

## **Thermal Performance of Annular Fins of Arbitrary Profile Subjected to Periodic Base and Environment Temperatures**

**Abdul Aziz M. Mujahid and Khalil Abu-Abdou**

*Mechanical Engineering Department, College of Engineering, King Saud University,  
P.O. Box 800, Riyadh 11421, Saudi Arabia*

(Received 16/10/1990; Accepted for Publication 3/8/1991)

**Abstract.** The thermal performance of an annular fin of arbitrary profile with periodic base and environment temperatures is considered. The temperature of the fin is taken as the sum of a steady and a steady-periodic components. The governing equation for the steady-periodic component is solved using the complex combination method.

The influence of the fin parameter, fin geometry, amplitudes and frequencies of the imposed periodic base and environment temperatures on the fin temperature, rate of heat transfer instantaneous efficiency and time-averaged efficiency is analyzed and discussed. The results show that for straight and annular fins, the performance of the rectangular shape is the best followed by the parabolic profile and the triangular shape is the least favourable. The fin parameter has a pronounced influence on the thermal performance of the fin. The results of the special case of straight fin agree well with those available in the literature.

### **Nomenclature**

A	Fin cross-sectional area
b	Fin thickness at the base
C	Amplitude, equation (16a)
D	Amplitude, equation (18a)
F	Functions, equation (6)
f	initial guess of $\phi'(0)$
g	initial guess of $\psi'(0)$
h	Heat transfer coefficient
k	Thermal conductivity
L	Fin radial length, $L = r_o - r_i$
m	Fin geometry factor

N	Fin parameter, $\sqrt{\frac{2hL^2}{kb}}$
n	Case identifier, equation (7)
p	Amplitude of temperature oscillation
r	radial distance along the fin, measured from the centerline of the fin
$r_i$	inner radius of the fin
$r_0$	outer radius of the fin
R	dimensionless parameter $\left(\frac{r_0 - r_i}{r_0}\right)$
q	Rate of heat transfer
Q	Dimensionless rate of heat transfer $(qL/kb\Delta\bar{T})$
$Q_s$	Steady-state dimensionless rate of heat transfer
S	Fin surface area
T	Temperature
$\bar{T}$	Mean temperature
t	Time
W	Complex function, equation (6)
z	Transformed distance, $\ln[(r_0 - r_i)/(r_0 - r)]$

### Greek

$\alpha$	Thermal diffusivity
$\varepsilon$	Phase angle of heat transfer rate, equation (29)
$\gamma$	Phase angle of the steady periodic temperature, equation (27)
$\theta$	Dimensionless temperature, $(T - \bar{T}_a)/(\bar{T}_b - \bar{T}_a)$
$\omega$	Frequency of temperature oscillation
$\Omega$	Dimensionless frequency $(\omega L^2/\alpha)$
$\tau$	Fourier number, $\alpha t/L^2$
$\eta$	Instantaneous fin efficiency
$\bar{\eta}$	Time-averaged fin efficiency

### Subscript

a	Ambient
b	Base
s	Steady state
p	Steady periodic

## Introduction

Thermal performance of straight fins of uniform thickness under various boundary and atmospheric conditions have been the subject of research of many researchers [1,2]. Straight fins with periodic variation of either the base or the environment temperature were studied by Aziz and Sofrata [3], Suryanarayana [4] and Aziz and Na [5]. Mujahid [6] considered the case where both the base heat flux and environment temperature are periodic.

Recently fins of arbitrary profile received some attention. Aziz and Na [5] considered straight fins with rectangular, parabolic and triangular profiles subject to either a periodic environment temperature or a periodic atmospheric temperature. Abu-Abdou and Mujahid [7] studied the general case of a straight fin of an arbitrary profile where both the environment and base temperatures are periodic.

Annular fins under periodic boundary conditions received a rather limited attention. Aziz [2] considered a numerical solution of an annular fin of uniform thickness subjected to a periodic base temperature and constant environment temperature. Mujahid [8] developed a closed form solution for an annular fin of uniform thickness exposed to a periodic rate of heat transfer at the base while the environment temperature is constant.

Solution techniques used for the analysis of the performance of fins range from separation of variables to complex temperature, Laplace transform, free-parameter, superposition and numerical techniques.

In the present work the thermal performance of a convecting annular fin of an arbitrary profile is undertaken. The fin is exposed to a periodic variation of both the base and environment temperatures. A general case treatment that accommodates the cases of rectangular, triangular and parabolic fins which includes also the case of a straight fin, is considered in the present study. The complex combination method is used and the resulting equations are solved numerically using the shooting method.

The effect of the fin parameter, amplitudes and frequencies of the periodic base and environment temperatures on the fin temperature distribution, rate of heat transfer, instantaneous and time-averaged efficiencies are presented for the three geometries. The results for the rectangular geometry are found to agree well with those of exact analytical solution [1, 3].

## Analysis

Consider an annular fin of inner radius  $r_i$ , outer radius  $r_o$  and height  $b$  at the base. The fin thickness  $2y$  varies radially according to the relation

$$\frac{2y}{b} = \left( \frac{r_0 - r}{r_0 - r_i} \right)^m$$

In this form, rectangular, convex parabolic and triangular fin profiles are described by  $m = 0, 0.5$  and  $1$  respectively. The fin is assumed to convect on both faces to the environment with heat transfer coefficient,  $h$ . Both the environment temperature  $T_a$  and the base temperature  $T_b$  have periodic variations. The heat transfer from the fin tip is assumed to be negligible (*i.e.*, insulated). The mathematical formulation of the problem also applies to a straight fin as a special case, where  $r_i$  and  $r_0 \rightarrow \infty$  while  $r_0 - r_i = L$  is still finite. Fig 1 shows the fin geometries considered in this study.

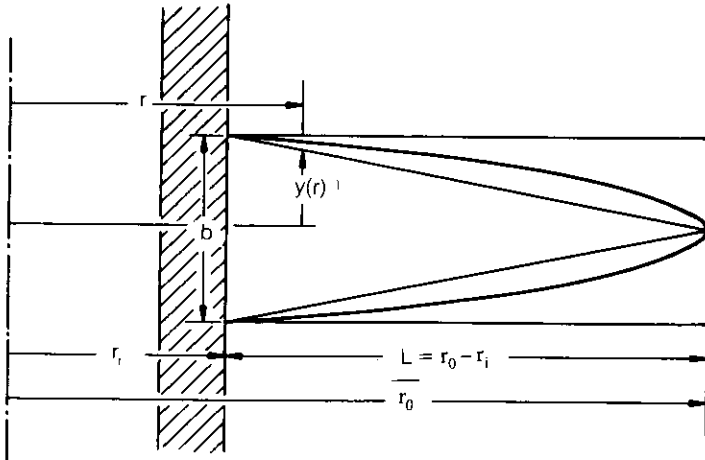


Fig. 1. Fin geometry.

Assuming one-dimensional conduction and constant thermal properties, the energy equation and boundary conditions for the fin problem can be put in the following form:

$$\frac{\partial^2 T}{\partial r^2} + \left( \frac{1}{r} - \frac{m}{r_0 - r} \right) \frac{\partial T}{\partial r} - \frac{2h}{kb} \left( \frac{r_0 - r_i}{r_0 - r} \right)^m (T - T_a) = \frac{1}{\alpha} \frac{\partial T}{\partial t} \tag{1}$$

$$r = r_0 \quad \frac{\partial T}{\partial r} = 0 \tag{1a}$$

$$r = r_i \quad T = T_b$$

The environment and base temperatures are given by the following equations:

$$T_a = \bar{T}_a + p_a \Delta \bar{T} \cos \omega_a t \quad (2a)$$

$$T_b = \bar{T}_b + p_b \Delta \bar{T} \cos \omega_b t \quad (2b)$$

Equation (1) has a singularity at the fin tip ( $r = r_0$ ) which can be removed by properly transforming the independent variable  $r$ . It is convenient also to use dimensionless coordinates. In dimensionless transformed form, equations (1) and (2) can be expressed as:

$$e^{2z} \left[ \frac{\partial^2 \theta}{\partial z^2} + \left\{ (1-m) + \frac{R}{e^z - R} \right\} \frac{\partial \theta}{\partial z} - N^2 e^{(m-2)z} (\theta - p_a \cos \Omega_a \tau) \right] = \frac{\partial \theta}{\partial \tau} \quad (3)$$

$$z = 0 : \theta = 1 + p_b \cos \Omega_b \tau ; z \rightarrow \infty : \frac{\partial \theta}{\partial z} = 0 \quad (3a)$$

$$\text{where } z = \ln[(r_0 - r_i)/(r_0 - r)]; \tau = \frac{\alpha t}{(r_0 - r_i)^2};$$

$$\theta = \frac{T - \bar{T}_a}{\Delta \bar{T}}; R = \frac{r_0 - r_i}{r_0}; N^2 = \frac{2h(r_0 - r_i)}{kb},$$

$$\Omega_a = \omega_a \frac{(r_0 - r_i)^2}{\alpha}; \Omega_b = \omega_b \frac{(r_0 - r_i)^2}{\alpha}$$

$$\Delta \bar{T} = \bar{T}_b - \bar{T}_a$$

### Solution

The solution  $\theta$  is assumed to be the sum of a steady component  $\theta_s$  and a steady-periodic component  $\theta_p$ ,

$$\theta(z, \tau) = \theta_s(z) + \theta_p(z, \tau) \quad (4)$$

The steady component is described by eqs. (3) and (3a) after dropping all terms containing  $\tau$ . Its solution is straight forward and will not be pursued here. The steady-periodic component is described by eq. (3) (replacing  $\theta$  with  $\theta_p$ ) and the modified boundary condition:

$$z = 0 : \theta_p = p_b \cos \Omega_b \tau ; z \rightarrow \infty : \frac{\partial \theta_p}{\partial z} = 0 \quad (3b)$$

The rest of the paper is devoted to the solution of the steady-periodic problem as formulated by eqs. (3) and (3b). According to the method of complex combination,  $\theta_p$  is set as the real part of a complex function  $W$ .

$$\theta_p(z, \tau) = \text{Re}\{(W(z, \tau))\} \tag{5}$$

Then eqs. (3), (3b) are rewritten to accommodate this generalisation. The differential equation and boundary conditions for the complex temperature  $W$  are:

$$e^{2z} \left[ \frac{\partial^2 W}{\partial z^2} + \left\{ (1-m) + \frac{R}{e^z - R} \right\} \frac{\partial W}{\partial z} - N^2 e^{(m-2)z} (W - P_a e^{i\Omega_a \tau}) \right] = \frac{\partial W}{\partial \tau} \tag{5a}$$

$$z = 0 : W = p_b e^{i\Omega_b \tau} ; z \rightarrow \infty : \frac{\partial W}{\partial z} = 0 \tag{5b}$$

Following ref. [7] the solution for  $W$  is set as follows:

$$W(z, \tau) = p_a F_a(z) e^{i\Omega_a \tau} + p_b F_b(z) e^{i\Omega_b \tau} \tag{6}$$

where the amplitude functions  $F_a(z)$  and  $F_b(z)$  are generally complex.

Upon substitution of eq. (6) into eqs. (3) and (3b) and collection of coefficients for each of  $e^{i\Omega_a \tau}$  and  $e^{i\Omega_b \tau}$  two sets of ordinary differential equations for the functions  $F_a(z)$  and  $F_b(z)$  are obtained. A common formulation for both sets is possible when a case identifier  $n$  is introduced:

$$F'' + \left[ (1-m) + \frac{R}{e^z - R} \right] F' - [N^2 e^{(m-2)z} + i\Omega e^{-2z}] F + (1-n) N^2 e^{(m-2)z} = 0 \tag{7}$$

$$z = 0 : F = n ; z \rightarrow \infty : F' = 0 \tag{7a}$$

( $n=0$  for  $F = F_a$ ,  $n=1$  for  $F = F_b$  and primes denote differentiation with respect to  $z$ .)

To obtain a solution for an  $F$  function, real notation is restored. Putting

$$F = \phi + i\psi \tag{8}$$

and substituting into equations (7) and (7a), the governing equations for  $\phi$  and  $\psi$  are reduced to two coupled linear boundary value problems as follows:

$$\left. \begin{aligned} \phi'' + \left[ (1-m) + \frac{R}{e^z - R} \right] \phi' - N^2 e^{(m-2)z} \phi + \Omega e^{-2z} \psi + (1-n)N^2 e^{(m-2)z} &= 0 \\ \psi'' + \left[ (1-m) + \frac{R}{e^z - R} \right] \psi' - N^2 e^{(m-2)z} \psi - \Omega e^{2z} \phi &= 0 \end{aligned} \right| \quad (9)$$

$$\left. \begin{aligned} z = 0 : \quad \phi &= n, \quad \psi = 0 \\ z \rightarrow \infty : \quad \phi' &= n, \quad \psi' = 0 \end{aligned} \right| \quad (9a)$$

The boundary value problem described by this system of equations can be solved using the method of superposition as in Ref. [9] for straight fins. However, for convenience, the method of shooting is adopted in the present work.

**Numerical Solution**

$$\text{let } y_1 = \phi, y_2 = \phi', y_3 = \psi, y_4 = \psi' \quad (10)$$

Equations (9) and (9a) can then be written as a system of first-order differential equations, viz:

$$\left. \begin{aligned} y_1' &= y_2 \\ y_2' &= -ay_2 + by_1 - cy_3 - d \\ y_3' &= y_4 \\ y_4' &= -ay_4 + by_3 + cy_1 \end{aligned} \right| \quad (11)$$

$$\left. \begin{aligned} y_1(0) &= n, \quad y_3(0) = 0 \\ y_2(\infty) &= 0, \quad y_4(\infty) = 0 \end{aligned} \right| \quad (11a)$$

$$\text{where } a = (1-m) + \frac{R}{e^z - R}, \quad b = N^2 e^{(m-2)z}$$

$$c = \Omega e^{-2z}, \quad d = (1-n) N^2 e^{(m-2)z}$$

Equations (11) and (11a) can only be treated as initial-value problem if the initial values  $y_2(0)$  and  $y_4(0)$  are known. In the method of shooting these values are assumed and an iterative procedure is followed to improve them. Let

$$y_1'(0) \equiv y_2(0) = f \text{ and } y_3'(0) \equiv y_4(0) = g \quad (11b)$$

Integration of the so constructed initial-value problem is done using the Runge-Kutta method and the solution is extended to sufficiently large values of  $z$ , i.e.  $z \rightarrow \infty$ . From

the solution, values of  $y_2(\infty)$  and  $y_4(\infty)$  are obtained and are compared with their correct values according to equation (11a). Regardless how careful the initial guesses of  $y_2(0)$  and  $y_4(0)$  are chosen, it is anticipated that more iterations are needed. Denoting  $f$  and  $g$  as  $f^{(1)}$  and  $g^{(1)}$ , and the resulting errors in the computed values  $y_2^{(1)}(\infty)$  and  $y_4^{(1)}(\infty)$  as  $E_2^{(1)}$  and  $E_4^{(1)}$ , where

$$E_2^{(1)} = y_2^{(1)}(\infty) - y_{2, \text{exact}}(\infty) = y_2(\infty, f^{(1)}, g^{(1)}) \tag{12}$$

$$E_4^{(1)} = y_4^{(1)}(\infty) - y_{4, \text{exact}}(\infty) = y_4(\infty, f^{(1)}, g^{(1)})$$

improved values  $f^{(2)} = f^{(1)} + \Delta f$  and  $g^{(2)} = g^{(1)} + \Delta g$  are sought such that the accompanying errors  $E_2^{(2)}$  and  $E_4^{(2)}$  vanish. By expanding  $E_2^{(2)}$  and  $E_4^{(2)}$  about  $(f^{(1)}, g^{(1)})$ , then setting  $E_2^{(2)} = E_4^{(2)} = 0$ , linear algebraic equations for  $\Delta f$  and  $\Delta g$  are obtained. In matrix form:

$$\begin{vmatrix} \frac{\partial E_2^{(1)}}{\partial f} & \frac{\partial E_2^{(1)}}{\partial g} \\ \frac{\partial E_4^{(1)}}{\partial f} & \frac{\partial E_4^{(1)}}{\partial g} \end{vmatrix} \begin{vmatrix} \Delta f \\ \Delta g \end{vmatrix} = - \begin{vmatrix} E_2^{(1)} \\ E_4^{(1)} \end{vmatrix} \tag{13}$$

$\Delta f$  and  $\Delta g$  can be calculated from equation (13) provided that the derivative terms are known. From equation (12) it is clear that the derivatives in the coefficient matrix of equation (13) can be replaced by the derivatives of the respective  $y$ -functions, *i.e.*

$$\frac{\partial E_2}{\partial f} = \frac{\partial y_2}{\partial f}, \quad \frac{\partial E_2}{\partial g} = \frac{\partial y_2}{\partial g}$$

$$\frac{\partial E_4}{\partial f} = \frac{\partial y_4}{\partial f}, \quad \frac{\partial E_4}{\partial g} = \frac{\partial y_4}{\partial g} \tag{14}$$

In the present work, the derivatives in equation (14) are obtained in a systematic way by solving a set of auxiliary differential equations generated from the system of equations (11) and the boundry conditions (11a) and (11b). For this purpose these equations are differentiated twice, once with respect to  $f$  and the other with respect to  $g$ . This results in two additional sets each of four equations. The new variables are denoted by:

$$y_5 = \frac{\partial y_1}{\partial f}, y_6 = \frac{\partial y_2}{\partial f}, y_7 = \frac{\partial y_3}{\partial f}, y_8 = \frac{\partial y_4}{\partial f}$$

$$y_9 = \frac{\partial y_1}{\partial g}, y_{10} = \frac{\partial y_2}{\partial g}, y_{11} = \frac{\partial y_3}{\partial g}, y_{12} = \frac{\partial y_4}{\partial g}$$

The solution of the overall initial-value problem for  $y_1, \dots, y_{12}$  is performed using a standard Runge-Kutta routine e.g. DGEAR from the IMSL library. Integration is carried to  $z \rightarrow \infty$ . The values of  $\Delta f$  and  $\Delta g$  are obtained from eq. (13) as:

$$\Delta f = - \left. \frac{y_2 y_{12} - y_4 y_{10}}{y_6 y_{12} - y_8 y_{10}} \right|_{z \rightarrow \infty}; \quad \Delta g = - \left. \frac{y_6 y_4 - y_8 y_2}{y_6 y_{12} - y_8 y_{10}} \right|_{z \rightarrow \infty} \tag{15}$$

The procedure is repeated until the ratios of  $\Delta f$  and  $\Delta g$  to the most recent values of  $f$  and  $g$ , respectively are equal to or smaller than a prescribed tolerance.

Once the boundary conditions of  $\phi'$  and  $\psi'$  at  $z \rightarrow \infty$  are satisfied as required by equation (11a), the complex function  $F(z)$  is then constructed according to equation (8). Now, with the appropriate values of  $n$  used, the solutions for the functions  $F_a$  and  $F_b$  are obtained, one at a time, and the steady periodic component,  $\theta_p$  is calculated from equation (5) as:

$$\theta_p = C_a \cos(\Omega_a \tau + \gamma_a) + C_b \cos(\Omega_b \tau + \gamma_b) \tag{16}$$

$$\text{where } C_a = p_a \sqrt{\phi_a^2 + \psi_a^2}, \gamma_a = \tan^{-1}(\psi_a/\phi_a) \tag{16a}$$

$$C_b = p_b \sqrt{\phi_b^2 + \psi_b^2}, \gamma_b = \tan^{-1}(\psi_b/\phi_b)$$

The dimensionless temperature of the fin is finally obtained by adding the steady-state solution  $\theta_s$  to the steady periodic component  $\theta_p$ .

### Rate of Heat Transfer

The instantaneous rate of heat transfer can be evaluated from the following equation:

$$Q = - \left. \frac{\partial \theta}{\partial z} \right|_{z=0} \tag{17}$$

where

$$Q = \frac{q(r_0 - r_i)}{2\pi r_i kb \Delta \bar{T}}$$

Equation (17) can be expressed as:

$$Q = Q_s - D_a \cos(\Omega_a \tau + \varepsilon_a) - D_b \cos(\Omega_b \tau + \varepsilon_b) \quad (18)$$

$Q_s$  = dimensionless, steady-state rate of heat transfer

$$= - \left. \frac{d\theta_s}{dz} \right|_{z=0}$$

$$D_a = p_a (\phi_a'^2 + \psi_a'^2)^{1/2}, \varepsilon_a = \tan^{-1}(\psi_a'/\phi_a') \quad (18a)$$

$$D_b = p_b (\phi_b'^2 + \psi_b'^2)^{1/2}, \varepsilon_b = \tan^{-1}(\psi_b'/\phi_b')$$

and all values are at  $z=0$

### Instantaneous Fin Efficiency

The instantaneous fin efficiency is defined as the ratio of the actual instantaneous heat transfer rate, as given by equation (17), to the ideal heat transfer rate for a fin with infinite thermal conductivity and can be expressed as:

$$\eta = \frac{q}{q_{ideal}} = \frac{q_s + q_p}{q_{ideal}}$$

In dimensionless form, the above equation may be expressed as:

$$\eta = \frac{1}{1 + p_b \cos \Omega_b \tau - p_a \cos \Omega_a \tau} \left[ \eta_s - \frac{1}{N^2} \cdot \frac{1-R}{1-\frac{R}{2}} \cdot \{D_a \cos(\Omega_a \tau + \varepsilon_a) + D_b \cos(\Omega_b \tau + \varepsilon_b)\} \right] \quad (19)$$

where  $\eta_s$  is the steady-state fin efficiency.

**Time-Averaged Fin Efficiency**

The time-averaged fin efficiency may be evaluated as:

$$\bar{\eta} = \frac{1}{\tau_0} \int_0^{\tau_0} \eta \, d\tau \tag{20}$$

where  $\tau_0$  is the time of an integral number of cycles for both the ambient and base temperatures,  $T_a$  and  $T_b$  respectively. For convenience, the ratio  $(\Omega_b/\Omega_a)$  is assumed to be an integer; thus  $\tau_0$  may be taken to be the period for one cycle of  $\Omega_a$  (i.e.,  $\tau_0 = 2\pi/\Omega_a$ ) and the corresponding  $(\Omega_b/\Omega_a)$  cycles of the  $\Omega_b$  oscillation.

The time-averaged fin efficiency can be evaluated by substituting equation (19) into equation (20) as:

$$\begin{aligned} \bar{\eta} = \eta_s \int_0^{\tau_0} \frac{d\tau}{1 + p_b \cos \Omega_b \tau - p_a \cos \Omega_a \tau} \\ - \frac{D_a}{N^2} \cdot \frac{1-R}{1-\frac{R}{2}} \cdot \int_0^{\tau_0} \frac{\cos(\Omega_a \tau + \epsilon_a) d\tau}{1 + p_b \cos \Omega_b \tau - p_a \cos \Omega_a \tau} \\ - \frac{D_b}{N^2} \cdot \frac{1-R}{1-\frac{R}{2}} \cdot \int_0^{\tau_0} \frac{\cos(\Omega_b \tau + \epsilon_b) d\tau}{1 + p_b \cos \Omega_b \tau - p_a \cos \Omega_a \tau} \end{aligned} \tag{21}$$

This equation is evaluated numerically using the Simpson's rule.

**Results and Discussion**

The temperature of the fin at any instant of time  $\theta(z, \tau)$  can be obtained from equation (4), whereas the rate of heat dissipation from the fin  $Q(\tau)$  is given by equation (18). The instantaneous fin efficiency is given by equation (19) and the time-averaged fin efficiency is represented by equation (21).

The performance of the fin as reflected by the equations of  $\theta(z, \tau)$ ,  $Q(\tau)$ ,  $\eta(\tau)$  and  $\bar{\eta}$  depends on the fin parameter  $N$ , amplitude and frequency of the base temperature ( $p_b, \Omega_b$ ), amplitude and frequency of the ambient temperature ( $p_a, \Omega_a$ ) and the geometry factor,  $m$ . The performance of the fin is represented in Figs (2–8) using the following values of parameters:

$m = 0, 0.5, 1$  (Rectangular, parabolic and triangular cross-sections).

$p_a = 0.1$  ,  $p_b = 0.25$

$(\Omega_b, \Omega_a) = (1.0, 0.5)$ , *i.e.* ratio of  $\Omega_b/\Omega_a = 2:1$

These rather limited parameters were used to generate some results to study the difference of performance between the three geometries.

### Temperature Distribution

The temperature at any point along the fin as represented by equation (4) is composed of a steady component and two superimposed steady periodic components which stem from the periodic nature of the ambient and base temperatures. Fig. 2 shows the variation of the temperature at the middle point along the annular fin with time for two values of the fin parameter  $N$  for the three types of fins considered in this study. As expected, the fin temperature shows an inverse proportionality with the fin parameter  $N$ . Fig. 2 shows also that for  $N > 4$  the fin temperature, approaches the ambient temperature, for all types of fins. It is also noted that the rectangular fin has a temperature that is always higher than that of the parabolic fin and this in turn is always greater than that of the triangular fin. When  $m$  is set equal to zero, this represents the special case of a straight fin. Fig. 3 represents the variation of the straight fin temperature at the middle point of the fin, with time, for the same base and environment conditions and fin parameter  $N$ , as represented in Fig. 2. The straight fin behaves in a similar fashion to that of the annular fin. However, the temperature of the straight fin is higher than that of the annular fin at any time for any given value of the fin parameter  $N$ .

It can be seen from Fig. 2 and Fig. 3 that the amplitude of the fin temperature decreases with the increase of the fin parameter  $N$  which is in agreement with the work of Mujahid [6] and Abu-Abdou and Mujahid [7].

### Heat Transfer Rate

The normalized value of the rate of heat transfer,  $Q$ , is represented by equation (18). Figure 4 shows the variation of rate of heat transfer with time for annular fins of rectangular, parabolic and triangular sections for two values of fin parameter  $N$ . In

this figure, it is clear that  $N$  has a pronounced effect on  $Q$ , where  $Q$  at any instant of time, increases with increase in the value of  $N$ . It can be seen also that the amplitude of variation of  $Q$  with time decreases with decrease in the value of  $N$ . For the special case of a straight fin where  $m = 0$ , the variation of  $Q$  with time for two values of the fin parameter  $N$  is shown in Fig. 5. A comparison between Fig. 4 and Fig. 5 shows that the rate of heat dissipated from the annular fin is greater than that of the straight fin when both have the same value of  $N$  as well as base and environment conditions.

### Fin Efficiency

The instantaneous fin efficiency is represented by equation (19). Figure 6 shows the variation of the fin efficiency of an annular fin with time for two values of  $N$ . The behavior of the fin efficiency is similar to that of the rate of heat transfer  $Q$ , as shown in Fig. 4. Figure 7 represents the variation of the instantaneous fin efficiency with time for the special case of a straight fin ( $m = 0$ ) subjected to the same conditions as the annular fin. The instantaneous efficiency of the straight fin is greater than that of the annular fin as can be seen from Figs 6 and 7.

The time-averaged efficiency,  $\bar{\eta}$ , is represented by equation (21). The variation of  $\bar{\eta}$  with  $N$  is shown in Fig. 8 for the annular and the straight fins. The time-averaged

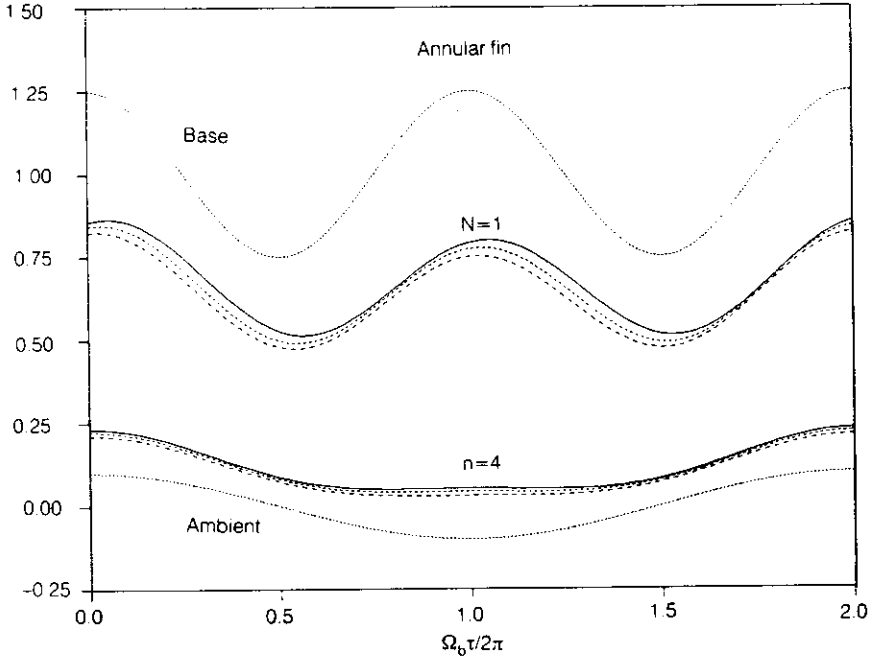


Fig. 2. Variation of rectangular (solid), parabolic (dotted) and triangular (dashed) annular fin midpoint temperature with time for  $\Omega_a = 0.5$ ,  $\Omega_b = 1.0$ .

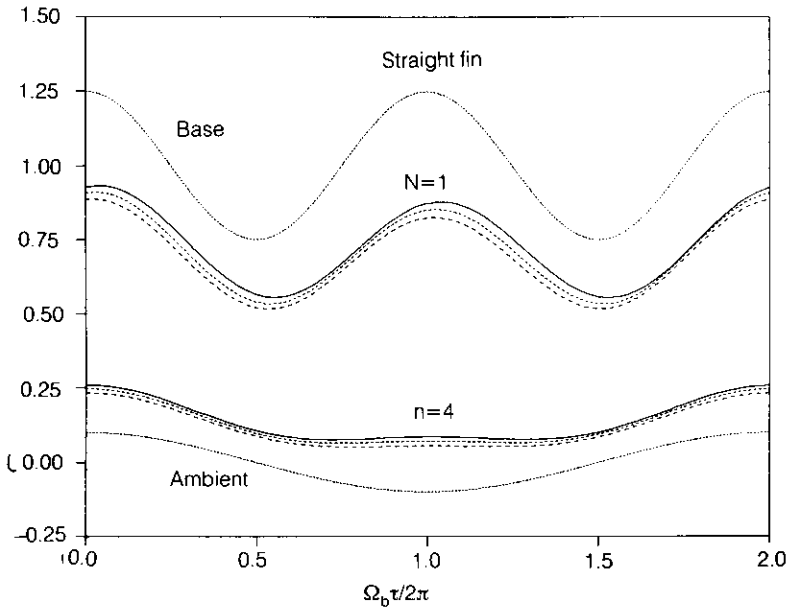


Fig. 3. Variation of rectangular, parabolic and triangular straight fin midpoint temperature with time for  $\Omega_a = 0.5$ ,  $\Omega_b = 1.0$ .

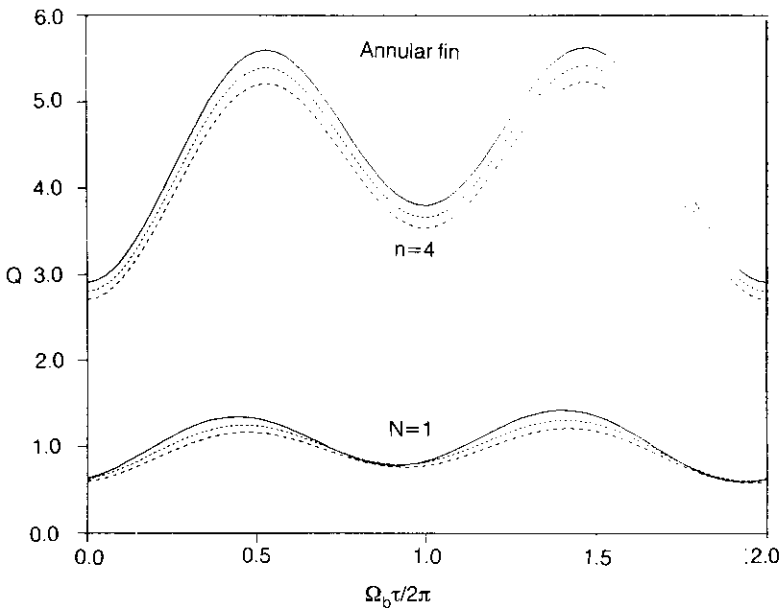
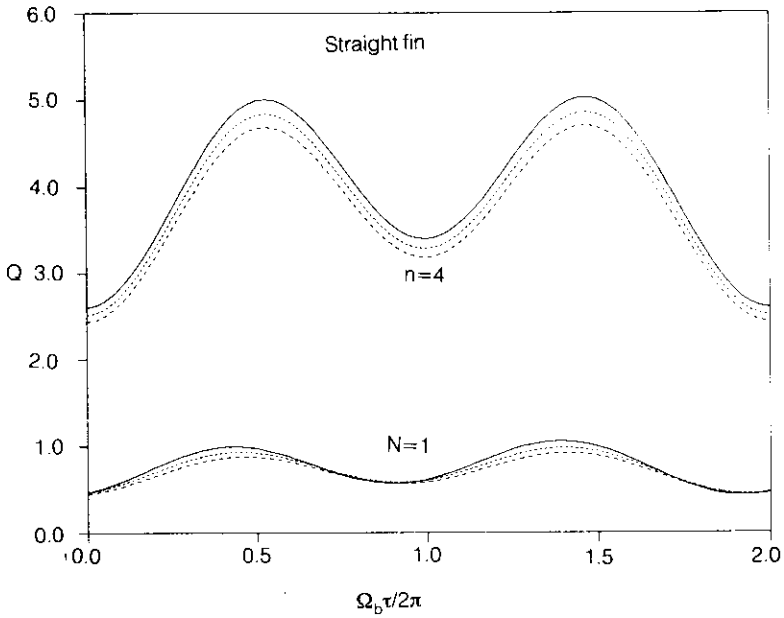
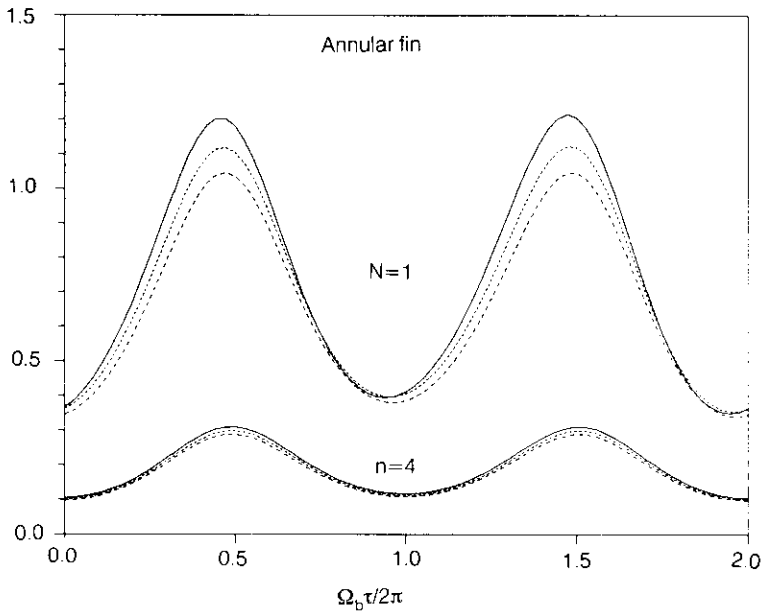


Fig. 4. Variation of rate of heat transfer of the rectangular, parabolic and triangular annular fin with time for  $\Omega_a = 0.5$ , and  $\Omega_b = 1.0$ .



**Fig. 5.** Variation of rate of heat transfer of the rectangular, parabolic and triangular straight fin with time for  $\Omega_a = 0.5$ , and  $\Omega_b = 1.0$ .



**Fig. 6.** Variation of the instantaneous efficiency of rectangular, parabolic and triangular annular fin with time for  $\Omega_a = 0.5$ , and  $\Omega_b = 1.0$ .

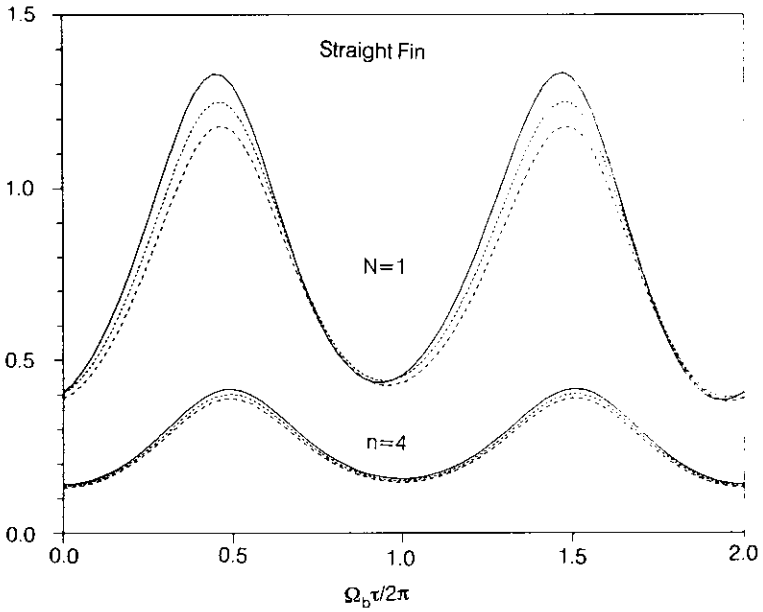


Fig. 7. Variation of the instantaneous efficiency of rectangular, parabolic and triangular straight fin with time for  $\Omega_a = 0.5$ , and  $\Omega_b = 1.0$ .

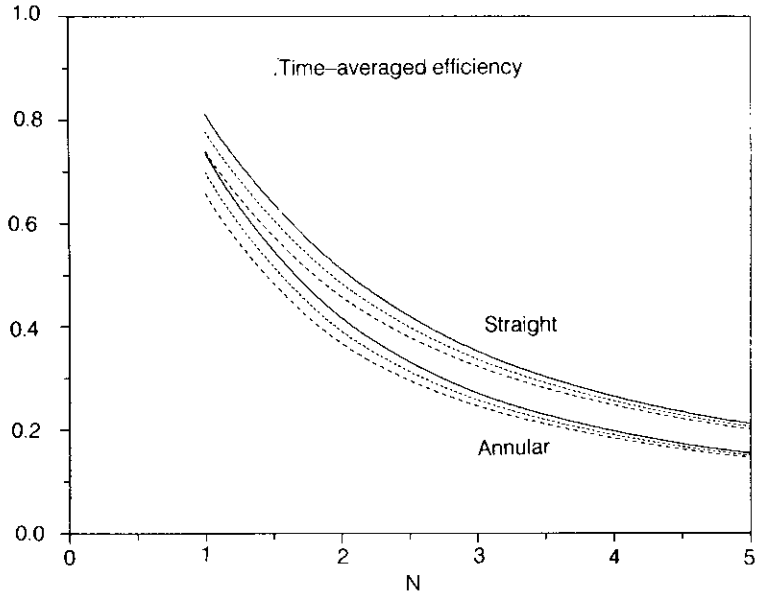


Fig. 8. Variation of the time-average efficiency of rectangular, parabolic and triangular annular fin with fin parameter  $N$  for  $\Omega_a = 0.5$ , and  $\Omega_b = 1.0$ .

efficiency,  $\bar{\eta}$ , decreases with the increase in the fin parameter  $N$ . It is also shown that the time-averaged efficiency of the straight fin is greater than that of the annular fin for any fixed value of the fin parameter  $N$ . When the efficiency of a periodically convecting fin is compared with the steady-state efficiency the two efficiencies are practically equal. This means that for purposes of efficiency estimation it is reasonable to consider a steady state problem with base and environment temperatures equal to the time-averaged values of the periodic case.

### References

- [1] Yang, J. W. "Periodic Heat Transfer in Straight Fins." *J. Heat Transfer*, 94 (1972), 310-314.
- [2] Aziz, A. "Heat Transfer in Annular Fins." *J. Heat Transfer*, 97 (1975), 302-303.
- [3] Aziz, A. and Sofrata, H. "Fin Performance in Oscillating Temperature Environment." *Applied Energy J.*, 9 (1981), 13-21.
- [4] Suryanarayana, N.V. "Transient Response of Straight Fins." *J. Heat Transfer*, 97 (1975), 417-423.
- [5] Aziz, A., and Na, T. Y. "Steady Periodic Heat Transfer in Fins of Arbitrary Profile." *Num. Heat Transfer*, 3 (1980), 331-344.
- [6] Mujahid, A. "Analysis of Performance of a Straight Fin with Oscillating Base Heat Flux and Environment Temperature." *J. Eng. Sci.*, King Saud University, Vol. 13, No.1 (1987), 25-37.
- [7] Abu-Abdou, K. and Mujahid, A. "Heat Transfer in Straight Fins of Arbitrary Profile Subjected to Periodic Base and Environment Temperatures." *Wärme-und Stoffübertragung* 24 (1989), 353-361.
- [8] Mujahid, A. "A Closed Form Solution for Period Heat Transfer in Annular Fins." *Wärme-und Stoffübertragung*, 24 (1989), 145-150.
- [9] Na, T. Y. *Computational Methods in Engineering Boundary Value Problems*. New York: Academic Press, 1979.

## الأداء الحراري للزعانف الحلقيّة ذات المقطع الاختياري تحت تأثير درجات حرارة متغيرةً دوريّاً

عبدالعزیز بن محمد المجاهد وخیل عمود أبو عبده  
قسم الهندسة الميكانيكية، كلية الهندسة، جامعة الملك سعود، ص.ب. ٨٠٠،  
الرياض ١١٤٢١، المملكة العربية السعودية  
(استلم في ١٠/١٠/١٩٩٠م؛ قبل للنشر في ٣/٨/١٩٩١م)

ملخص البحث. تمت دراسة الأداء الحراري للزعانف الحلقيّة ذات المقطع الاختياري المُعرّضة لدرجات حرارة متغيرةً دوريّاً عند القاعدة والمحيط المجاور. وقد اعتبرت درجة الحرارة اللحظية في الزعنف كمجموعة مركبتين هما درجة الحرارة المستديمة ودرجة الحرارة الدورية. واستخدمت طريقة التعميم المركّب لحل معادلة درجة الحرارة الدورية.

جرى تحليل ومناقشة تأثير كل من: معامل الزعنف، شكل مقطع، ذروة وتردد الذبذبة (لكل من درجتي حرارة القاعدة والمحيط المجاور) على درجة الحرارة اللحظية، معدّل انتقال الحرارة، الكفاءة اللحظية والكفاءة المتوسطة زمنياً وقد أوضحت النتائج أن الزعانف ذات المقطع المستطيل هي الأفضل أداءً تليها زعانف القطع المكافئ وأقلها أداءً الزعانف مثلثة المقطع. وتبين أن لمعامل الزعنف تأثير واضح على الأداء الحراري. وقد وجد أن النتائج الخاصة بالزعنف المستقيمة مُتّفقة مع مثيلاتها في المراجع المختلفة.