MECHANICAL ENGINEERING

Analytical Solution of Heat Conduction in a Multi-Layer Orthotropic Cylinder Subject to Periodic and Asymmetric Heat Flux

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Abstract. The problem of heat conduction in a multi-layer two-dimensional orthotropic cylinder subject to asymmetric and periodic heat flux on the outer wall was solved analytically. The dimensional analysis of the problem revealed that the heat conduction through the cylinder is a function of the Biot number (Bi) and four non-dimensional parameters per layer. These layer-dependant parameters include: frequency ratio (α^*_m), thickness ratio (x^*_m), radial conduction ratio (K^*_{rm}), and tangential conduction ratio (K^*_{tm}). The derivation is valid for an arbitrary number of layers. The number of possible combinations increases dramatically with the number of layers. Thus as an example, the results for a cylinder composed from three layers, are presented and discussed. The results showed that the magnitude of heat conduction in a multi-layer orthotropic cylinder can be significantly different from those of an isotropic cylinder subject to the same externally imposed heat flux. The solution could be extended to an arbitrary varying imposed heat flux through the use of Fourier series and the principle of superposition. The solution also includes the analytical periodic temperature distribution across the cylinder. This could be used to study the effect of thermal stress fatigue in each layer and at the interface of adjacent layers.

Nomenclature

A	=	Complex constant used in Eq. (31)
B	=	Complex constant used in Eq. (31)
B_i	=	Biot number
C_p	=	Specific heat of layer material
c	=	Complex constant used in Eq. (31)
exp	=	Exponential to the base e

h	=	Convection heat transfer coefficient
J	=	Complex Bessel function of first kind
K_i	=	Non-dimensional conduction coefficient in the i direction
k_i	=	Conduction coefficient in the i direction
ln	=	Logarithm to the base e
N	=	Number of layers
Q	=	Non-dimensional heat conduction in the radial direction
Qm	=	Magnitude of the maximum radial heat conduction irrespective of the tangential location. $Qm(R) = Max[Q(R,\theta) \forall \theta]$
q	=	Dimensional heat conduction in the radial direction
R	=	Non-dimensional radius
Re[ ]	=	Real part of the complex value between brackets
r	=	Radius
T	=	Temperature
t	=	Time
x	=	Layer thickness relative to the inner layer thickness
Y	=	Complex Bessel function of the second kind.

Greek letters

α	=	Non-dimensional frequency
γ	=	Non-dimensional time
ζ	=	Auxiliary non-dimensional temperature
ξ	=	Spatial part of the complex non-dimensional temperature
θ	=	Angle
ρ	=	Density of layer material
ϕ	=	Non-dimensional temperature
ψ	=	Complex non-dimensional temperature
ω	=	Frequency of fluctuating temperature.

Superscripts

*	=	Parameter's value relative to its counterpart at the inner layer

Subscripts

m	=	m-th layer
N+1	=	Cylinder outer radius
r	=	Radial direction
t	=	Tangential direction
1	=	Cylinder inner radius
o	=	Magnitude of the fluctuating heat flux at the cylinder outer radius
∞	=	Temperature at which convection occurs at the cylinder inner radius

Introduction

There exist many materials that exhibit anisotropic behavior. This includes natural materials such as wood, crystals, and sedimentary rock. More important is the anisotropic behavior of composite materials and alloys made using unidirectional crystal solidification techniques. Interest in anisotropic materials dates back to the late nineteenth and early twentieth centuries. Earlier work in this field focused on one-dimensional heat conduction in crystals [1,2]. Carslaw and Jaeger [3] discussed the general anisotropic heat conduction problem with emphasis on orthotropic materials. Analytical solutions to unsteady anisotropic heat conduction problems relied on the use of Green's function but still proved extremely difficult [4].

With the increase of composite materials use in most engineering applications, it is becoming more important to study the effects of the anisotropic nature of such materials. Still, researchers tended to concentrate their work on isotropic materials due in part to their predominant use. Researches also tended to shy away from tackling anisotropic problems due to their complex mathematical nature. The analytical solution of a fully anisotropic problem is extremely difficult due to the cross direction conduction coefficient.

The use of composite materials has increased exponentially in the last two decades. The main advantage of composite materials is the ability to tailor its properties as needed. For the most part, engineers are interested in the mechanical properties of composite materials to achieve the required tensile strength and bending moment characteristics to support the imposed loads while using the smallest and lightest structure possible. Tailoring the thermal properties of composite materials could prove as important when used as insulation material. The difference in conduction coefficient between different directions can also lead to uneven material expansion and contraction. This leads to what is known as thermal stresses [5]. Such stresses add to the stresses imposed by external loads and could result in failure if not taken into account. To the best of the author's knowledge, all recent papers relevant to this work dealt with either a steady [6-8] or transient [9-13] heat transfer mainly across orthotropic materials [6,8-13] and some anisotropic materials [7].

This paper discusses the effect of several parameters on the radial heat conduction in a two-dimensional multi-layer orthotropic cylinder subject to steady periodic and asymmetric heat flux on the cylinder outer surface. It is of great importance to be able to determine the amount of heat being conducted to the cylinder inner surface under different conditions. Such a problem can be encountered in the space shuttle, the proposed space station, storage tanks, pipes, and chemical reactors. Other practical engineering applications could be treated using the problem at hand. The results will also be used to compare the numerical results of heat conduction in fully anisotropic materials subject to similar conditions. In case of anisotropic materials, an analytical solution is not possible.

Although the derivation enclosed is for a heat flux function composed of a single frequency in both the tangential and time, the resulting equations could be extended to include a much larger class of problems. The use of Fourier series expansion allows any function to be written as the sum of sine and cosine functions. Thus the effect of any imposed heat flux variation can be studied using the equations derived herein, including random fluctuations. Using the idea of superposition, the effect of each frequency can be studied separately and the sum of the individual effects will be the result of the original signal. The results will show that the frequency effect on the heat conduction through the cylinder is limited to a specific range. Thus, the Fourier decomposition needed only be considered in this effective range.

As a by-product of the analysis, the time dependant temperature distribution within the cylinder can be calculated. This could be used to study the thermal stress and fatigue problems in a cylinder made from several orthotropic layers. Such stresses could arise due to the dynamic thermal loading in a cylinder subject to an asymmetric and cyclic heat flux.

Mathematical Model

The problem at hand is that of a 2-D cylinder made from different orthotropic layers. The cylinder outer surface is subjected to an externally imposed steady and periodic heat flux. Figure 1 shows a schematic of the cylinder physical parameters and the imposed

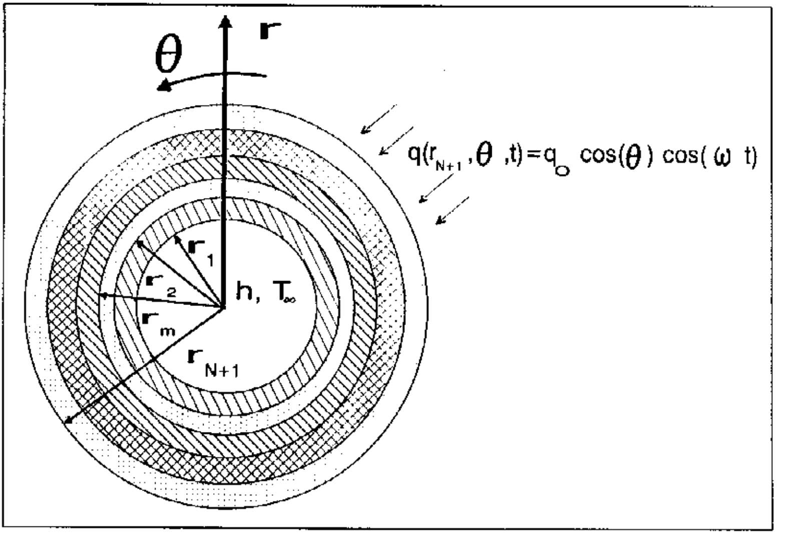

Fig. 1. Schematic of the cylinder layers and boundary conditions.

boundary conditions. The two-dimensional unsteady heat conduction equation in the m-th orthotropic layer of the cylinder is [14]:

$$\rho_m C_{\rho m} \frac{\partial T_m}{\partial t} = k_{rm} \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T_m}{\partial r} \right) + k_{tm} \frac{1}{r^2} \frac{\partial^2 T_m}{\partial \theta^2}; N \geq m \geq 1 \tag{1}$$

The heat transfer problem in the cylinder is governed by a system of N partial differential equations. The required boundary conditions for this system are:

$$k_{r1} \frac{\partial T_1(r_1,\theta,t)}{\partial r} = h[T_1(r_1,\theta,t) - T_\infty] \tag{2}$$

$$T_m(r_m,\theta,t) = T_{m-1}(r_m,\theta,t);\ N \geq m > 1 \tag{3}$$

$$k_{rm} \frac{\partial T_m(r_m,\theta,t)}{\partial r} = k_{rm-1} \frac{\partial T_{m-1}(r_m,\theta,t)}{\partial r}; N \geq m > 1 \tag{4}$$

$$k_{rN} \frac{\partial T_N(r_{N+1},\theta,t)}{\partial r} = q_o \cos(\theta)\cos(\omega t); 2\pi \geq \theta \geq 0 \tag{5}$$

$$T_m(r,\theta,t) = T_m(r,\theta + 2\pi,t); N \geq m \geq 1 \tag{6}$$

The asymmetric heat flux given by Eq. (5) would result from the cylinder being exposed to an external heat source situated far from the cylinder wall. Thus, the view factor for the cylinder wall will change with θ. A cylinder exposed to the sun would exhibit an asymmetric and periodic heat flux. Since we are interested in the steady state periodic heat transfer problem, no initial condition is required. This point will become clear later in the derivation.

The following non-dimensional groups will be defined in order to simplify the analysis:

$$R_m \equiv \frac{r_m}{r_1} \tag{7}$$

$$\phi_m(R,\theta,\gamma) \equiv \frac{T_m\left(\frac{r}{r_1},\theta,\omega t\right) - T_\infty}{\frac{q_o r_1}{k_{r1}}} \tag{8}$$

$$\gamma \equiv \omega t \tag{9}$$

$$K_{rm} \equiv \frac{k_{rm}}{k_{r1}} \tag{10}$$

$$K_{tm} \equiv \frac{k_{tm}}{k_{r1}} \tag{11}$$

$$B_i \equiv \frac{h r_i}{k_{r1}} \tag{12}$$

$$\alpha_m \equiv \frac{\rho_m C_{\rho m} \omega (r_1)^2}{k_{r1}} \tag{13}$$

Using these groups, Eq. (1) is rewritten as:

$$k_m \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \phi_m}{\partial R}\right) + K_{tm}\frac{1}{R^2}\frac{\partial^2 \phi^m}{\partial \theta^2} = \alpha_m \frac{\partial \phi_m}{\partial \gamma} \quad (14)$$

Equation 2 through 5 can be written, respectively, as:

$$\frac{\partial \phi_1(1,\theta,\gamma)}{\partial R} = \mathrm{Bi}\,\phi_1(1,\theta,\gamma) \quad (15)$$

$$\phi_m(R_m,\theta,\gamma) = \phi_{m-1}(R_m,\theta,\gamma)\ ;\ N \ge m > 1 \quad (16)$$

$$K_{rm}\frac{\partial \phi_m(R_m,\theta,\gamma)}{\partial R} = K_{rm-1}\frac{\partial \phi_{m-1}(R_m,\theta,\gamma)}{\partial R}\ ;\ N \ge m > 1 \quad (17)$$

$$K_{rN}\frac{\partial \phi_N(R_{N+1},\theta,\gamma)}{\partial R} = \cos(\theta)\,\cos(\gamma) \quad (18)$$

The insulation effect of the symmetry plane passing through $\theta = 0$ and $\theta = \pi$ is used to replace Eq. (6) with new boundary conditions in the tangential direction. The new conditions can be written as follows:

$$\frac{\partial \phi_m(R,0,\gamma)}{\partial \theta} = 0\ ;\ N \ge m \ge 1 \quad (19)$$

$$\frac{\partial \phi_m(R,\pi,\gamma)}{\partial \theta} = 0\ ;\ N \ge m \ge 1 \quad (20)$$

The solution of ϕ_m (R,θ,γ) can be found indirectly by defining an auxiliary problem ζ_m (R, θ, γ). The governing equation for ζ is the same as that for ϕ_m given by Eq. (14). The boundary conditions are also similar to those in Eqs. (15-17) and (19,20). The only difference is the boundary condition on the cylinder outer wall, Eq. (18), which is now expressed by the following form:

$$\xi_N(R_{N+1},\theta,\gamma) = \cos(\theta)\sin(\gamma) \quad (21)$$

A new function will now be defined as follows:

$$\psi_m(R,\theta,\gamma) = \phi_m(R,0,\gamma) + i\zeta_m(R,\theta,\gamma)\ ;\ i \equiv \sqrt{-1} \quad (22)$$

The boundary conditions for ψ_m are similar to those of ϕ_m and ζ_m, except for the boundary condition on the cylinder outer wall. This condition is written as:

$$\psi_N(R_{N+1},\theta,\gamma) = \cos(\theta)\,\exp(i\gamma) \quad (23)$$

The solution for ψ_m can be obtained using the complex temperature method [15]:

$$\psi_m(R,\theta,\gamma) = \xi_m(R,\theta)\exp(i\gamma) \quad (24)$$

The exp (i γ) term satisfies the time periodicity condition. This is why solving the system of equations did not require an initial condition. Using the solution defined by Eq. (17) in the governing equation for ψ_m and after dividing both sides of the equation by exp (i γ), we get the following equation:

$$K_{rm}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \xi_m}{\partial R}\right)+K_{tm}\frac{1}{R^2}\frac{\partial^2 \xi_m}{\partial \theta^2}=i\alpha_m\xi_m \quad (25)$$

The boundary conditions become as follows:

$$\frac{\partial \xi_1(1,\theta)}{\partial R}=Bi\,\xi_1(1,\theta) \quad (26)$$

$$\xi_m(R_m,\theta)=\xi_{m-1}(R_m,\theta);N\geq m>1 \quad (27)$$

$$K_{rm}\frac{\partial \xi_m(R_m,\theta)}{\partial R}=K_{rm-1}\frac{\partial \xi_{m-1}(R_m,\theta)}{\partial R};N\geq m>1 \quad (28)$$

$$K_{rN}\frac{\partial \xi_N(R_{N+1},\theta)}{\partial R}=\cos(\theta) \quad (29)$$

$$\frac{\partial \xi_m(R,0)}{\partial \theta}=\frac{\partial \xi_m(R,\pi)}{\partial \theta}=0;N\geq m\geq 1 \quad (30)$$

The solution of ξ_m (R, θ) is found using the method of separation of variables. After some mathematical manipulation and applying the tangential boundary conditions, the solution of the m-th layer is written in the following form:

$$\xi_m(R,\theta)=\cos(\theta)\left[A_m J_{\sqrt{\frac{K_{tm}}{K_{rm}}}}(C_m R)+B_m Y_{\sqrt{\frac{K_{tm}}{K_{rm}}}}(C_m R)\right] \quad (31)$$

$$C_m\equiv\sqrt{\frac{\alpha_m}{K_{rm}}}\exp(-i\frac{\pi}{4}) \quad (32)$$

J and Y are the complex first and second kind Bessel functions, respectively. In general, J and Y are of fractional order. The values of the complex constants A_m and B_m are found by the simultaneous solution of the 2N boundary condition equations, i.e., Eqs. (25-30). The solution sought, ϕ_m (R, θ, γ), is the real part of ψ_m (R, θ, γ). The final form of the periodic temperature distribution in the m-th layer becomes:

$$T_{m\ periodic}(R,\theta,\gamma)=\left(\frac{q_o r_1}{k_{r1}}\right)Re[\xi_m(R,\theta)\exp(i\gamma)]+T_\infty \quad (33)$$

where Re [ ] represents the real part of the complex value between the brackets. The non-dimensional periodic heat flux, $Q_{m\ periodic}$ (R, θ, γ) is calculated as follows:

$$Q_{m\ periodic}(R,\theta,\gamma)=-K_{rm}\frac{\partial(Re[\xi_m(R,\theta)\exp(i\gamma)])}{\partial R} \quad (34)$$

Results and Discussion

The non-dimensional analysis of the problem showed that the periodic radial heat conduction in a cylinder made from N layers is dependent on the following 4N parameters: α_m, x_m, K_{rm}, K_{tm}, and Bi. Note that K_{r1}=1.0 by definition. Since the number of possible combinations of the significant parameters increases rapidly with the number of layers, the case of a three-layer cylinder will be discussed in this section. The inner and outer layers will be assumed to be of the same material. Such a case could be used to model structures made from two layers of an orthotropic material with a third intermediate layer made from a different orthotropic material. The inner and outer layers makeup the hull of structure while the intermediate material could be used to provide thermal insulation. This is similar to the use of a honeycomb material between the structure's inner and outer layers. Buildings and storage tanks use such fabrication techniques.

The values of the inner and outer layer parameters, m=1 and 3, were fixed at: $\alpha_1 = \alpha_3 = 2.0$, $K_{r1} = K_{r3} = 1.0$, and $K_{t1} = K_{t3} = 2.0$. The inner and outer layers' thickness, R_2-R_1 and R_4-R_3 were also fixed at 0.5 and 0.25, respectively. The effect of changing the values of the intermediate layer parameters will be represented as a ratio relative to their inner layer counterparts. Thus four new parameters will be defined as follows:

$$x = \frac{R_3 - R_2}{R_2 - R_1} \tag{35}$$

$$K_r^* = \frac{K_{r2}}{K_{r1}} \tag{36}$$

$$K_t^* = \frac{K_{t2}}{K_{t1}} \tag{37}$$

$$\alpha^* = \frac{\alpha_2}{\alpha_1} \tag{38}$$

The new parameters will be referred to hereafter as: thickness ratio (x), radial conduction ratio (K^*_r), tangential conduction ratio (K^*_t), and frequency ratio (α^*). The use of a ratio allows the study of the effect of the order in which the layers are stacked. The k_r value of an insulation material is small. If the inner and outer layers are used to provide thermal insulation then K^*_r will be greater than 1.0. If the intermediate layer is used to provide the thermal insulation K^*_r will be less than 1.0. Similar arguments would apply to K^*_t and the effect of thermal storage capacity in α^*.

The results will detail the effect of changing each of the four new parameters and the Biot number (Bi) on the radial heat conduction through the cylinder. In particular, this section will report on the magnitude of maximum radial heat conduction at the inner radius (Qm_1). The values of α^*, Bi, K^*_r, and K^*_t were varied separately over the range in which each exhibited a noticeable effect on the value of Qm_1. The effect of layer thickness is of great importance for design purposes. Thus several thickness ratios (x) were used while studying the effect of each of the other four parameters. A reference value for each of the four parameters was used for all cases except when the effect of the parameter was under investigation. The reference values used were fixed at: $\alpha^* = 0.5$, $Bi = 0.7$, $K^*_r = 0.5$, and $K^*_t = 2.0$. The reference values correspond to the case where the intermediate layer is the insulation material. Thus, the intermediate layer has a lower thermal capacity.

Figure 2 shows the effect of frequency ratio (α^*) on the magnitude of the maximum radial heat conducted to the cylinder inner surface (Qm_1). The curves show that the effective range of α^* is strongly dependent on the thickness of the intermediate layer up to x = 2.0. For x values greater than 2.0, the effective frequency range was small. This suggests that the intermediate layer is acting as a low pass filter/capacitor. A thicker intermediate layer will allow less radial heat transfer to the cylinder inner wall.

Figure 3 shows the change of Qm_1 versus Biot number (Bi). A high value of Bi would indicate that the inner layer has a low radial heat conduction coefficient (k_r) and/or high convection heat transfer coefficient (h). Thus a high Bi value would result in a lower radial heat conduction resistance in the inner layer. The cylinder's total radial heat resistance is the sum of the three layers' resistance. Since the value of K^*_r was fixed, the intermediate layer resistance would increase with layer thickness (x). Thus, at low x values, the radial

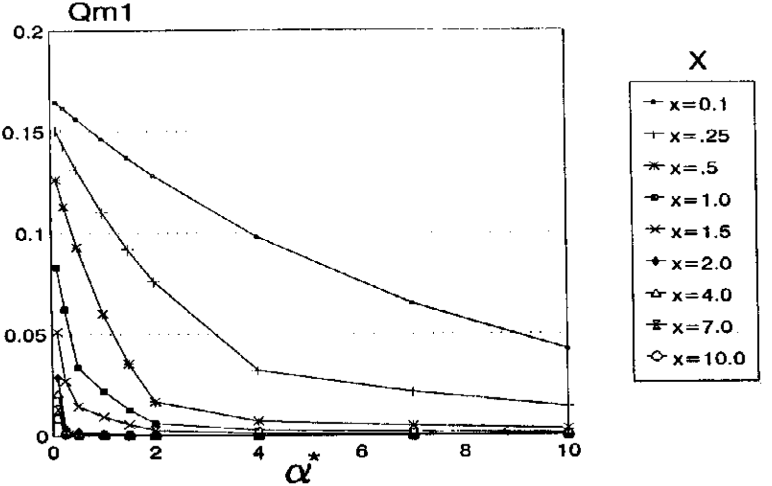

Fig. 2. Effect of the frequency ratio (α^*) on the radial heat conduction to the cylinder inner radius (Qm_1)

is the sum of the three layers' resistance. Since the value of K^*_r was fixed, the intermediate layer resistance would increase with layer thickness (x). Thus, at low x values, the radial resistance is due mainly to the inner layer. That is why the value of Qm_1 changes significantly with Bi at low thickness ratios (x). As the value of x is increased beyond 1.5, the resistance of the intermediate layer becomes the dominant resistance, and thus the effect of Bi on the radial heat conduction is reduced.

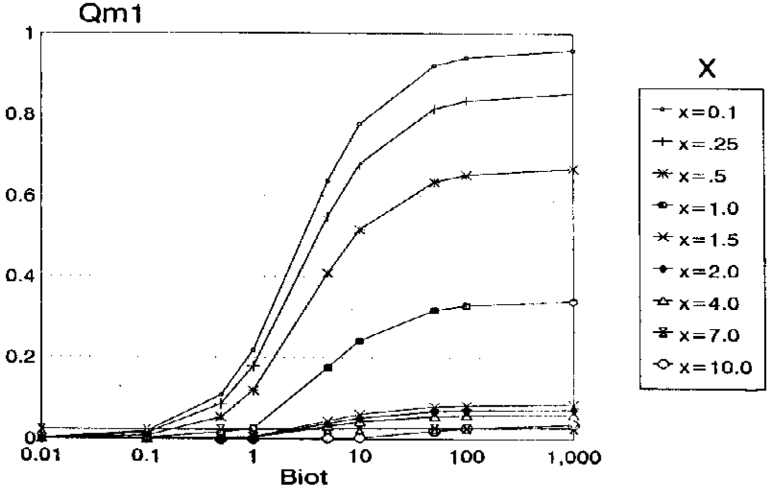

Fig. 3. Effect of theBiot number (Bi) on the radial heat conduction to the cylinder inner radius (Qm_1).

Figure 4 shows the effect of the radial conduction ratio (K^*_r) on Qm_1. A K^*_r value less than 1.0 indicates that the intermediate layer acts as a thermal insulation layer while a value more than 1.0 will indicate that the inner and outer layers are the insulators. That is why the effect of thickness ratio is greatest at low K^*_r values. A layer's radial heat conduction resistance is directly proportional to the thickness of the layer and inversely proportional to its radial thermal conductivity. Since all values at the inner and outer layers were fixed, their respective radial heat conduction resistance remained constant, irrespective of the K^*_r value. Thus, the cylinder total resistance will change with the intermediate layer resistance. A low value of K^*_r would indicate that the intermediate layer is a poor conductor of heat. Thus a low K^*_r value coupled with a relatively thick outer layer would result in a significant decrease in the value of Qm_1. The reduction in the radial heat conduction values increases as the intermediate layer thickness increases. At high K^*_r values, the contribution of the intermediate layer to the cylinder total radial heat resistance is significant only if coupled with a high x value. Thus, the value of Qm_1 becomes insensitive to K^*_r values greater than 4.0.

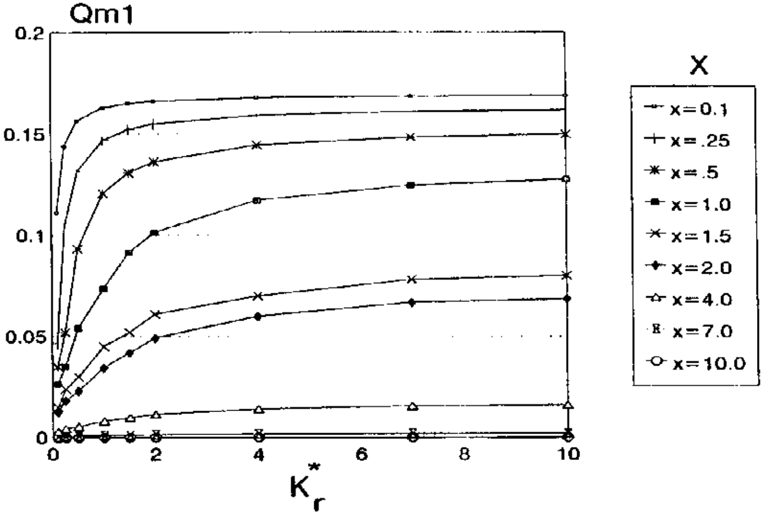

Fig. 4. Effect of the radial conduction ratio (K^*_r) on the radial heat conduction to the cylinder inner radius (Qm_1).

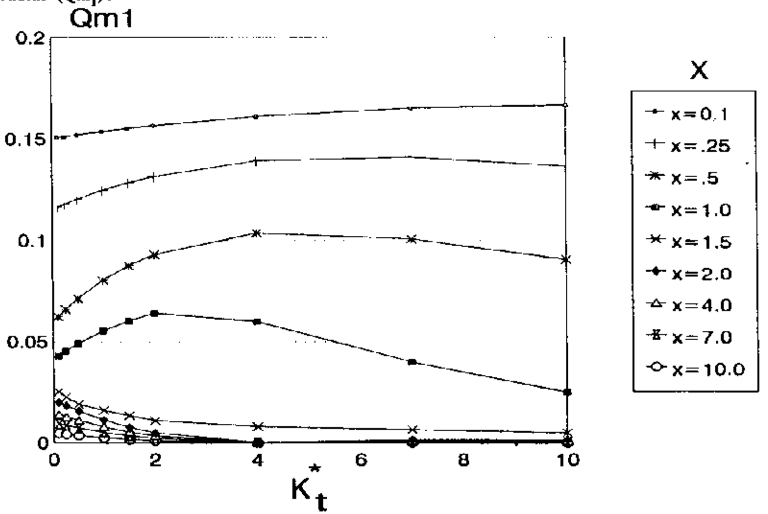

Fig. 5. Effect of the tangential conduction ratio (K^*_r) on the radial heat conduction to the cylinder inner radius (Qm_1).

Figure 5 shows the effect of the tangential conduction ratio (K^*_t) on Qm_1. As the value of K^*_t increases, the tangential heat conduction resistance decreases. The radial heat conduction resistance is not affected by the value of K^*_t but increases with x. Thus a higher K^*_t value would result in a reduction in the magnitude of heat energy conducted radially to the cylinder inner wall. The reduction in Qm_1 value due to higher K^*_t value is amplified at high x values. This can be seen clearly in Fig. 5 for all x values > 1.0. At small values of x, Qm_1 initially increased then decreased. The K^*_t value at which Qm_1 reached its maximum value decreased as the thickness ratio value increased. This is due to the fact that the radial heat conduction resistance increases with x. Thus, a smaller value of K^*_t value is needed for the tangential heat conduction resistance to become less than the radial heat conduction resistance. At which point the value of Qm_1 will decrease with at higher K^*_t values.

Conclusions

An analytical solution of the problem of heat conduction in a cylinder made from N orthotropic layers subjected to an asymmetric and periodic heat flux on the cylinder outer wall was obtained. The non-dimensional form of the equation showed that the heat conduction across the cylinder was dependent on 4N parameters: α_m, x_m, K_{rm}, K_{tm}, and Bi. A three-layer example was studied in detail. The effect of each parameter on the magnitude of radial heat conduction to the cylinder inner radius, Qm_1, was investigated. The results showed that the magnitude of the radial heat conducted can be altered, to different degrees, based on the combination of the aforementioned parameters. Low values of Bi and/or K^*_r tend to reduce the magnitude of the heat energy conducted radially across the cylinder. High values of x, α^*, and/or K^*_t also tend to reduce the magnitude of the heat energy conducted radially across the cylinder. A cylinder designed with the judicious choice of the properties and size of orthotropic layers would exhibit excellent insulation characteristics compared with an isotropic cylinder subject to the same conditions.

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حلّ تحليلي لانتقال الحرارة في إسطوانة مكوّنة من طبقات مختلفة الخواص تحت تأثير تدفقّ حراري دوري وغير متماثل

بسام عبد الكريم أبو حجله

قسم الهندسة الميكانيكية ، جامعة العلوم والتكنولوجيا الأردنية ،

ص.ب. ٣٠٣٠ ، إربد ٢٢١١٠ ، الأردن

(استُلم في ١٩٩٥/١٠/٣٠ ؛ وقُبل للنشر في ١٩٩٦/٦/١٩م)

ملخص البحث . تمّ اشتقاق حلّ تحليلي لعمليّة انتقال الحرارة في إسطوانة ثنائية الأبعاد مكوّنة من طبقات مختلفة الخواص تحت تأثير تدفق حراري دوري وغير متماثل . تحليل الوحدات أظهر أنّ عمليّة انتقال الحرارة عبر الإسطوانة تعتمد على قيمة مجموعة "البيوت" بالإضافة إلى أربع مجموعات عديمة الوحدات لكل طبقة و هي : الذبذبة النسبية ، السماكة النسبية ، معامل التوصيل الحراري النسبي في الاتّجاه الشعاعي ومعامل التوصيل الحراري النسبي في الاتجاه المماسي . الحلّ التحليلي صالح لأي عدد من الطبقات ولكن عدد الاحتمالات الممكنة يتزايد بشكل مطّرد مع عدد الطبقات . و عليه فقد تمّ دراسة انتقال الحرارة في إسطوانة مكوّنة من ثلاث طبقات على سبيل المثال . تظهر النتائج أن مقدار انتقال الحرارة عبر إسطوانة مكوّنة من طبقات مختلفة الخواص تختلف كثيرًا عن مقدار انتقال الحرارة عبر إسطوانة مكوّنة من مادة متجانسة تحت تأثير نفس التدفق الحراري المسلّط خارجيًا. يمكن استخدام الحل لتمثيل حالات متعدّدة عن طريق استخدام مبدأ تحليل الذبذبات و مبدأ التجميع . يوفّر الحل التوزيع التحليلي للحرارة عبر الإسطوانة والذي يمكن أن يستخدم لدراسة تأثير الإجهاد الحراري والكلل الحراري في كل طبقة و بين الطبقات المختلفة .