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Accuracy of Stadia Tacheometry with Optical Theodolites and Levels

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Abstract. Five optical theodolites, Wild T16, T1, T2, Kern DKM-1, DKM-2 and one automatic level, a Wild NA2, were tested for horizontal distance and height accuracy measurement. Two cases were considered. In the first, nominal values of scale factor and additive constants (K and C) were used (typically 100 and 0). In the second, these two parameters were determined in a least squares solution and applied. In the first case, the horizontal accuracy obtained ranged from ± 28 mm with the NA2 to ± 38 mm with the T16. The equivalent figures in height are ± 20 mm with the NA2 to ± 34 mm with the Wild T16. In the second case, the best results in horizontal distance accuracy was obtained with the T1 and the DKM-2 (± 24 mm), followed by the T2 (± 25 mm), the NA2 (± 26 mm), the DKM-1 (± 30 mm) and finally the T16 (± 36 mm). The height accuracy also improved, the biggest improvement being obtained by the Wild T16 *i.e.* from ± 34 mm to ± 30 mm (*i.e.* 12% improvement). In both cases, however, the results obtained are much better than what is generally believed to be attainable with stadia tacheometry.

Introduction

Conventionally, tacheometry is the procedure by which horizontal distances and differences of elevation are determined using the optical properties of the telescope of the measuring instrument (transit, theodolite, level or tacheometer). The method has long been acknowledged as a simple and inexpensive tool for mapping areas of limited extent. Depending on the distance being measured (and some other factors), the horizontal accuracy obtained with tacheometric surveys ranges from 1/1,000 with stadia and self-reduction instruments to perhaps 1/10,000 with subtense bar tacheometry. The accuracy of height measurement generally falls in the range ± 32 mm to ± 56 mm in 100 m [1]. These accuracy figures have been obtained with rather old versions of equipment (theodolites and staves); perhaps those manufactured in the 1950's and 1960's [2]. With the advent of modern high quality optics and higher resolution mensuration, in particular the somewhat modern theodolites, optical tacheometry needs to be further assessed for topographic surveys of limited areas.

Although, tacheometric mapping with conventional optical methods has decreased considerably in prominence in the last two decades in favour of digital electronic tacheometry, the case may arise, even in well developed countries, in which optical tacheometry makes the only directly available alternative to modern electronic means.

The advantages of electronic tacheometry over stadia tacheometry are many. These include increased accuracy (up to 20 times better), increased range (up to 3 km as opposed to only 100 - 150 m for stadia), less operational effort and possibility of automatic data recording for subsequent processing in a computer. However, electronic tacheometers are expensive devices and in many cases where a conventional theodolite is available and where results are not very urgently needed, optical tacheometry can well be cost-effective in mapping areas of small extent.

Aim of the Test and Procedure

The purpose of this article is to report results of an evaluation of the accuracy of distance and height measurement using selected "modern" optical theodolites and levels as tacheometers. Since diagram (or self-reducing) tacheometers, subtense bars and optical wedge telemeters are special tacheometric systems that were widely used before but now completely replaced by electronic tacheometers, emphasis will be placed on the more "general-purpose" optical theodolites; and hence stadia techniques will be employed.

Basically, the approach is to measure known lines using each of the test instruments. The discrepancies (or residual errors) between the known and computed distances (and differences in elevation) are then calculated and used to derive standard deviations in distance and height measurement. These were compared with accuracy figures generally assumed to be attainable with stadia tacheometry. Finally, a comparison of the results of the present test with the results obtained by a number of other investigators is made; and a conclusion is drawn as regards the use of optical theodolites in tacheometric surveys.

For the purpose of the present experiment, a ten-section line of a total length of approximately 100 m was established. The line lies on a reasonably protected site with trees and buildings all round. This was believed to be an advantage since it would tend to lessen the effects of wind on the measuring process. Each of the sections was approximately 10 m long. The actual distances and differences in elevation between one end of the line and each of the ten points were precisely established using a Kern DM-501 distance meter. The distance meter was calibrated before the test and the additive constant, the scale factor, the short-periodic (cyclic) error and the long-periodic error were taken into account. The accuracy of the established test line is believed to be better than ± 3 mm in 100 m. The height differences between the one end of the line and the other test points were also established using precise levelling. A recently-checked Wild NK2 tilting level with a parallel-plate micrometer and a recently calibrated Wild GPLE3 precise levelling staff were used for this purpose. According to the U.S.-based Federal Geodetic Control Committee (FGCC), the accuracy of the derived heights of the points satisfy the requirements for first order class I levelling standards. The derived distances and differences in elevation were assumed to be the "true" values with which computed distances and elevations, as obtained with the test instruments, are compared.

Each of the test instruments was then used to measure these distances and differences in elevation using stadia techniques. A standard Wild GNLE12 levelling staff graduated in feet was used with all test instruments. The choice of the imperial length unit of this staff is believed to increase staff reading accuracy [3]. It is to be pointed out here that all European publications refer to metric staffs which are estimated to 1.00 mm only. In an attempt to minimize the effects of heat waves and shimmer on the measuring process, observations were carried out in early mornings (06-08 hr) or late afternoons (15-17 hr). Other precautional measures taken during observation include careful centring of theodolite, use of bond level with the staff, placing staff on exact point on the steel peg, observing part of staff well above ground level (to avoid grazing rays), and shading instrument and staff during observations.

For each point, staff intercept and zenith angle were measured five times on different days and the mean values used to compute distance and difference in elevation. During the time span of the test (5 days) the average temperature ranged from 32°C to 35°C which is a rather small variation and one would expect less adverse effects on the results due to this variation.

Computations and Results

Instruments used in the present experiment were one each of the following types, Wild T16 (scale-reading), Wild T1 (single-reading micrometer), Wild T2 (double-reading micrometer) theodolites, Wild NA2 automatic level, Kern DKM-1 (single-reading micrometer) and Kern DKM-2 (double-reading micrometer) theodolites. All instruments have stadia hairs etched on their diaphragms. Before being used in the test, each of the test theodolites was made to undergo a series of tests *e.g.* plate-bubble dislevelment, diaphragm orientation, collimation, and optical plummet adjustment, and adjustments were made when found necessary. The NA2 was subjected to a two-peg test before being used in the experiment. Table 1 shows some of the characteristics of the test instruments.

Table 1. Some of the characteristics of the test instruments

Instrument	Serial No.	Telescope magn.	Angle reading system	Direct angle reading
Wild T16	284199	30x	Centesimal	10 mgon (1mgon)*
Wild T1	268593	30x	Centesimal	3 mgom (1 mgon)*
Wild T2	286593	30x	Centesimal	0.1 mgon
Wild NA2	128843	30x	–	–
Kern DKM-1	204375	20x	Centesimal	1 mgon (0.5 mgon)*
Kern DKM-2	235329	32x	Centesimal	0.1 mgon (0.5 mgon)*

( )* = by estimation

Note: All theodolites were equipped with compensators.

The basic equation used in determining horizontal distance (D) using stadia tacheometry is:

$$D = KS + C \qquad (1)$$

where

K = multiplication constant (scale factor);
C = additive constant; and
S = staff intercept.

In the general case, with vertical staff and inclined line of sight, equation (1) is modified to take the form:

$$D = KS \sin^2 z + C \cos z \sin z \qquad (2)$$

where z is the zenith angle.

For computing the difference in elevation between instrument station (P) and staff position (X), the following equation is used:

$$R_{L_X} = R_{L_P} + h_i \pm H - m \qquad (3)$$

where

R_{L_X} = reduced level of staff position (required);
R_{L_P} = reduced level of instrument station (known);
h_i = height of trunnion axis of instrument;
m = middle hair reading (made equal to h_i during observations);
H = vertical component of slope distance = $(1/2)\,KS\cos^2 2z + C\cos z$; and
z = zenith angle.

In all cases, the method of observation was to point the middle hair to the height of the instrument (rounded to next 0.02 ft line). This is followed by reading upper and lower hairs to 1/10th of graduation (*i.e.* 0.61 mm) thus increasing reading accuracy of the staff [3].

The index error of the vertical circle could contribute significantly to the final accuracy values. In this test, the index error was determined by summing face left and face right readings of the vertical circle. The misclosure of the sum of the index error, which was then halved and a correction (equal to the half index error and opposite in sign) was applied to both angular measurements in order to bring the misclosure to the zero value and subsequently obtain the correct vertical angle. For the sake of increased accuracy, zenith angles were, in all cases, read to the estimation value of the instrument.

The first stage of the experiment consists of computing horizontal distances and differences of elevation using equations (2) and (3) respectively assuming K = 100 and C = 0. For each instrument, the discrepancies between computed distances (or reduced levels) as obtained using equation (2) or (3) and their known equivalents as derived from EDM measurements (or precise levelling) were computed and used to derive root-mean-square errors in the form of standard deviation σ_j using the standard formula:

$$\sigma_j = \pm \left[\frac{\sum P_i V_i^2}{n} \right]^{1/2}$$

where

V_i = discrepancy between true and computed values of distance (reduced level) i using instrument j;

n = number of distances (reduced levels); and

P_i = a weighting function = $100/D_i$ [4].

It is to be noted that distance D_i has to be taken in metres; and the accuracy σ_j relates to a measurement of weight 1.0 (*i.e.* over a distance of 100 m).

In equation (4), a rejection criterion was adopted. In this criterion, for a sample having a size of 10 (as is the case in this test), observations showing discrepancies by more than 3 σ were rejected [5]. In fact this happened only once in the test with one of the T16 observations. The results are shown on Table 2.

Table 2. Results of the experiment using K = 100, C = 0

Instrument	σ_D (mm)	σ_V (mm)
Wild T16	± 38	± 34
Wild T1	± 31	± 21
Wild T2	± 33	± 29
Wild NA2	± 28	± 20
Kern DKM-1	± 37	± 31
Kern DKM-2	± 32	± 26

In the next stage, a combined least squares program to solve for the two parameters K and C was written and implemented. The derived values of K and C are shown on Table 3 together with their standard deviations. These were used again to compute a new set of distances and differences in elevation using equations (2) and (3). The discrepancies between the true and the newly-computed values were then derived and used to calculate root-mean square errors (rmse) using a modified form of equation (4) (*i.e.* denominator = (n-2)). The results are shown on Table 4.

Discussion and Analysis

Tables 2 and 4 are largely self-explanatory. However, it is appropriate to supplement them with additional comments. With the values of scale factor and additive

Table 3. Values of K and C as determined in this test

Instrument	K	C (mm)	σ_K	σ_C (mm)
Wild T16	99.998	− 0.10	± 0.11	± 0.012
Wild T1	100.004	− 0.24	± 0.034	± 0.004
Wild T2	99.949	− 0.04	± 0.037	± 0.003
Wild NA2	99.997	− 0.09	± 0.120	± 0.008
Kern DKM-1	100.022	− 0.31	± 0.044	± 0.004
Kern DKM-2	99.985	− 0.16	± 0.025	± 0.009

Table 4. Results of the experiment using computed values of K and C (100 m distance)

Instrument	σ_D (mm)	σ_V (mm)
Wild T16	± 36	± 30
Wild T1	± 24	± 20
Wild T2	± 25	± 26
Wild NA2	± 26	± 18
Kern DKM-1	± 30	± 28
Kern DKM-2	± 24	± 23

constants assumed 100 and 0 respectively, the accuracy in horizontal distance (100 m) ranged from ± 28 mm with the NA2 to ± 38 mm with the T16. This range of accuracy is noticeably better than what is generally assumed to be attainable with stadia tacheometry (*i.e.* a maximum accuracy of ± 50 mm in 100 m, [1;6;7].

In height measurement, the accuracy (over 100 m) ranged from ± 20 mm for the NA2 to ± 34 mm with the T16. Again, this is better than what is believed to be obtained with stadia tacheometry (*i.e.* ± 35 mm to ± 50 mm depending on the length of the line of sight).

When additive constants and scale corrections were determined and applied, horizontal distance (100 m) accuracy improved noticeably; with the T1 and the DKM-2 giving the best results (*i.e.* $\sigma = \pm 24$ mm). These are followed by the T2 with a standard deviation of ± 25 mm, the NA2 (± 26 mm), the DKM-1 (± 30 mm) and finally the T16 (± 36 mm).

The improvement experienced in height measurement was also noticeable (*i.e.* ranging from 5% to 12%). In both cases considered, the height accuracies obtained are better than horizontal distance accuracy values. This is in general agreement with theory and experience gained elsewhere.

A number of investigators carried out similar tests on various tacheometric systems, the general aim being to evaluate the accuracy of the various tacheometric techniques *e.g.* [6-8] and many others. For distances less than 150 m and using stadia or self-reduction methods, almost all investigators reported accuracy values in the range 1/600 - 1/2100, with elevations being determined to an accuracy ranging from ± 36 mm to ± 59 mm. It is clear, therefore, that the horizontal and vertical accuracy figures obtained in this experiment, even without determining and applying additive constants and scale factors, are noticeably better than those reported by other investigators.

In a similar investigation using self-reduction tacheometers and subtense bar, Ali reported that subtense tacheometry gave the best results over 100 m distance (around 1/4700 *i.e.* ± 21 mm in 100 m) [6]. This means that relatively modern optical theodolites, if in good adjustment, could produce horizontal accuracy figures compatible with those obtained by subtense tacheometry, the advantage of using such theodolites being rapid measurement, high accuracy and possibility of producing contoured site plans. The accuracy figures shown on Table 4 are compatible with accuracy requirements for third order traversing in civil engineering applications (*e.g.* see [9]).

It is to be pointed out, however, that since the nature of this test is only experimental, some observational procedures followed in the test do not represent standard techniques *e.g.* use of imperial staffs, measurement carried out during early morning or late afternoon (most European/American investigations refer to normal daytime observations) and shading of the staff. However, these procedures were adopted in an attempt to minimize some anticipated systematic errors. In most practical situations, therefore, expected accuracy values will be worse than those shown in Tables 2 and 4.

Conclusions

Six modern optical instruments (five theodolites and one automatic level) were evaluated for accuracy in stadia surveys over a base-line using two methods. In the first, nominal values of K and C were used (typically 100 and 0). In the second, K and C were determined using least squares. The results show that in both cases, the horizontal and vertical accuracy figures obtained are better than what is generally assumed attainable with stadia tacheometry. Therefore, for application areas requiring horizontal accuracies of around ± 30 mm in 100 m, modern optical theodolites and levels could be suitable for the job. This conclusion may serve a useful purpose in situations where optical tacheometry is the only alternative to electronic techniques.

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دقة تاكومترية شعرات الأستاديا بالثيودوليتات والموازين البصرّية

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ملخص البحث. أخضعت خمسة من الثيودوليتات البصريّة وهي فيلد T2، T1، T16، كيرن DKM-2 و DKM-1 وميزان آلي واحد هو فيلد NA2 لتجربة عمليّة لاختبار دقتها في قياس المسافة الأفقيّة وفرق الارتفاعات وذلك باستعمال طريقة تاكومترية شعرات الأستاديا. وقد اتّبعت طريقتان في ذلك. في الطريقة الأولى استعملت القيمة ١٠٠ لمعامل المقياس والقيمة صفر للثابت الجمعي. أما في الطريقة الثانية فقد تم حساب القيم الأكثر احتمالاً لهذين المتغيرّين بوساطة نظرية ضبط التربيعات الأقل.

الدقة التي تُحِصِّلَ عليها في الطريقة الأولى تراوحت ما بين ± ٢٨مم بجهاز NA2 إلى ± ٣٨مم بجهاز T16. والدقة الرأسية تراوحت مابين ± ٢٠مم بجهاز NA2 إلى ± ٣٤مم بجهاز T16. وكانت نتائج التيودوليت T1 هي أحسن نتائج دقة أفقية حُصل عليها في الطريقة الثانية (± ٢٤مم) يليه T2 (± ٢٥مم) ثم NA2 (± ٢٦مم)، DKM-1 (± ٣٠مم) وأخيرًا T16 (± ٣٦مم). وتحسّنت أيضًا الدقة الرأسية عما كانت عليه في الطريقة الأولى بحيث كانت أكبر نسبة تحسن في نتائج T16 (من ± ٣٤مم إلى ± ٣٠مم أي بنسبة تحسن ١٢٪). وعليه فقد أثبتت هذه التجربة أن نتائج كلتا الحالتين أحسن من تلك المتعارف عليها عادةً في المسوحات التاكومترية بوساطة شعرات الأستاديا.