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A Numerical Model for Predicting the Behavior of Rehabilitated and/or Strengthened RC Beams

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Abstract. The work presented herein uses the compatibility of deformations, equilibrium of forces, and a more rational stress-strain relationship for concrete in compression to suggest a numerical model that is capable of estimating, for any load level, the forces, moments, stresses, strains, and curvatures of the reinforced concrete beams before and after repairing, upgrading and/or strengthening. The model considers the variation of stresses, strains, and curvatures along the length of the beam. It also accepts different material properties, cross sectioal shapes, and loading and reinforcement configurations. Further, it takes into account effects of shrinkage and creep on the deflection and the strain of concrete and internal and external reinforcement. Thus, it can be used with different techniques that are available for rehabilitation and/or strengthening of reinforced concrete beams.

The analytical prediction using the proposed model was checked against the published experimental data and found in good correlation with the corresponding measured results. It also accorded better prediction than other models available in the literature.

It is believed that availability of such a model is of great value to the structural engineer especially through the design and decision process. It can be used to evaluate the strength and serviceability criteria of the existing damaged or undamaged structure. Then, if needed, the model can be utilized to help in selecting the type and size of the material that best fulfills the conditions to rehabilitate and/or strengthen the structure. This will, of course, influence the work strategy and may lead to appreciable saving in both time and cost.

Notation

A_c = area of the concrete compressive zone (Ignoring steel)
$\overline{A}_c$ = area of the aged adjusted transformed section (using $\bar{n}$ in lieu of n)

B_c = first moment area of concrete compressive zone about the top fiber of the cross-section
$\overline{B}_e$ = first moment area of the aged adjusted transformed section about the top fiber concrete
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of the steel
$\overline{E}_e$ = aged adjusted effective modulus = $E_c/(1 + \chi \nu)$
I_c = second moment area of the concrete compressive zone about the top fiber
$\overline{I}_e$ = second moment area of the aged adjusted transformed section about the top fiber concrete
K = the initial or elastic stiffness
K_p = the final or plastic stiffness
ΔM = $\overline{E}_e[\nu(-B_c\varepsilon + I_c\phi) - \varepsilon_{sh}B_c]$
ΔN = $\overline{E}_e[\nu(A_c\varepsilon - B_c\phi) + \varepsilon_{sh}A_c]$
n = the curve shape parameter
n_1 = modular ratio = E_s/E_c
$\overline{n}$ = aged modular ratio = $E_s/\overline{E}_c$
σ = the applied stress
σ_0 = the reference stress
ε = instantaneous elastic strain in the top fiber of concrete
ε_{cf} = strain at the extreme concrete compression fiber corresponding to the applied stresses
ε_{sh} = shrinkage strain
ϕ = curvature due to short time loading
ν = ratio of creep strain at time t to instantaneous elastic strain
χ = aging coefficient = 0.8
$\Delta\varepsilon$ = increase in ε due to shrinkage and creep
$\Delta\phi$ = increase in ϕ due to shrinkage and creep

Introduction

Rehabilitation and/or strengthening (RS) of concrete structural elements may become necessary to retain the structural integrity, meet the current standards, increase the ultimate strength, improve the serviceability performance, and to remedy the unintentional design errors. Several techniques are now recognized to be effective and convenient methods for repairing, upgrading, and strengthening of reinforced concrete beams including ferrocement encapsulation and laminates [1-3], and externally bonded steel or glassfiber plates [4;5]. These methods and others have already been successfully used to rehabilitate and strengthen buildings and bridges in many countries around the world [6-8]. However, the current available design methods cannot be used to reasonably estimate the effect of RS on the responses of the reinforced concrete (RC) element when subjected to different load levels. Prediction of the strength and ductility of the structural elements before and after the RS process, however, greatly influence their strategy and economy.

This paper presents a numerical model that is capable of predicting the responses of the repaired concrete beams and its components before and after performing the RS. The model is based on the compatibility of deformation, equilibrium of forces, and a true stress-strain relationship for concrete. It accounts for the variation of moments, strains, and stresses along the length of the beam and considers the influence of shrinkage and creep of the concrete on deflection and the strain distribution. Further, it accounts for the different mateial properties that constitute the composite, different shapes of cross sections, external and internal reinforcement configurations, and different loading types. The model can be adjusted to account for other material properties and design requirements. Thus, it can be utilized with different methods available to repair, strengthen, and/or upgrade reinforced concrete beams.

Stress-Strain Relationship

Although the program can fit different material properties, only those pertaining to the materials used to verify the model and presented herein are discussed. These are concrete, cold worked deformed bars, mild steel, epoxy resin, and glass fiber reinforced plastics (GFRP).

It seems, however, that the most versatile part of the RC constituents is the concrete itself. Many models are available in the literature to simulate the stress-strain relationship of the concrete when subjected to axial compression. It is well known also that the accuracy of the prediction of the behavior of the RC element is highly dependent on the concrete stress-strain relationship (CSSR). In this study the CSSR at different load levels was generated using a rational representation of the stress-strain relationship. The relationship is based on the model developed by Richard and Abbott [9] and modified by Almusallam and Alsayed [10] to be used for concrete.

$$\sigma = \frac{(K - K_p)\,\varepsilon_{cf}}{\left(1 + \left[\dfrac{(K - K_p)}{\sigma_0}\right]^n\right)^{1/n}} + K_p\,\varepsilon_{cf} \qquad (1)$$

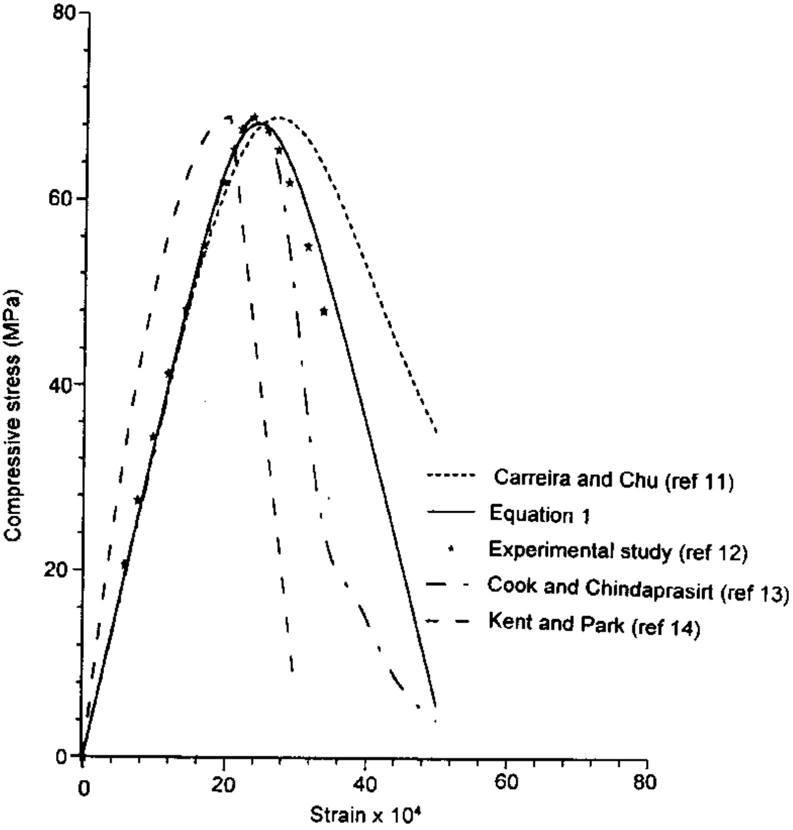

Fig. 1. Stress-strain curve for concrete loaded in uniaxial compression, f_c = 68 MP_a.

The previous equation is defined by the four parameters K, K_p, σ_0, and n. These parameters are only a function of the compressive strength of the concrete, f'_c. Figure 1 shows an example of the stress-strain relationship, generated by the above model as well as those generated by previous proposed models that were cited in references [11-14], for a concrete with a compressive strength, f'_c of 68 MPa. Evaluation of parameters of the previous model for normal, high strength and light weight concrete along with further comparisons with prediction by models proposed by other researchers are given elsewhere [10].

The stress-strain relationship of the cold worked deformed bars was represented by a trilinear curve. It exhibits an initial linear elastic portion up to strain of 0.001 with a modulus of elasticity of 190 KN/mm^2, a second linear portion up to a strain of 0.0026 with a modulus of elasticity of 150 KN/mm^2, and a yield plateau thereafter. The stress-strain relationship of the mild steel was represented by a bilinear elastic perfectly plastic curve. The stress-strain curves of the GFRP and the epoxy resin were represented by a linear line up to the failure point [4;15].

Shrinkage and Creep Effects

There are many methods available to estimate shrinkage and creep (SC) of concrete. Unfortunately the predicted SC obtained from the different methods vary widely [16]. Therefore, the accuracy of predicting the time dependent behavior for the concrete element is not expected to be as accurate as that for the short-term behavior.

However, a powerful method to predict the variation in the compressive strain at the outermost fiber of the cross section ($\Delta\varepsilon$) and the change in the curvature ($\Delta\phi$) due to the shrinkage and the creep effects is the relaxation procedure which was developed by Bresler and Seina [17].

The prediction equations are given by:

$$\Delta\varepsilon = \frac{\overline{B}_e\,\Delta M + \overline{I}_e\,\Delta N}{\overline{E}_e\,(\overline{B}_e\,\overline{I}_e - \overline{B}_e^2} \tag{2}$$

$$\Delta\phi = \frac{\overline{A}_e\,\Delta M + \overline{I}_e\,\Delta N}{\overline{E}_e\,(\overline{B}_e\,\overline{I}_e - \overline{B}_e^2} \tag{3}$$

The method has previously been adopted in many numerical studies and provided satisfactory correlation with the measured results [16;18].

Description of the Model

The computer model considered herein was developed on the basis of the strain compatibility and the equilibrium of forces. It has the advantages over other similar ones in that it uses a more realistic stress-strain relationship for different types of concrete (normal, high strength and lightweight concrete). The model considers the effects of stress, strain, and curvature variations along the length of the beam (based on equally spaced 40 sections along the span) and the responses of composite constituents. It also accounts for different types of reinforcing materials, (*i.e.* FRP and steel. Further, it takes into account the influence of shrinkage and creep of the concrete on strains, stresses and deflection. The main features of the algorithm suggested is shown in Fig. 2. It can be summarized as follows:

A. Input data

Data required to run the model are: Dimensions of the beam (height, width, depth to bottom and top reinforcing steel or FRP, and external plate dimensions); the span of the beam; the external load points; the concrete model parameters; the quantity and the stress-strain relationship for all reinforcing materials; and, if needed, the time-dependent parameters which include shrinkage strain, ε_{sh}, ratio of creep to short-term strain, v, and curvature, ϕ, corresponds to the load level where the shrinkage and creep effects are to be estimated. When actual values of ε_{sh} and ϕ are not available, some imperical formulas such as those recommended by the ACI-committee 209 [19] may be used to estimate them.

B. Iteration technique

The iteration technique is carried out as follows:

1. Assume a small initial strain at the extreme compression fiber, e_{cf}, and the depth of the neutral axis, $d_{N.A.}$, from the extreme compression fiber. Then apply the strain compatibility requirements, check the strain limitations, and iterate until the equilibrium condition of all forces is satisfied.
2. Compute the values and the locations of the compression and tension forces, moment, curvature and any other needed composite response such as strain in the reinforcing bars and in the external reinforcement, if any. In the next steps these outputs are referred to as the first set of output.
3. Increase the concrete strain in the extreme compression fiber by a small value and repeat steps one and two to establish the next set of output.
4. Repeat step 3 until the maximum allowable concrete strain at the extreme concrete compression fiber is reached.
5. Use boundary conditions and load configuration of the beam and also the outputs obtained in step 4 to establish the M-ϕ relationship for 40 sections along the length of the beam for different levels of loading.
6. Use conjugate beam method or any other equivalent method to calculate the load-deflection relationship for any point along the length of the beam.

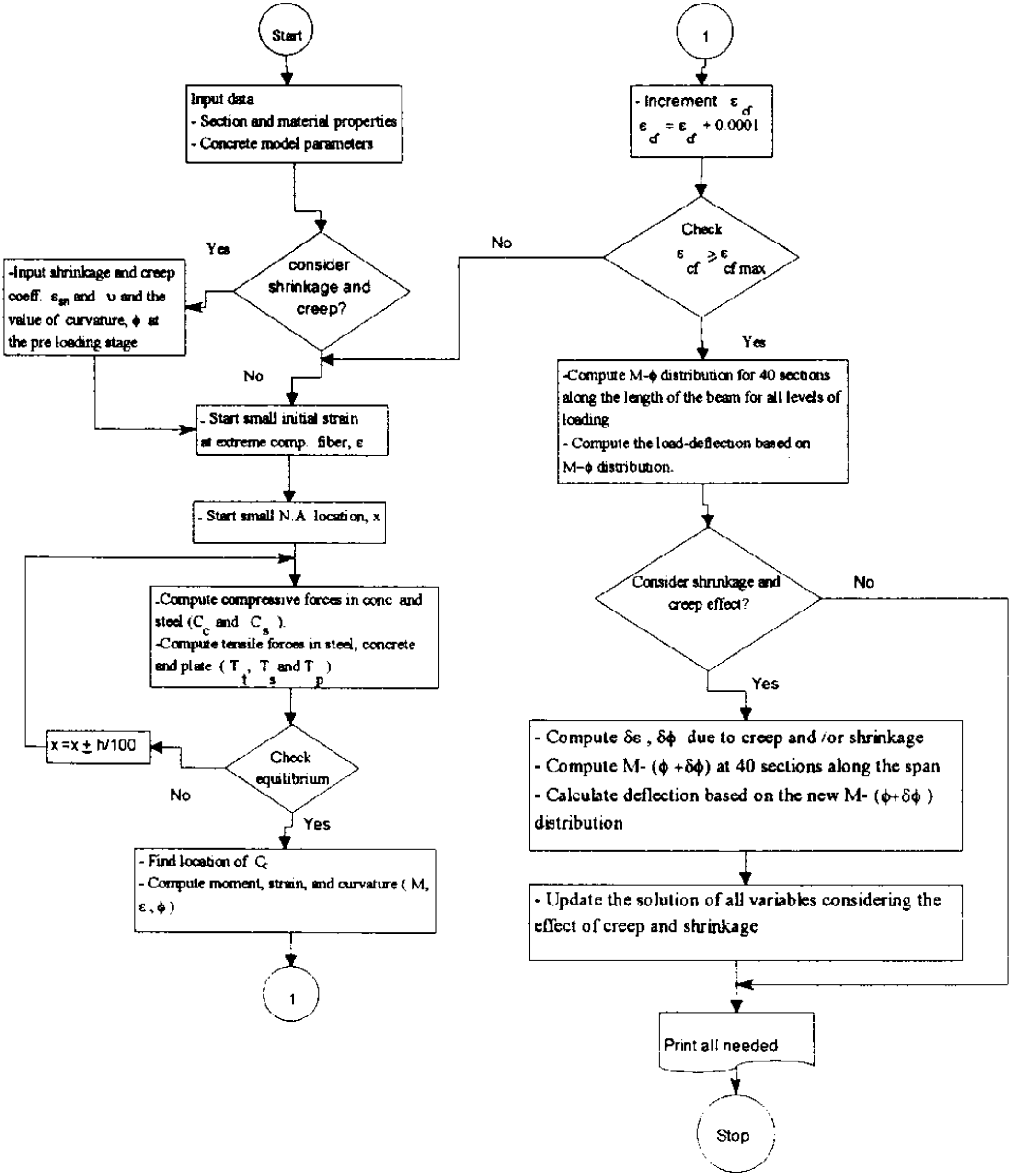

Fig. 2. Flow chart of the suggested model.

7. When time-dependent effects are required, specify the M-ϕ values over the 40 locations along the length of the span for which Δε and Δϕ are to be estimated.
8. Use the time-dependent parameters, equations 2 and 3, and data from step 7 to estimate Δε and Δϕ correspond to the M-ϕ values specified in step. 7.
9. Use the conjugate beam method or any other equivalent method to calculate the total deflection due to short and long time loading.
10. Update the load-deflection relationship calculated in step 6 to account for the increase in the deflection caused by the shrinkage and creep effect.

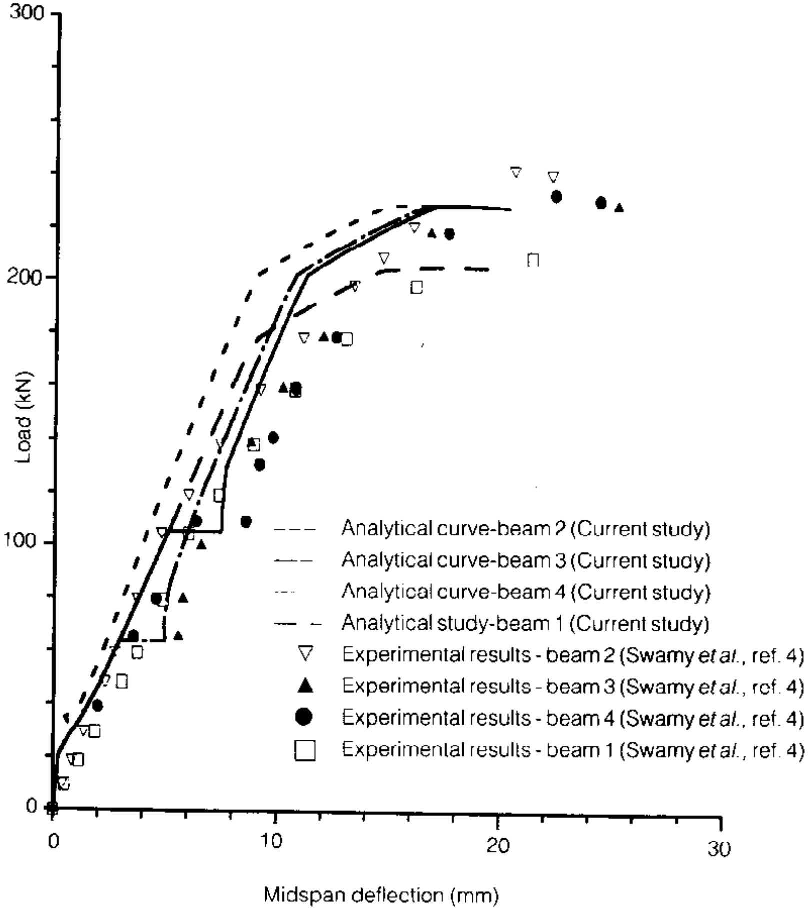

Fig. 3. Analytical versus experimental results of load-deflection curves.

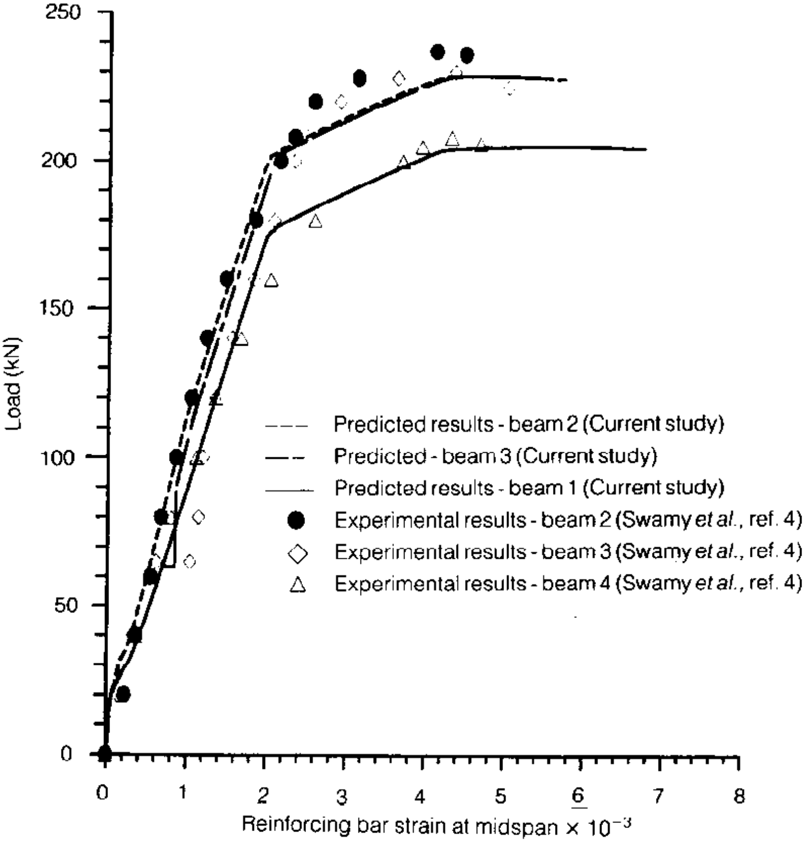

Fig. 4. Prediction versus measured load-reinforcing bar strain.

Measured Versus Predicted Results

To check the validity of the suggested model, the predicted values were checked against the relevant published data.

The measured and predicted load-deflection curves for four reinforced concrete beams that were tested by Swamy *et al.* [4] are shown in Fig. 3. The first beam did not have a plate, the second one was externally reinforced by 125 × 1.5 mm (width × thickness) mild steel plate, and the other two were also externally reinforced by mild steel plates and preloaded to 30%, and 50%, respectively, of their ultimate flexural capacity (strengthening process was performed while beams were held under constant load). Beams were 155 × 255 × 2500 mm in size and reinforced with 3ϕ20 mm diameter cold worked deformed bars in the tension zone. All beams were simply supported and subjected to two concentrated loads placed at equal distance from the centerline of the beam (766 mm apart). Other details of material properties and test setup of the beams are given elsewhere [4]. The corresponding measured and predicted load-strain curves of the reinforcing bars for the first three beams are shown in Fig. 4.

It can be seen in these two figures that the proposed model reasonably predicted the measured results. This is also true for beams that were subjected to the sustained load effect (creep). It appears, however, that there is small difference between the measured and the predicted values. This is true even at the start of loading. This may be due to measuring devices errors (dial gages). Such deviations between the measured and the predicted values disappeared in Fig. 4 where strain measurements were performed using electrical strain gages. Results on the same figures also show that the model slightly underestimated the ultimate load of the composite. This can be ascribed to the fact that, due to the lack of actual data, the stress-strain relationship of the cold worked deformed bars was idealized without considering the strain hardening effect.

The adequacy of the model was also investigated by considering the reinforced concrete beam tested by Ritchie *et al.* [20]. The beam had a 152 × 305 mm cross section, externally reinforced by a 4.8 × 152.4 mm GFRP plate, simply supported over a span of 2438 mm, and subjected to two concentrated loads symmetrically placed about the mid-span (610 mm apart).

The experimental and the analytical mid-span moment-deflection curves are shown in Fig. 5. The agreement between the proposed model and the experimental results is excellent. Figure 5 also shows the moment-deflection curves for the same beam when deflections were computed using the ACI-318 effective flexural rigidity of the beam [21] and the model suggested by Ritchie *et al.* [20]. The curves show that the current ACI formula underestimates the actual deflection of the beam. This suggests that the current ACI formula to estimate the deflection of a conventional RC beam may need some modification before it can be used to estimate the deflection of RC with external reinforcement. However, more data are needed before any suggestion can be proposed. Results also show that the model proposed in this study is more accurate in predicting the test results than the model proposed by Ritchie *et al.* [20].

Figure 6 shows the curve that represents the mid-span moment-versus-plate stress for the same beam. Again the results show that the proposed model accurately estimates the experimental results.

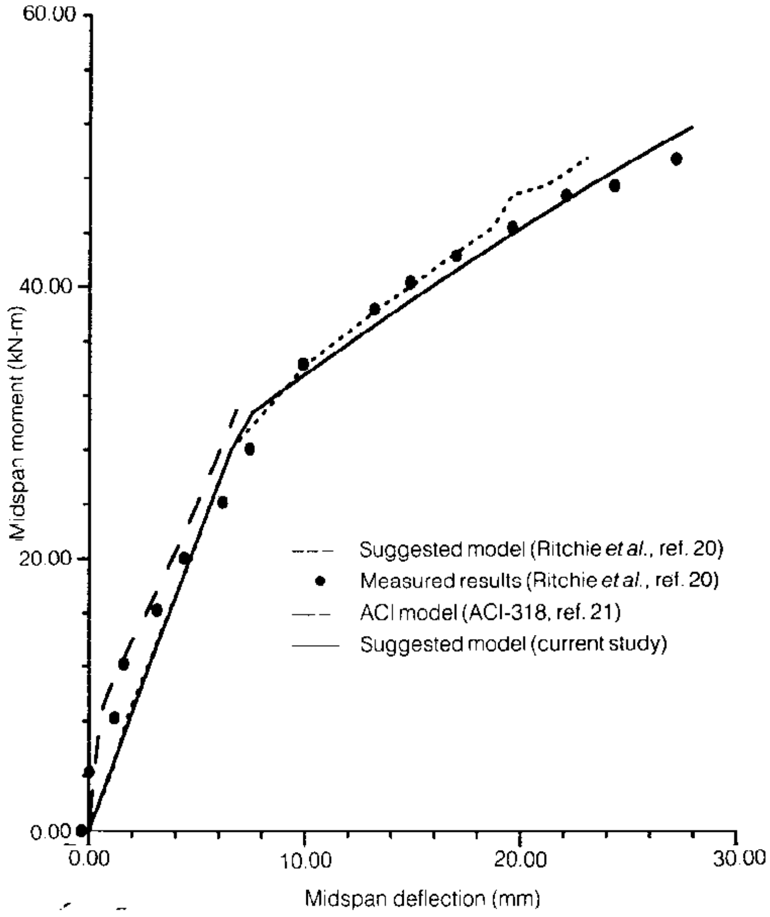

Fig. 5. Analytical versus experimental results of moment-deflection curves.

To check the capability of the proposed model in predicting the responses of the constituents of the concrete composite, Figs 7,8, and 9 show the measured versus the predicted load-strain curves in the steel bars, concrete, and the GFRP plate, respectively, of beam A that was tested by Saadatmanesh and Ehsani [22]. The beam had 455 × 205 mm in cross section and 4880 mm in length. It was reinforced by 3 ϕ 25 mm in the tension side, 2 ϕ 13 mm in the compression side, and 6 × 152 mm GFRP plate glued to the tension side of the beam. The beam was simply supported on a clear span of 4570 mm and subjected to two concentrated loads symmetrically placed about midspan (610 mm apart). The values of strains in concrete, reinforcing steel bars, and the external GFRP plate were measured by electrical resistance strain gages. For the purpose of comparison between the model suggested in the current study and that

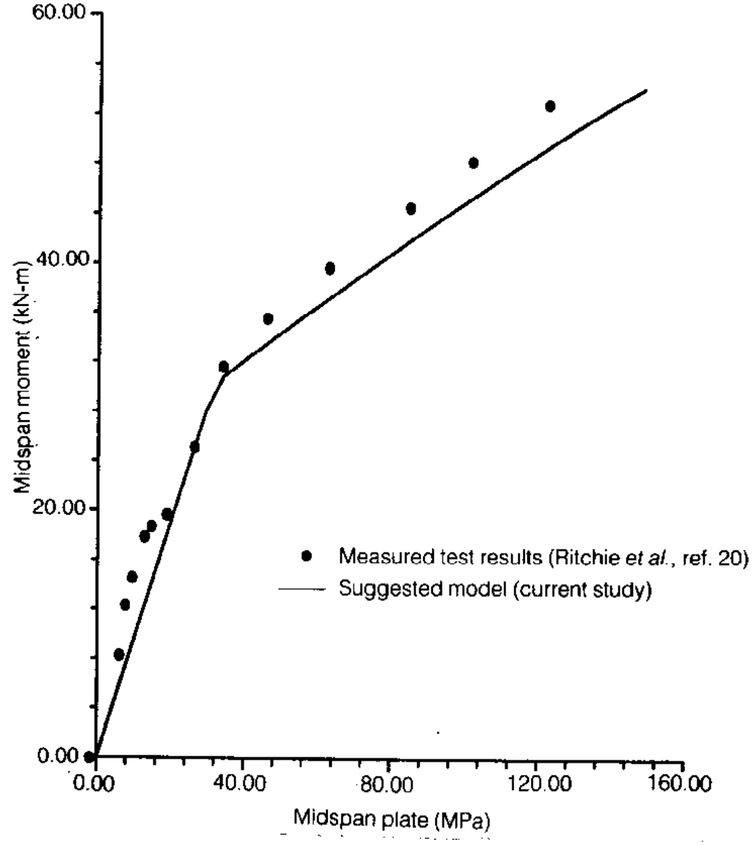

Fig. 6. Analytical versus experimental results of moment-plate midspan stress.

proposed by Saadatmanesh and Ehsani [22], it was assumed that the concrete carried no tensile strength.

The results in the three figures clearly show that the proposed model predicted well the measured strains in the different constituents of the composite and for all load levels. The small difference between the measured and the estimated curves was attributed by Saadatmanesh and Ehsani [22] to result from the fact that the tensile strength of the concrete was not considered in the analytical model. Here again, the curves shown in the figures indicate that values predicted by the model suggested here are in better agreement with the experimental results than those predicted by the model proposed by Saadatmanesh and Ehsani [22].

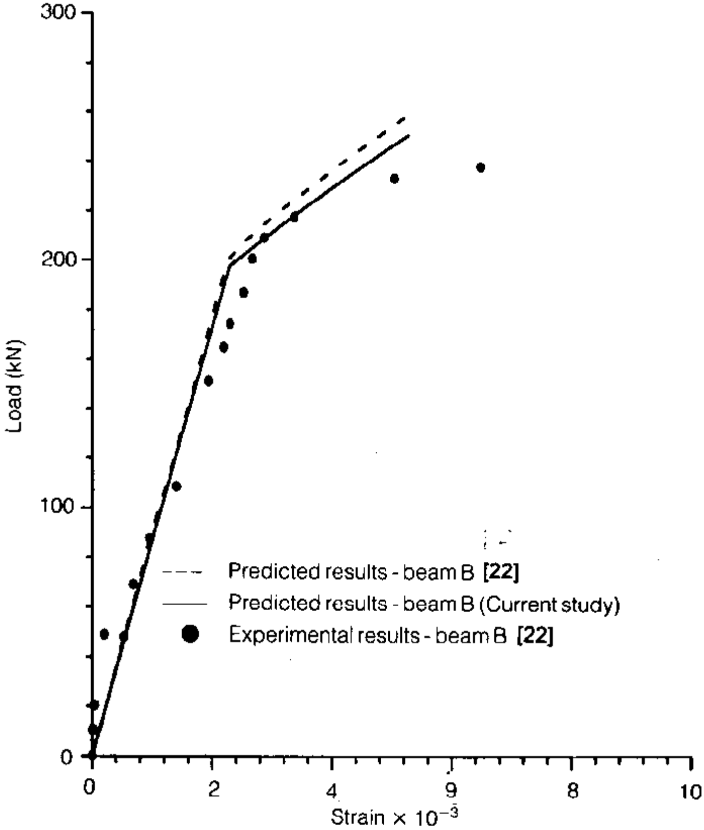

Fig. 7. Measured versus predicted load-reinforcing bar strain.

Summary and Conclusions

This study presented a numerical model that is capable of developing complete internal responses of reinforced concrete beam subjected to external stresses. The model, for all levels of loading, calculates the responses of the RC constituents to external stresses in 40 sections along the length of the beam. Thus, it accounts for the variation of stresses, strains, and curvature along the span of the beam. It also accounts for the influence of shrinkage and creep in these variations. Furthermore, it accepts different material and geometry properties and the presence of different tension, compression, and external reinforcement.

The adequacy of the model was established by comparing the published test data with corresponding predicted ones. The comparisons showed the ability of the proposed model to predict, with good accuracy, the test results. The same data were also

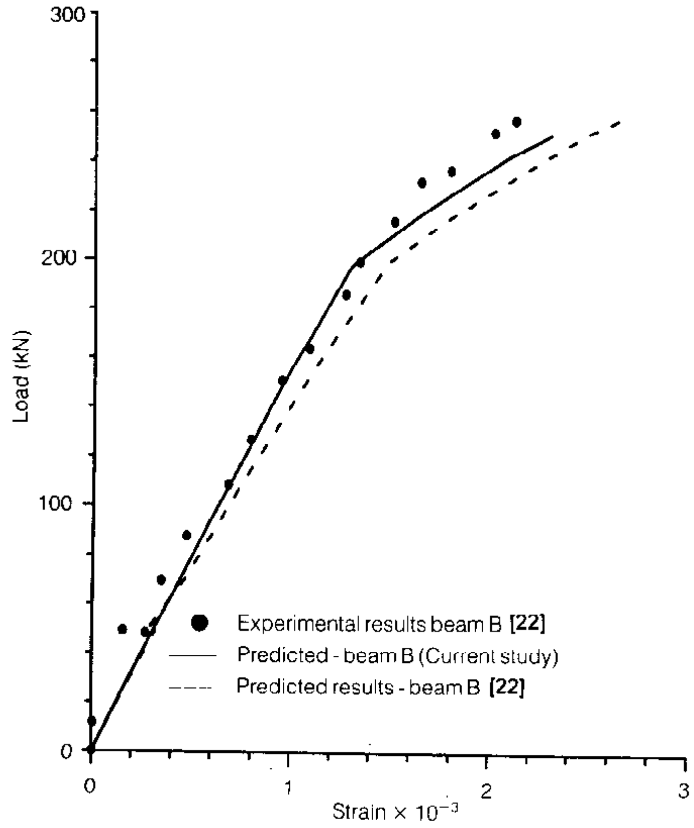

Fig. 8. Measured versus predicted load-concrete strain.

used to compare the accuracy of the prediction by the currently proposed model and other available models. The comparisons showed that, in most cases, the currently proposed model provided better prediction to the experimental results than other models. Therefore, it is believed that availability of such model can be useful in selecting and sizing the material needed to rehabilitate and/or strengthen the existing reinforced concrete beams.

Furthermore, the current ACI formula for estimating the deflection of conventionally RC beams underestimates the actual deflection when external reinforcement was added to the RC beams. Therefore, the ACI model may need some modification to account for the presence of external reinforcement. However, more data are needed before any modification can be suggested.

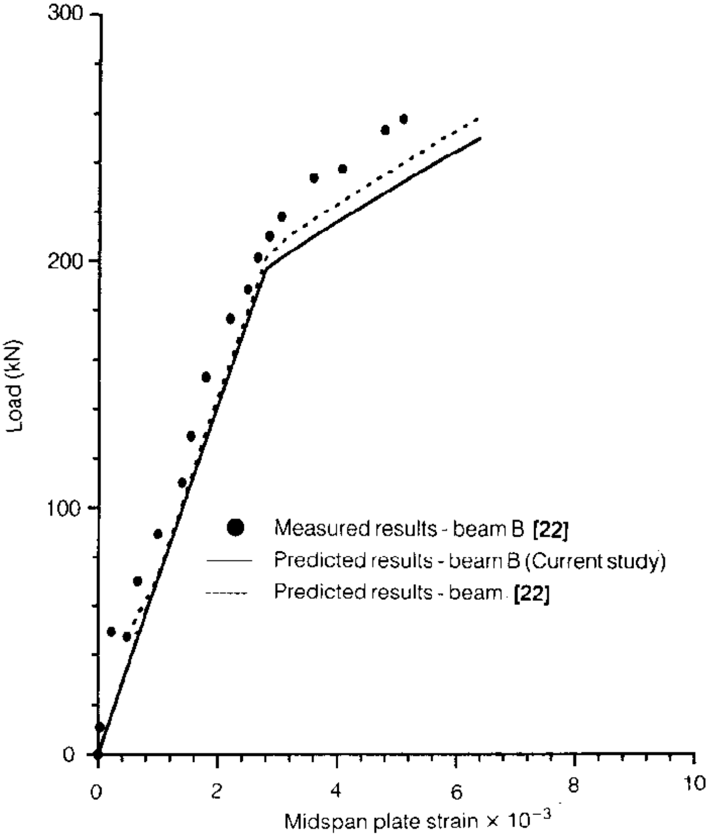

Fig. 9. Measured versus predicted load-GFRP plate strain.

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أنموذج عددي للتنبؤ بأداء العوارض الخرسانية المسلحة بعد إعادة تأهيلها و / أو تقويتها

صالح حامد السيد وطارق حمود المسلم

قسم الهندسة المدنية، كلية الهندسة، جامعة الملك سعود، ص. ب ٨٠٠،
الرياض ١١٤٢١، المملكة العربية السعودية

(سُلِّم في ٥/١٢/١٩٩٣م؛ قُبل للنشر في ١٣/٢/١٩٩٥م)

ملخص البحث. تقترح هذه الدراسة أنموذجًا عدديًا للتنبؤ بالعزوم والإجهادات والانفعالات والانحناءات في العوارض الخرسانية المسلحة قبل أو بعد تقويتها أو زيادة تحملها أو إعادة إصلاحها. يعتمد هذا الأنموذج على مبدأ تطابق الإزاحات، توازن القوى، ومحاكاة علاقة الإجهاد بالانفعال للخرسانة تحت تأثير الضغط بعلاقة أكثر دقة. كما أنه يأخذ في الاعتبار التغيّرات في الإجهادات والانفعالات والانحناءات في الاتجاه الطولي للعوارض. وهو أيضا يقبل الخواص المختلفة للمواد، الأشكال المختلفة للمقاطع والأوضاع المختلفة للأحمال وحديد التسليح. وبالإضافة إلى ذلك فإنّ هذا الأنموذج المقترح يأخذ في الاعتبار تأثير الزحف والانكماش على التشوه الذي يحدث في الخرسانة والتسليح الداخلي والخارجي. وعليه فإنّه يمكن استعمال هذا الأنموذج مع الطرق المختلفة المتوافرة لتقوية وإعادة تأهيل العوارض الخرسانية المسلحة.

تمّ تحليل نتائج التنبؤ باستعمال النموذج المقترح ومقارنتها بالنتائج المعملية المنشورة، وتبين أن هناك توافقا جيدا بين النتائج المعملية والنظرية، كما أنّ هذا التوافق كان أفضل من ذلك الذي تمّ الحصول عليه عندما استعملت النماذج الأخرى المتوافرة في أدبيات الموضوع.

إنّ توافر مثل هذا الأنموذج يعتبر ذا فائدة قيّمة للمهندس الإنشائي، وخاصة في مرحلتي التصميم واتخاذ القرار. ومن الممكن الاستفادة منه لتقويم خواص المتانة والخدمة للمنشآت القائمة سواءً المتضررة أو غير المتضررة. عند ذلك يمكن الاستفادة من هذا الأنموذج لاختيار نوع وحجم المادة الأكثر ملاءمة للظروف القائمة لتقوية و / أو إعادة تأهيل المنشأ. بطبيعة الحال فإن ذلك سيؤثر إيجابًا على خطة العمل وقد يؤدي إلى توفير المال والوقت.