs (2a,b). It should be pointed out that the analysis given here is valid only in the neighbourhood of the organisms (Stokes region). However, if a uniform flow field valid over the entire flow region is obtained the stream line pattern schematically will resemble the Figs (3a,b) in situations specified in the legends.

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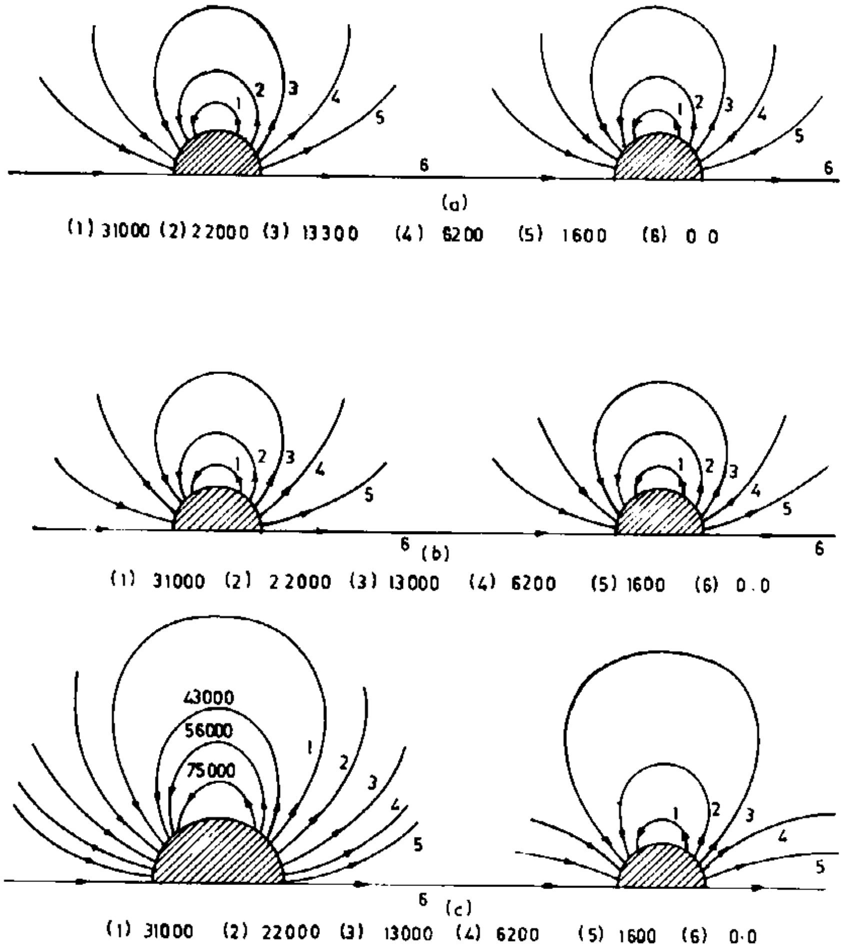

Fig. 2. Streamlines $\psi' \times 10^5$ for self propelling sphere: (a) $A_1 = C_1 - 0.01$, $B_1 = D_1 = -1.505$; (b) $A_1 = -C_1 = 0.0$, $B_1 = -D_1 = -1.5$; $\beta = b/a = 1.0$; (c) $A_1 = C_1 = 0.0$, $B_1 = D_1 = -1.5$, $\beta = 1.5$

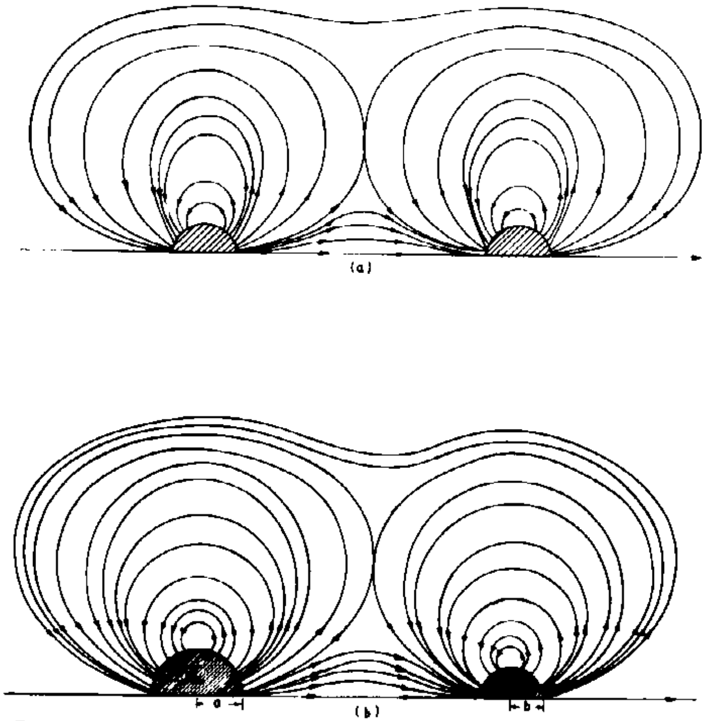

Fig. 3. Schematic diagram of stream lines: (a) Identical spheres moving with zero relative velocity; (b) Non identical spheres moving with different velocities

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الدفع لكائنين حيين ذوي أهداب عبر خط مراكزهم

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ملخص البحث. لقد تم دراسة نموذج مائع ميكانيكي لعملية ميكانيكية الدفع لكائنين حيين دقيقين يتحركان على خط مراكزهما والتأثير الكلي للأهداب استبدل بالسرعات والتي توصف بأنها دالة للزاوية القطبية (θ) على السطوح الكروية. وتم الحصول على التصحيحات اللازمة لسرعة السريان الحر نتيجة لوجود كائن حي آخر. وتم تحليل أنظمة خط السريان لأنواع معينة.