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Harmonic Minimization in a Chopper-type AC Voltage Controller

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Abstract. An AC voltage regulator of the chopper-type presents an attractive alternative for AC voltage control. This is attributed to its extended range of control coupled with independence of the switching angles with respect to load phase angle. Moreover, using microprocessors to determine firing instances makes it possible to change regulator's ON and OFF periods at will with appreciable ease. This adds a new merit for this type of controllers since it would be possible to alter the magnitudes of harmonics present in load voltage or even eliminating selected orders.

In this paper the values of switching angles yielding minimum harmonic content in load current are determined. Profiles of these angles at different values of load phase angle are presented for cases of output load voltage with three, five and seven pulses per haf cycle. Optimal values of switching angles are evaluated using an algorithm based on optimization technique of the direct search method. Validity of the results obtained is confirmed by comparing it with relevant values obtained by direct manipulation for the possible combinations of switching angles. Experimental results obtained from a microprocessor-controlled AC chopper confirming the theoretical analysis are presented.

The approach presented is general and can be well extended for harmonic minimization with any number of pulses per half cycle and specific values of load phase angle.

Introduction

Solid-state switches have fast response, compactness, lossfree control and low power demands for their control circuitries. Therefore, developments aimed at increasing their switching power capabilities, made it possible to realize numerous applications with improved performance.

In general, solid-state devices are used either in supply or load conditioning. Supply conditioning involves changing its frequency (Inverters/Cyclo-converters), altering its nature (Converters/Investers) or controlling its level (Choppers/Voltage regulators). Regarding load conditioning, solid-state devices are utilized in changing load active and or reactive power requirements (FCTCR, for example) [1-3].

On the other hand, utilization of solid-state devices involves generation of harmonics due to switching action. These harmonics are responsible for increased system loss, low power factor and associated interference problems with other nearby telecommunication lines. Therefore, a reasonable amount of research work has been devoted to overcome the effects of these harmonics through either harmonic elimination or minimization whenever the application allows.

The AC voltage controller presents an attractive method for stepping down a given AC voltage level in an efficient and simple way. Conventional types of solid-state voltage regulators operate with phase angle control strategy with a capability to control a large amount of AC power with low level control power. However, it suffers from the drawback associated with low order harmonics produced in load voltage and current. Moreover, the switching instant is dependent on load phase angle, thus presenting difficulties in control under varying load conditions [4-5]. These drawbacks could be overcome by using a chopper-type AC voltage regulator.

In this type, the series switch along with the free-wheeling one can control the amount of power delivered to the load by simply varying the ON/OFF time ratio. Moreover, using self-commutated devices such as GTOs [6-8] and transistors [9-10] as the controlled switches along with a microprocessor as a controller makes it possible to achieve different control strategies. The simplest control strategy is the mark to space ratio control [3-6,7-9]. PWM techniques have been proposed in [7]. Harmonic elimination technique gives an improvement in performance as a result of eliminating some selected harmonics in the output voltage [8].

However, regulator performance can be further improved if switching angles are so adjusted to minimize the distortion in the load current. The present paper shows that there exists an optimal combination of switching angles which yields this minimal condition. Moreover, profiles of these optimal switching angles are presented and relevant values of HF of load current showed improvement when compared to that of the time ratio control and harmonic elimination strategies. Experimental results are given for a practical system confirming the theoretical analysis.

Chopper Type AC Voltage Regulator

The chopper type AC voltage regulator is schematically shown in Fig. 1. Control of output load voltage is achieved by using series and parallel bidirectional controlled switches SW1 and SW2. SW1 allows power flow from mains to load whilst SW2 is used for free-wheeling. GTO's are used as switches to relieve the system from the burden of building commutation circuits.

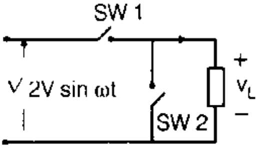

Fig. 1. Chopper-type a.c. voltage controller.

Firing signals are generated using a microprocessor, which generates supply synchronized pulses with predetermined timings according to ON and OFF periods.

The system described is practically realized and its associated performance characteristics under different modes of control strategies were reported in Refs. [6-8].

Analysis

A typical output load voltage waveform is shown in Fig. 2, for the case of three pulses per half cycle. Triggering is adjusted to ensure quarter wave symmetry so that only sine components of odd order exist in the output voltage. As a result no phase shift exist between the fundamental component of the load voltage and the supply voltage.

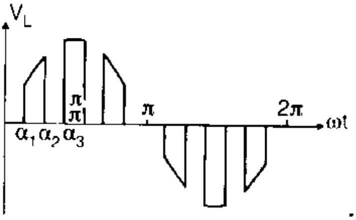

Fig. 2. Load voltage waveform of the a.c. chopper.

The normalized value of the fundamental component of load voltage is then given by:

$$V_1 = \frac{2}{\pi}\sum_{i=1}^{N+1}(-1)^i\left(\alpha_i - \frac{\sin 2\alpha_i}{2}\right) \tag{1}$$

where, N is the number of pulses per half cycle

$$\alpha_{N+1} = \pi / 2$$

Consequently, normalized amplitudes of harmonic voltages will be then given by:

$$V_n = \frac{2}{\pi}\sum_{i=1}^{N+1}(-1)^i\left[\frac{\sin(n-1)\alpha_i}{n-1} - \frac{\sin(n+1)\alpha_i}{n+1}\right] \tag{2}$$

where, n = 3, 5, 7, ...

One salient feature of this type of AC voltage control is that the total harmonic content in its output load voltage waveform is independent of firing angles and is mainly determined by the value of the fundamental component. This can be easily visualizede by determining the effective value of load voltage which is given by:

$$\begin{aligned} V_{rms} &= \left\{\frac{2}{\pi}\int_{\alpha_i,\alpha_3 \ldots \alpha_N}^{\alpha_2,\alpha_4 \ldots \pi/2} \sin^2\theta d\theta\right\}^{1/2} \\ V_{rms} &= \left\{\frac{1}{\pi}\sum_{i=1}^{N+1}(-1)^i\left[\alpha_i \frac{\sin 2\alpha_i}{2}\right]\right\}^{1/2} \\ V_{rms} &= \sqrt{V_1 / 2} \end{aligned} \tag{3}$$

and since:

$$V_{rms}^2 = \sum_{n=1,3,\ldots}^{\infty} \frac{V_n^2}{2} = \frac{V_1^2}{2} + \sum_{n=3,5,7,\ldots}^{\infty} \frac{V_n^2}{2}$$

Hence:

$$\sum_{n=3,5,7,\ldots}^{\infty} \frac{V_n^2}{2} = \frac{V_1}{2}\left(1 - v_1\right) \tag{4}$$

The previous equation indicates that harmonic content is function of V_1, with a maximum value at $V_1 = 0.5$ p.u. It follows then, that changing the values of firing angles will be of no value as far as load voltage total harmonic minimization is concerned. On the other hand, switching angles do affect the value of harmonic content in load current since various combinations of these angles will yield different amplitudes for different harmonics, maintaining the same fundamental, which are not equally suppressed.

Analytically, harmonic content in load current can be determined using superposition as follows:

(a) Assume the load to be passive inductive one with an impedance of 1.0 p.u. and phase angle ϕ_L with respect to the fundamental frequency; ***i.e.***

$$Z_{L_1} = 1.0./\phi_L = \cos\phi_L + j\ \sin\phi_L \quad \text{p.u.} \tag{5}$$

(b) The impedance of such a load for the nth harmonic will be then given by;

$$Z_{L_n} = \sqrt{\cos^2\phi_L + n^2\sin^2\phi_L} \,/\, \tan^{-1}\left(n.\tan\ \phi_L\right) \quad \text{p.u.} \tag{6}$$

(c) Consequently, the amplitude of harmonic current of order "n" will be then given by:

$$I_n = V_n / Z_{L_n} \quad \text{p.u.} \tag{7}$$

(d) The performance criterion U is the square of the rms value of the distortion component in the output current as the additional losses on the copper windings are proportional to this value. Also, minimization of that value gives near optimum results for all other types of harmonic losses such as iron losses, stray load losses, peak current, torque ripple etc. [11-12].

$$U = I_{dis}^2 = I^2 - I_1^2$$

$$= \sum_{n=3,5,\ldots}^{\infty} I_n^2 \tag{8}$$

where,

I and I_n are the rms values of the output current and the *nth* component in the output current respectively.

Variation of Harmonic Content in Load Current Versus Values of Firing Angles

The influence of changing switching instants on the value of harmonic content in load current can be well demonstrated by evaluating all possible combinations of switching angles producing a certain value of fundamental component, then determining the associated values of "U". It is evident that evaluating all possible combinations with a large number of switchings per half cycle will involve quite a large number of manipulations. Therefore, a case of three pulses per half cycle with a wave form schematically shown in Fig. 2. was considered. The manipulations start by assigning a value for α_1 (starting value = 0), then α_2 is chosen such that $\alpha_2 > \alpha_1$, and α_3 is evaluated in such a way to yield a certain value of fundamental component. Evaluation of α_3 is achieved by solving the following equation:

$$\sin(2\alpha_3) - 2\alpha_3 = \pi(K-1) + 2(\alpha_1 - \alpha_2) + \sin(2\alpha_2) - \sin(2\alpha_1)$$

where, $K = V_1$

The previous equation is solved using Newton-Raphson method maintaining $\alpha_2 < \alpha_3 < \pi/2$. Having determined the values of firing angles, harmonic content in load current is then monitored by calculating "U". Accuracy of the results obtained is ensured by considering a large number of harmonics in computations (500 were taken). The procedure is repeated for new value of α_2 and ultimately terminated when $\alpha_3 = \pi/2$. The whole procedure is repeated again for a new value of α_1 till calculations ended when α_1 reaches its limit corresponding to $\alpha_2 = \alpha_3$, for all values of α_2, which corresponds to an output voltage waveform with a single pulse per half cycle. Scanning the values of "U" obtained, the combination of firing angles corresponding to minimum "U" can be then allocated.

Sample output results relating "U" to α_1 and α_2 is shown in Fig. 3, for a load having a phase angle $\phi_L = 60°$ and K = 0.5 p.u. It is clear that there exists a single value for α_1 and α_2 yielding minimum "U". The same outcome can be easily viewed by observing the contours of equal values of "U" for different values of α_1 and α_2, under the same coditions, see Fig. 4. The areas enclosed by the contours decrease as the values of "U" is decreased, ultimately reaching a point representing conditioin of

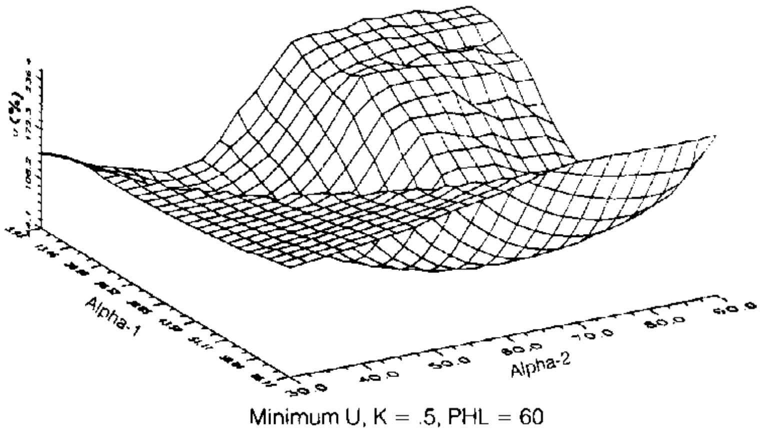

Fig. 3. Variation of U(%) versus α_1 and α_2 for K = 0.5 3 pulses/half cycle and ϕ_L = 60°.

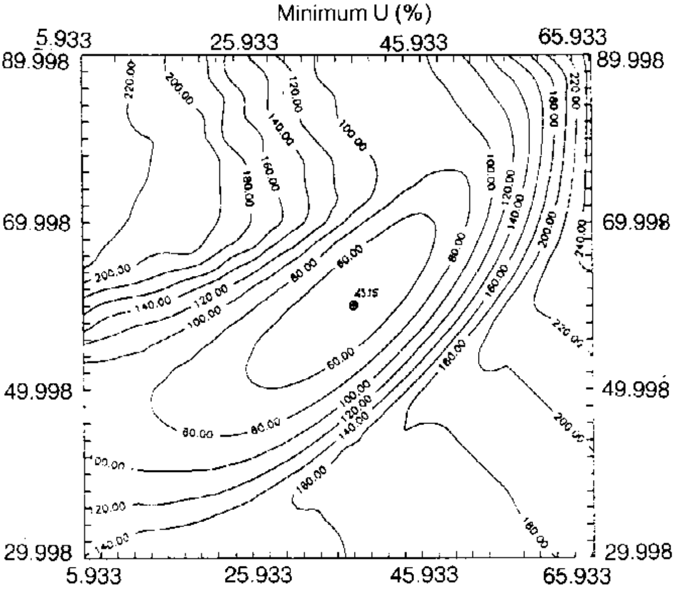

Fig. 4. Profile of "U" versus α_1 and α_2 for K = 0.5 and ϕ_L = 60°. (3 pulses/hafl cycle case).

optimality. For the case considered in Figs. 3 and 4, optimum values of switching angles are $\alpha_1 = 39.6$ and $\alpha_2 = 60°$ for $\phi_L = 60°$ and $K = 0.5$.

It is then obvious that evaluating a single combination of firing angles involves a great deal of manipulations. Moreover, with increased number of switchings, the problem may become very difficult and time consuming. However, direct evaluation of optimal switching angles can be achieved with well known optimization techniques. Consequently, the problem ought to be reformulated in a form acceptable for such an approach.

Evaluation of Firing Angles for Minimum Value of Harmonic Content in Load Current

The problem of evaluating optimum firing instances can be formulated as follows:

Find Min "U"

where:

$$U(\alpha_1, \alpha_2, \alpha_3, \ldots \phi_L) = \sum_{n=3,5,\ldots}^{500} I^2(n)$$

subject to:

$$K = \frac{2}{\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_{\alpha_1, \alpha_3, \ldots, \alpha_N}^{\alpha_2, \alpha_4, \ldots, \pi/2} \tag{9}$$

And satisfying the following inequality constraints:

$$0 < \alpha_1 < \alpha_2 < \alpha_3 ------ < \pi/2$$

Constraints are placed into the performance criterion via a penalty parameter which incur a positive penalty for infeasible points and no penalty for feasible points. A suitable penalty function is defined by:

$$\beta(\alpha) = 4(k - V_1)^2 + 4\sum_{i=0}^{N+1}\left(\max\left(\left[\alpha_i - \alpha_{i+1^0}\right]\right)\right)^2 \tag{10}$$

where

$$\alpha_0 = 0$$

$$\alpha_{N+1} = \pi / 2$$

A program that minimizes the sum of the squares of a number of functions using the Levenberg – Marquard algorithm [13-14] was employed. Functions are:

$$\begin{aligned}
F &= \left[F_1 F_2 \ldots\ldots F_{252+N+1}\right] && && (11)\\
F_{p'} &= 1_{2p-1} && p = 1{:}250 &&\\
F_{251} &= 2\left(K - V_1\right) && &&\\
F_p &= 2\max\left[\left(\alpha_i - \alpha_{i+1}\right)\ 0\right] && p = 252{:}252 + N + 1 &&
\end{aligned}$$

The problem is stated as

$$\text{minimize} \sum_{1}^{252+N+1} F_n^2 \tag{12}$$

Gradients of the functions are given by

$$\frac{\partial F_p}{\partial \alpha_i} = (-1)^i \frac{2}{\pi}\left[\cos(n-1)\alpha_i - \cos(n+1)\alpha_i\right] / \sqrt{\cos\theta^2 + \left(n\ \sin\theta^2\right)}$$

$$p = 1{:}250$$

$$\frac{\partial F_{251}}{\partial \alpha_i} = (-1)^{i+1} \frac{2}{\pi}\left[1 - \cos\left(2\alpha_i\right)\right]$$

$$\frac{\partial F_p}{\partial \alpha_i} = 2\max\left[\left(\alpha_i - \alpha_{i+1}\right)\ 0\right] / \left(\alpha_i - \alpha_{i+1}\right) \tag{13}$$

$$p = 252{:}252 + N + 1$$

It has been found that speed of execution of program enhanced if started with a very small value of V_1. Then V_1 incremented by a very small value such as 1×10^{-3}, with the result of previous iteration is used as an initial guess. Validity of the results obtained from such an algorithm was ascertained by comparing optimal values with corresponding values obtained by manipulations detailed in the previous section. Correlation between results is observed to be excellent. For example, at $\phi_L = 60°$ and $K = 0.5$ for three pulses/half cycle case, $\alpha_1 = 38.5°$ compared to 39.6° and $\alpha_2 = 61°$ compared to 60° as being determining by the direct manipulation method.

Results and Discussion

One of the simple strategies for determining the firing instances in such a regulator is the time ratio control (TRC) [6]. In such a case, a time ratio is assumed and given by:

$$\text{Time Ratio TR} = \frac{t_{ON}}{t_{ON} + t_{OFF}} \leq 1.0$$

The advantages offered by such a method of control are:

(1) Firing instances can be determined by calculations and hence a look-up table in programming the microprocessor can be dispensed with.

(ii) For number of pulses per half cycle greater than 3, fundamental component in load voltage changes almost linearly with TR.

(iii) For $0 \leq TR \leq 1$, the fundamental component in load voltage can be controlled from zero up to full supply voltage.

In spite of the simplicity encountered in programming the microprocessor to achieve such a mode of control, harmonic content in load current is no longer optimized. Based on the optimization algorithm developed, profiles showing variation of optimum combinations of firing angles at different values of V_1 with an output load voltage having three pulses per half cycle were evaluated. Figure 5 shows profiles for different load phase angles, namely for $\phi_L = 15°$, 30°, 45°, 60° and 90° respectively. It is clear that profiles for $\phi_L = 45°$, 60° and 90° are almost identical. Figure 6 shows a comparison between the output current harmonic factors (HF) of the following strategies:

○ TRC
○ harmonic elimination (HARELM) with the 3rd and the 5th harmonics eliminated.
○ optimal

when connected to a load with $\phi_L = 15°$.

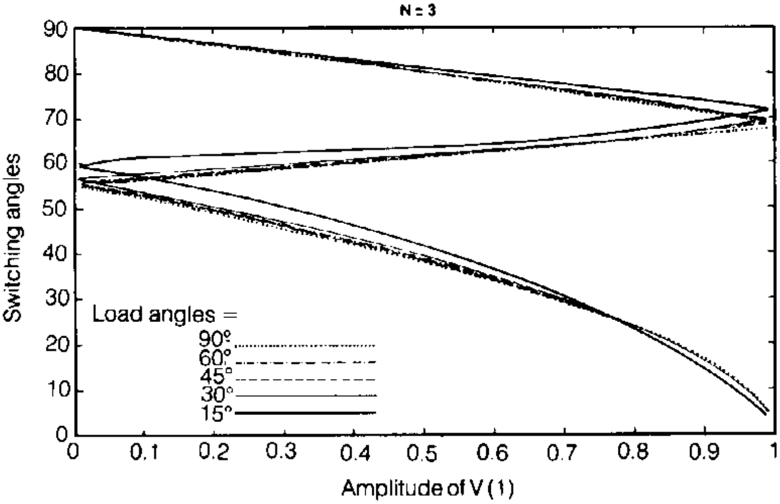

Fig. 5. Optimal firing angles profiles for minimum load current harmonic content, (3 pulses/half cycle a-case).

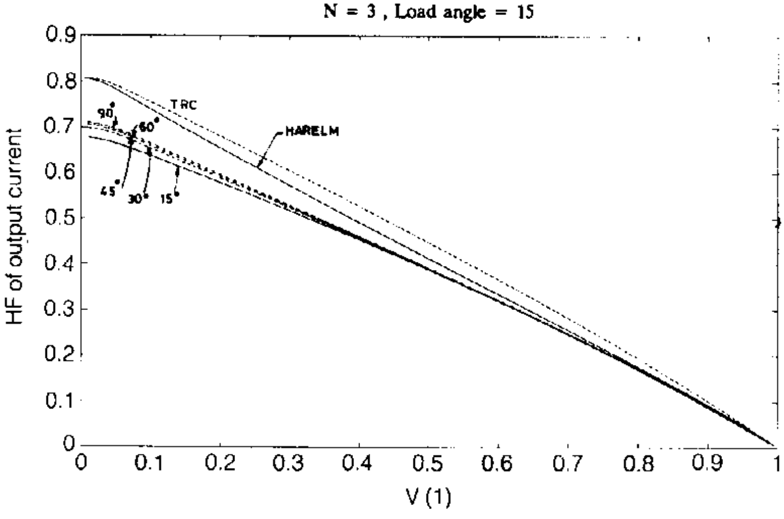

Fig. 6. HF of output current (3 pulses/half cycle and ϕ_L = 15°).

In the case of the optimal strategy the HF is improved as compared with that of TRC and harmonic elimination strategies. Although, the switching angles for optimal strategy for $\phi_L = 15°$ are not identical to that of of other values of ϕ_L, HF in the case of optimal strategy is less than that of harmonic elimination and TRC strategies and near to that for $\phi_L = 15°$ and almost equal to it for $V_1 > 0.4$. Therefore, it is possible to use switching angles of one angle for all loads. The shown profiles can be stored in a look-up table in the microprocessor memory and intermediate values can be estimated by interpolation with insignificant error since the profiles are continuous.

The approach was then extended to cover the five and seven pulses per half cycle cases. Relevant optimal firing strategies for same values of ϕ_L are shown in Fig. 7. Corresponding harmonic contents in load voltage subject to these firing strategies are shown in Fig. 8. It is evident that optimal firing angles resulted in appreciable reduction in lower order harmonics. Such a reduction, however, is associated with consequent increase in the amplitudes of higher order harmonics. This is anticipated, since harmonic content in load voltage is constant and dependent on the value of "V_1", as has been previously outlined. As far as load current is concerned, higher order harmonics will be suppressed by virtue of load inductance and thus a condition of minimum harmonic content is realized. Moreover, harmonics under this optimal firing are dominated by the 13th order while time ratio control, the dominant harmonic is the 11th [6]. On the ohter hand, the 11th harmonic in the latter case reaches a maximum value of 0.4 p.u. at $K = 0.5$ compared to a maximum value of 0.3 p.u. for the 13th harmonic in the former. Such a redistribution of load voltage harmonics under optimal firing strategies is the main reason for minimization of load current harmonics. Again the influence of such a choice of switching angles can be observed through the reduction encountered in the low order harmonics.

Comparison between HFs for TRC, harmonic elimination with the 3rd, 5th, 7th and 9th harmonics eliminated and optimal for $\phi_L = 90°$ and $\phi_L \mp 15°$ when connected to a load with $\phi_L = 15°$ is shown in Fig. 9. HF in the case of optimal for $\phi_L = 90°$equals to that for $\phi_L = 15°$ for $V_1 > 0.45$ and with a value near to it for other values. Again, the deviation of switching angles for 15° from other angles is not significant.

In confirmation of the validity of angles profiles obtained theoretically, the output load voltage and load current waveforms and spectrum are shown in Fig. 10 for $K = 0.4$, $\phi_L = 30°$ and $N = 5$.

On the other hand, it can be observed that switching angles profiles for a given number of pulses are not very sensitive to load phase angle variations. The profiles are close to each other and even coincident for values of $\phi_L \geq 30$. Moreover, with

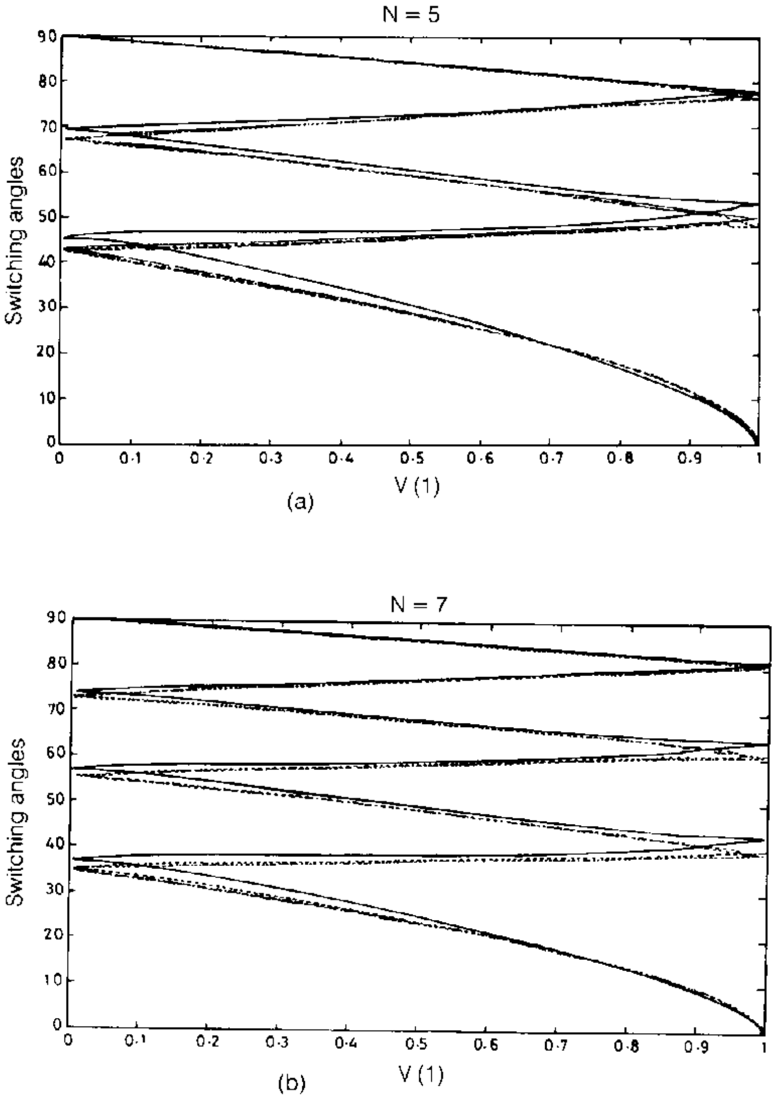

Fig. 7. Optimal firing angles profiles for minimum load current harmonic content.
(a) 5 pulses/half cycle. (b) 7 pulses/half cycle.

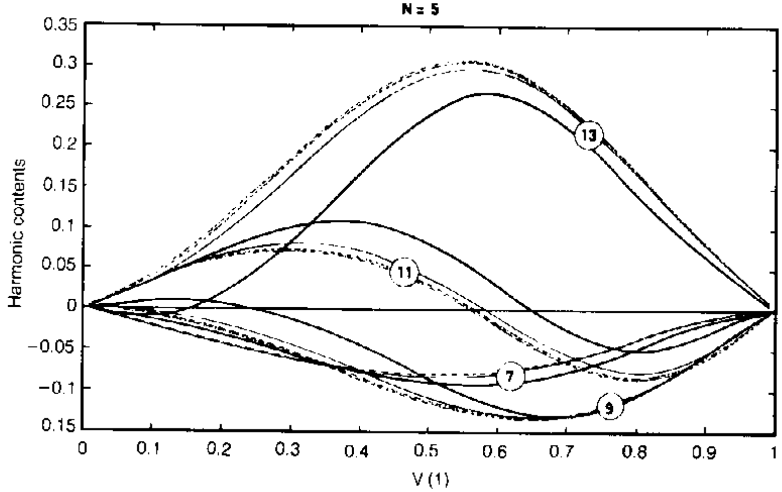

Fig. 8. Variation of harmonic contents in load voltage.

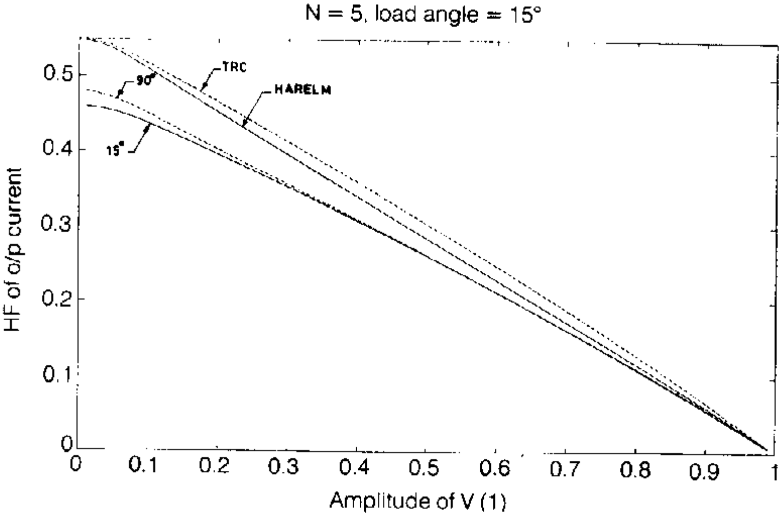

Fig. 9. HF of output current (5 pulse/half cycle and $\varphi_L = 15°$).

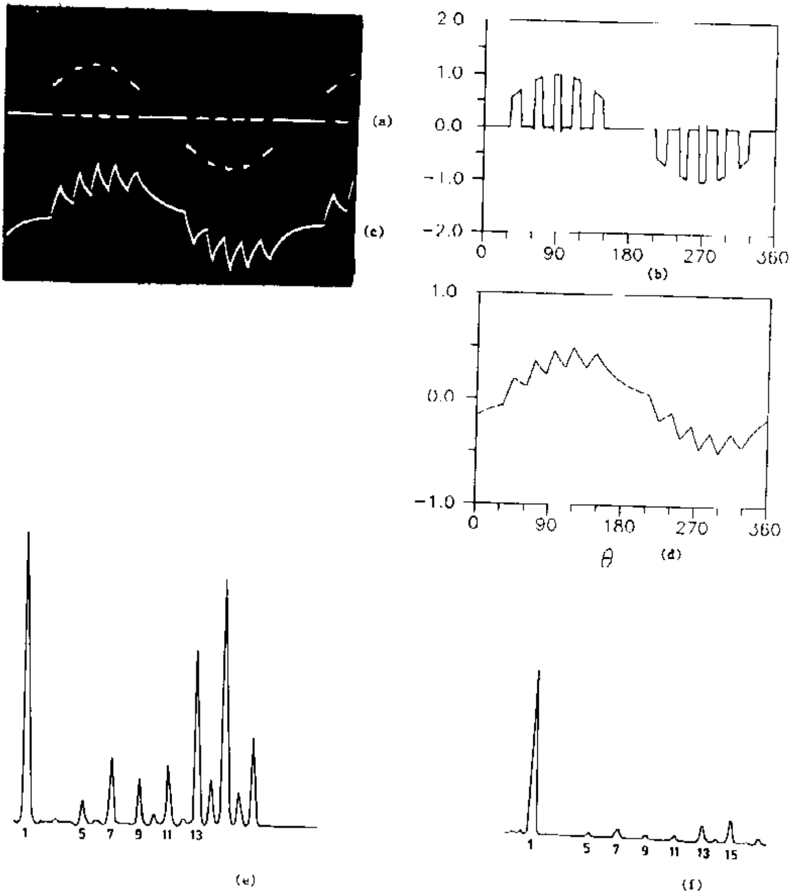

Fig. 10. Experimental and simulated results for K = 0.4, ϕ_L = 30° and N = 5.
(a) Experimental load voltage, (b) Simulated load voltage, (c) Experimental load current, (d) Simulated load current, (e) Spectrum and load voltage, (f) Spectrum of load current.

increased number of pulses per half cycle, the profiles for different number of pulses per half cycle, the profiles for different values of ϕ_L (in the practical range of load phase angle) will be so close to each other that a single profile could be used with varying load parameters. Such a behavior is quite useful since it makes AC voltage control with variable load impedance very simple and straight forward regarding microprocessor programming.

Conclusions

In this paper, it has been shown that there exists a certain combination of firing angles in a chopper-type AC voltage regulator which yields minimum values of load current harmonic content, for a given value of fundamental component. Such a condition was realized by evaluating harmonic content for all possible combinations and minimum value was allocated. This optimum condition was then determined using a least squares optimization method whereupon, results obtained showed good correlation with those evaluated by the first approach.

Profiles of switching angles for various values of load phase angle are then given for three, five and seven pulses per half cycle. Proof of minimality was given by comparing HFs of output current as being produced by optimal, harmonic elimination and TRC strategies. Optimum profiles are found to be insensitive to load phase angle variations thus adding a merit to using this model of control. Profiles given are general and are considered useful for users of such a type of regulators.

Experimental results obtained by a microprocessor based AC Chopper were observed to be in a good agreement with relevant theoretical ones.

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References

[1] Shepherd, W. *Thyristor Control of AC Circuits.* London: William Clowes and Sons, Ltd., 1976.

[2] Bird, B.M. and King, K.G. *An Introduction to Power Electronics.* England: John Wiley and Sons, 1983.

[3] Rashid, M.H. *Power Electronics: Circuits, Devices and Applications.* U.S.A.: Prentice Hall International Editions, 1988.

[4] Mozdzer, A. and Bose, B.K. "Three Phase AC Power Control Using Power Transistors." *IEEE Transactions on Industry Applications,* IA, No. 12 (Sept./Oct. 1976), 499-505.

[5] Kenly, W.L. and Bose, B.K. "Triac Speed Control of Three-Phase Induction Motor with PLL Regulation." *IEEE Transactions on Industry Applications,* IA, No. 12 (Sept./Oct. 1976), 492-498.

[6] Ad'doweesh, K.E.; Mohamadein, A.L. and Al-Ghalban, H.A. "Time-Ratio Control of Chopper-Type AC Voltage Regulators." *Journal of King Saud University,* Vol. 3, *Eng. Sci* (1), (1991), 19-43.

[7] Mohamadein, A.L. and Ad'doweesh, K.E. "Evaluation of the Performance of Chopper-Type AC Voltage Controller." *International Journal of Electronics,* 67, No. 4 (1989), 669-683.

[8] Ad'doweesh, K.E. and Mohamadein, A.L. "Microprocessor-Based Harmonic Elimination in Chopper Type AC Voltage Regulators." *IEEE Transactions on Power Electronics,* 5, No. 2 (April 1990), 191-200.

[9] Nishimura, T., *et al.* "Reduction of Vibration and Acoustic Noise in Induction Motor Driven by Three Phase PWM AC Chopper Using Static Induction Transistors." *IEEE Power Electronics Specialists Conf.* Rec., June (1987), 625-630.

[10] Ahmed, S. "Steady-State Modeling, Analysis, and Performance of Transistor-Controlled AC Power Conditioning Systems." *IEEE Trans on Power Electronics,* 5, No. 3 (July 1990), 305-313.

[11] Buja, G. and Indri, G. "Optimal Pulsewidth Modulation for Feeding AC Motors." *IEEE Trans. on Ind. Appl.,* IA-13, No. 1 (Jan./Feb. 1977), 38-44.

[12] Bowes, S.R. and Midoun, A. "Suboptimal Switching Strategies for Microprocessor - Controlled PWM Inverter Drives." *IEE Proc.,* Vol. 132, Pt. B., No. 3 (May 1985), 133-148.

[13] Bazaraa, M. and Shetty, C. *Nonlinear Programming, Theory and Algorithms.* U.S.A.: John Wiley, 1979.

[14] More, M. "The Levenberg - Marquardt Algorithm: Implementation and Theory." *Spring,* Vol 7, (1977).

تقليل التوافقيات في منظم الجهد المتردد بطريقة التقطيع

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(استُلم في ٣/١/١٩٩٣م؛ قُبل للنشر في ٣/٥/١٩٩٣م)

ملخص البحث. يقدم منظم الجهد المتردد بطريقة التقطيع بديلاً جذابًا لمنظمات الجهد التقليدية. حيث تعطي توسيعًا لنطاق التحكم إضافة إلى عدم اعتماد زوايا القطع على زاوية الطور للحمل. إن استخدام المعالج الدقيق لتحديد أوقات القطع يجعل تغيير فترات الفتح والقفل سهلاً. وذلك يعطي مَزِيَّةً إضافية إلى هذا النوع من منظمات الجهد حيث يمكن تغيير قيم التوافقيات الموجودة في الجهد الناتج أو إلغاء بعض الرتب المنتقاة.

في هذا البحث حُددِت قيمُ زوايا القطع التي تعطي تقليلاً في محتوى التوافقيات في تيار الحمل. وعرضت منحنيات لتلك الزوايا عند قيم مختلفة من زوايا الطور للحمل عند جهد ناتج مكون من ثلاث وخمس وسبع نبضات في نصف الدورة.

استخدمت خوارزمية مبنية على تقنية أداء أمثل ذات طريقة بحث مباشر. وتم التأكد من صحة النتائج بمقارنتها بقيم مثيلة مستقاة من معالجة مباشرة لتَوْليفات من زوايا القطع. كما عرضت نتائج معملية من مُقَطِع جهد متحكم به بوساطة معالج دقيق وأظهرت توافقًا مع التحليل النظري.

الطريقة التي يعرضها هذا البحث ذات صفة عمومية ويمكن توسيعها لتقليل التوافقيات في أي عددٍ من النبضات في الجهد الناتج عند أي قيمة لزاوية طور الحمل.