

## **Prediction of Critical Pipe Running Speed During Tripping in Drilling Operations**

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**Abstract.** Blowout and loss of circulation are two serious accidents that can happen while drilling without the control on drillstring running speed during tripping operations. Exceeding the critical running speed during tripping-out (high swabbing pressure) and lack of control of mud rheology are the main causes for blowout. Also, exceeding the critical running speed while tripping-in (high surge pressure) and lack of control of mud rheology are the main causes for loss of circulation. Many factors which affect surge and swab pressures must be precisely selected in order to control kicks or blowouts and to prevent loss of circulation. Prediction of the critical pipe running-in and -out speeds during tripping operations is therefore very important. This can be done by the evaluation of several basic fluid flow equations. A computer program has been developed to simplify these calculations. The developed program requires fairly simple input data which can be measured in laboratory in addition to hole and drillstring dimensions. The output of this program then is transformed into graphical form from which the safe running in and -out speeds during tripping can be predicted. As an alternative and direct way to predict the critical pipe running speed during tripping operations two correlations have been developed. These correlations account for the governing factors which affect the tripping-in and -out speeds including mud properties and drillstring and hole configuration. A comparison between the critical pipe running speed computed using the two methods has been outlined and very good accuracy based on the coefficient of linear correlation ( $r^2$ ) and the standard error of estimate (SEE) ( $r^2 = 0.995$  ; SEE = 0.39 for running-out correlation and  $r^2 = 0.875$  ; SEE = 0.817 for running-in correlation) have been obtained. Critical tripping speeds were found to be greatly dependent on mud weight and rheology, hole diameter and drillcollars-to-drillpipe length ratio.

### **Nomenclature**

- $a_0, a_1, a_2, a_3, a_4, a_5, a_6$  = Surge correlation coefficients
- $b_0, b_1, b_2, b_3, b_4, b_5, b_6$  = Swab correlation coefficients
- $d_h$  = Hole diameter, inch
- $d$  = Outside diameter of drillcollars or drillpipe, inch
- $f$  = Fannings friction factor, fraction

$G_f$	= Formation fracturing pressure gradient, psi/ft
$G_p$	= Formation pore fluid pressure gradient, psi/ft
$h$	= Well depth, feet
$L$	= Drillcollars or drillpipe length, feet
$R$	= Drillcollars-to-drillpipe length ratio
$r^2$	= Coefficient of linear correlation
$Re$	= Reynolds number, dimensionless
$V_a$	= Upward flow speed, ft/sec
$V_c$	= Critical running speed, ft/sec
$V_p$	= Pipe running speed, ft/sec
$P_t$	= Formation fracturing pressure
$P_p$	= Formation pore fluid pressure
$P_{surge}$	= Surge pressure, psi
$P_{swab}$	= Swab pressure, psi
$P_{ac}$	= Pressure drop around drillcollars, psi
$P_{ap}$	= Pressure drop around drillpipe, psi
SEE	= Standard error of estimate
$Y_p$	= mud yield point, lb/100 sq. ft
$\mu_p$	= Drilling fluid plastic viscosity, cp
$\rho$	= Drilling fluid density, ppg
$\sigma_{ov}$	= Overburden stress, psi
$\sigma_t$	= Formation tensile strength, psi
$\nu$	= Poisson's ratio, fraction

### Introduction

Pipe running-in or -out of a wellbore can cause pressure fluctuations (surge and swab respectively) caused by the piston-cylinder action of the pipe and the borehole. This action can contribute to loss of circulation and blowouts or kicks. Prediction of surge

and swab pressures has always been an essential part of well control. Excessive surge or swab pressure can lead to serious problems. Pressure reduction due to swabbing is a major source of blowouts.

### **Blowouts and kicks**

A kick occurs as a result of a formation pore fluid pressure being greater than the mud hydrostatic pressure which causes fluids to flow from the formation into the wellbore. The reasons of this pressure imbalance could be one of the following [1]:

- (i) Insufficient mud weight is one of the predominant causes of kicks. In this case a permeable zone is drilled while using a low mud weight that exerts less pressure than the formation pore pressure within the zone. As a result of this imbalance, fluids begin to flow into the wellbore and kick or blowout occurs.
- (ii) Improper hole fill-up during tripping is another cause of kicks. As the drillstring is pulled out of the wellbore, the mud level falls because the drillstring steel had displaced some amount of mud. With the pipe no longer in the hole the overall mud level will decrease and as a consequence the hydrostatic pressure of the mud will also decrease leading to a kick.
- (iii) Swabbing pressure is the pressure caused by pulling the drillstring from the wellbore. Such pressure is negative and reduce the effective mud hydrostatic pressure leading to a kick. Among the factors controlling swab pressure are pipe pulling speed, mud properties and hole and drillstring configuration.

### **Loss of circulation**

Loss of circulation is the loss of an appreciable fraction of the entire volume of drilling fluid through the borehole into highly porous formations. Loss of circulation can take place while lowering of drillstring or casing into the borehole. As a result of loss of circulation, the level of the drilling fluid in the annulus is lowered causing hydrostatic pressure in the annulus to become lower than the formation pore fluid pressure which may lead to a disastrous blowout. If the running-in speed is too high, weak formations in the open hole could be fractured leading to a serious loss of circulation which results in a blowout [2]. Therefore, the prediction of safe running-in and -out speeds can minimize the magnitude of surge and swab pressures and dangerous hole problems can be avoided especially in areas where huge amounts of money is invested to increase oil production capacity such as in Saudi Arabia. Huge drilling activities will take place in Saudi Arabia in the near future. Thus any interruption to these activities such as blowouts or kicks are not required and must be predicted during the planning phase. These surge and swab pressures have been

studied extensively by many researchers [3-13]. Unfortunately, most of these studies have developed models which lack in simplicity and require complex input parameters. In this study critical pipe running speeds required to minimize surge and swab pressures were predicted by two methods, firstly using a graphical technique and secondly using correlations. These methods account for drilling mud properties and hole and drillpipe sizes. Furthermore, the developed correlations require only simple data which can be measured in the laboratory using the API testing equipment. Using the developed correlations, long calculations can be avoided.

### Calculation Method and Results

Surge and swab pressures can be calculated using a procedure based on basic fluid flow equations [14]. The systematic calculation steps are shown below:

- (1) The flow critical velocity ( $V_c$ ) around the drillcollars and around the drillpipe can be calculated as follows:

$$V_c = \frac{1.08\mu_p + 1.08\sqrt{\mu_p^2 + 9.3\rho(d_h - d)^2} Y_p}{\rho(d_h - d)} \quad (1)$$

- (2) The average flow velocity ( $V_a$ ) around the drillcollars and around the drillpipe can be calculated as follows:

$$V_a = V_p \left( \frac{1}{2} + \frac{d^2}{d_h^2 - d^2} \right) \quad (2)$$

- (3) If the flow is laminar, the pressure drop around the drillcollars and the drillpipe can be calculated as follows:

$$\Delta P = \frac{L}{300(d_h - d)} \left[ Y_p + \frac{\mu_p V_a}{5(d_h - d)} \right] \quad (3)$$

- (4) If the flow is turbulent, the pressure drop around the drillcollars and the drillpipe can be calculated as follows:

$$\Delta P = \frac{f L \rho V_a^2}{25.8(d_h - d)} \quad (4)$$

(5) The friction factor can be calculated using the following equations:

$$R_e = \frac{2790 \rho V_a (d_h - d)}{\mu_p} \quad (5)$$

$$f = e^{\left( C_1 + \frac{C_2}{R_e} \right)} + C_3 \ln(R_e) \quad (6)$$

Where,

$$C_1 = -3.5378591164$$

$$C_2 = 300.26609292$$

$$C_3 = -0.126153971$$

(6) Surge and swab pressures can be calculated as follows:

$$P_{\text{surge}} = \Delta P_{\text{ap}} + \Delta P_{\text{ac}} + 0.052 \rho h \quad (7)$$

$$P_{\text{swab}} = 0.052 \rho h - \Delta P_{\text{ap}} - \Delta P_{\text{ac}} \quad (8)$$

A basic computer program (see Fig. 1) has been developed to do all the above mentioned calculations. Based on the above procedure a sensitivity analysis was performed to investigate the effect of fluctuation of the model parameters on surge and swab pressures (see Table 1) including:

- (i) Mud properties (density, plastic viscosity and yield point).
- (ii) Hole diameter.
- (iii) Drillcollars-to-drillpipe length ratio.
- (iv) Formation pore fluid pressure.
- (v) Formation fracturing pressure.

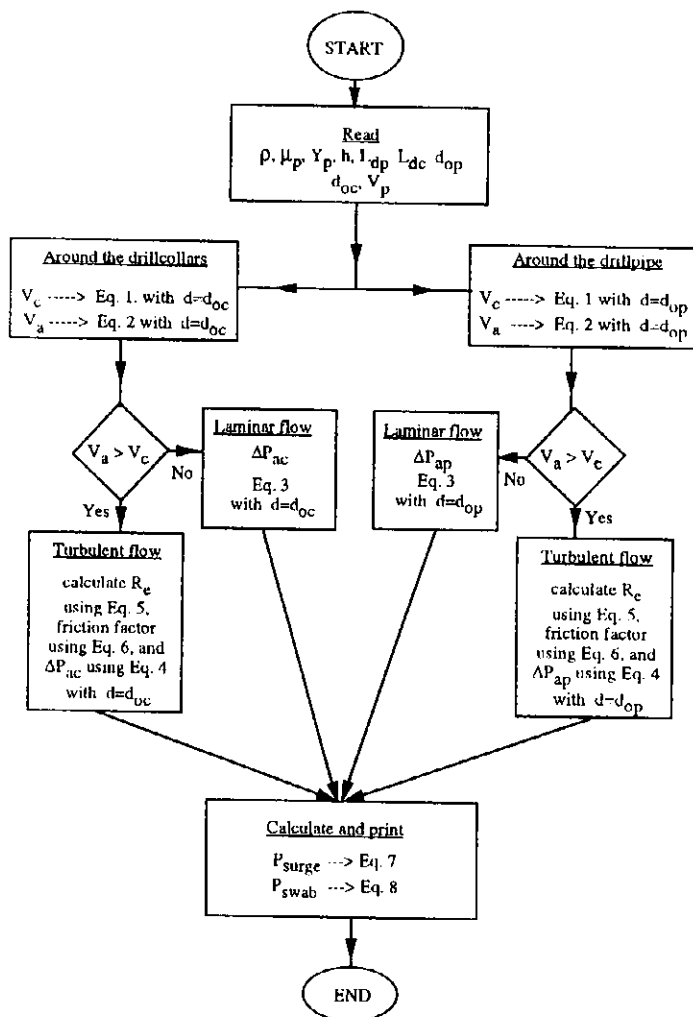


Fig. 1. Flow chart of the developed computer program.

**Table 1. Computer program input data**

Mud weight	= 8 -12	PPG
Mud plastic viscosity	= 10 - 50	cp
Mud yield point	= 10 - 200	lb/100 sq.ft.
Well depth	= 5000 -15000	ft.
Hole size	= 7.875 - 9.875	inches
Drillcollars-to drillpipe ratio	= 0.0364 - 0.1273	fraction
Drillpipe size (ID, OD)	= 3.826 , 4.5	inches
Drillcollars size (ID, OD)	= 2.813 , 6.75	inches
Pipe running speed during tripping-in and -out	= 0 - 14	ft/sec
Formation pore fluid pressure gradient	= 0.45	psi/ft
Formation fracturing pressure gradient	= 0.60	psi/ft
Poisson's ratio	= 0.23	fraction

Formation fracturing pressure can be estimated using the following equation [15]:

$$P_f = \frac{2\nu}{1-\nu} \sigma_{ov} + \sigma_t \quad (9)$$

Normally, the tensile strength of reservoir rocks is neglected as a worst case could be encountered (equal zero). The overburden stress for normally stressed formations can be evaluated as follows:

$$\sigma_{ov} = 1 \frac{\text{psi}}{\text{ft}} * h \quad (10)$$

Formation fracture gradient for deep wells can be estimated using the following equation:

$$G_f = \frac{2\nu}{1-\nu} \quad (11)$$

Assuming that average Poisson's ratio for most reservoir rocks equal to 0.23, the formation fracturing pressure gradient (calculated using Eq. 11) equals to 0.60 psi/ft. Formation pore pressure gradient normally constant and assumed to be 0.45 psi/ft in this study. Using data presented in Table 1 and the developed computer program shown in Table 2, a sensitivity analysis was conducted. Based on the results of the sensitivity analysis, Figs. 2 to 6 were plotted. The critical pipe running speeds for both surge and swab cases were clarified in Figs. 2 to 6. The following general correlations for critical pipe running-in and -out speeds are assumed:

**Table 2. Listing the developed computer program**

```

5      REM***** Surge and swab pressures calculation *****
7      REM
10     h      = 6000
20     ldp    = 5500
30     ldc    = 500
40     dpod   = 4.5
50     dcod   = 6.75
60     mw     = 10
70     yp     = 10
80     pv     = 30
90     dh     = 7.875
100    vp     = 9
110    pfg    = 0.60
120    ppg    = 0.45
121    pf     = pfg * h
122    pp     = ppg * h
130    PRINT
140    PRINT "-----Pressure drop around the drillpipe-----"
145    PRINT
150    vc = ((1.08 * pv) + (1.08 * (pv^2 + 9.3 * mw * (dh - dpod)^2 * yp)^.5)) / (mw * (dh - dpod))
160    vaap = vp * (.5 + (dpod^2 / (dh^2 - dpod^2)))
170    IF vaap > vc GOTO 210
180    dpadp = (ldp / (300 * (dh - dpod))) * (yp + ((pv * vaap) / (5 * (dh - dpod))))
190    REM print "The flow around the drillpipe is laminar"
200    GOTO 290
210    re = 2970 * mw * vaap * (dh - dpod) / pv
220    PRINT "The flow around the drillpipe is turbulent"
230    PRINT "Reynolds number = ", re
240    c1 = -3.5378591164
250    c2 = 300.26608292
260    c3 = -126153971
270    f = EXP(c1 + c2 * (1/re) + c3 * LOG(re))
280    PRINT "Friction factor = ", f
290    dpadp = (f * ldp * mw * vaap^2) / (25.8 * (dh - dpod))
300    PRINT "Pressure drop= ", dpadp, "psi"
310    PRINT "-----Pressure drop around the drillcollars-----"
320    ve = ((1.08 * pv) + (1.08 * (pv^2 + 9.3 * mw * (dh - dcod)^2 * yp)^.5)) / (mw * (dh - dcod))
330    vaac = vp * (.5 + (dcod^2 / (dh^2 - dcod^2)))
340    IF vaac > ve GOTO 380
350    dpadc = (ldc / (300 * (dh - dcod))) * (yp + ((pv * vaac) / (5 * (dh - dcod))))

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360 PRINT "The flow around the drillpipe is laminar"
370 GOTO 460
380 re=(2970*mw*vaac*(dh-dcod))/pv
390 PRINT "The flow around the drillcollars is turbulent"
400 PRINT "Reynolds number = ",re
410 c1= -3.5378591164
420 c2= 300.26608292
430 c3= -.126153971
440 f=EXP(c1+c2*(1/re)+c3*LOG(re))
450 PRINT "Friction factor =",f
460 dpadc = (f*ldc*mw*vaac^2)/(25.8*(dh-dcod))
470 PRINT "Pressure drop- ",dpadc,"psi"
480 PRINT "-----"
490 surge = (.052*mw*h)+dpadp+dpadc
500 swab = (.052*mw*h)-dpadp-dpadc
510 PRINT "Surge pressure =",surge,"psi"
520 PRINT "Swab pressure =",swab,"psi"
530 PRINT "-----"
540 z =swab-pp
550 zz=pl-surge
560 IF zz<=0 GOTO 600
570 PRINT "At a pipe speed = ",vp,"ft/sec"
580 PRINT "Safe tripping-in operation at this pipe speed."
590 GOTO 945
600 PRINT "At a pipe speed = ",vp,"ft/sec"
610 PRINT "Loss of circulation may occur during tripping-in operation."
620 IF z<=0 GOTO 650
630 PRINT "Safe tripping-out operation at this pipe speed"
640 GOTO 660
650 PRINT "Blowout may occur during tripping-out at this pipe speed. "
660 PRINT "-----"
670 PRINT
680 END

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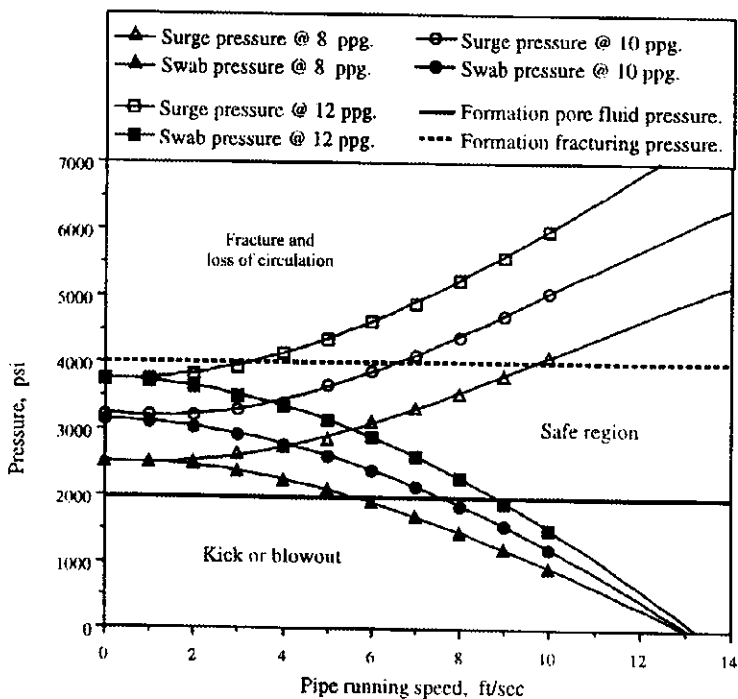


Fig. 2. The effect of mud weight on surge and swab pressures magnitude.

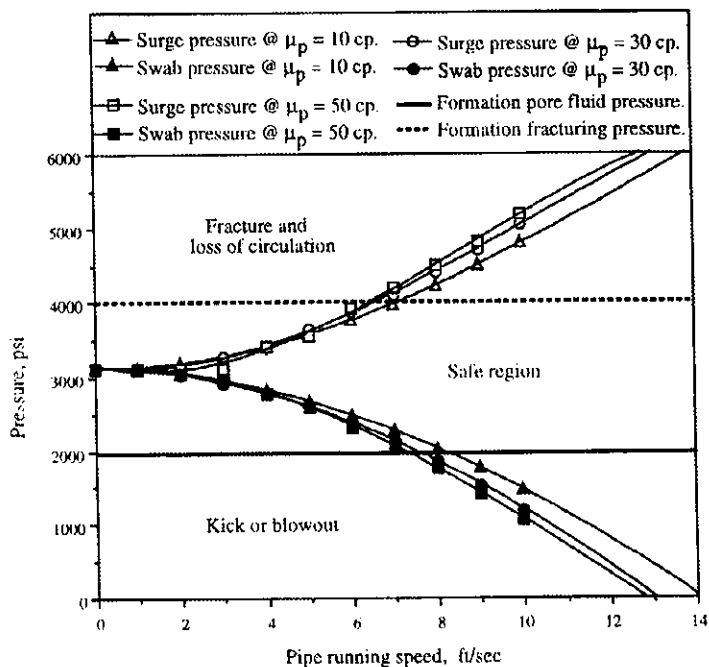


Fig. 3. The effect of mud plastic viscosity on surge and swab pressures magnitude.

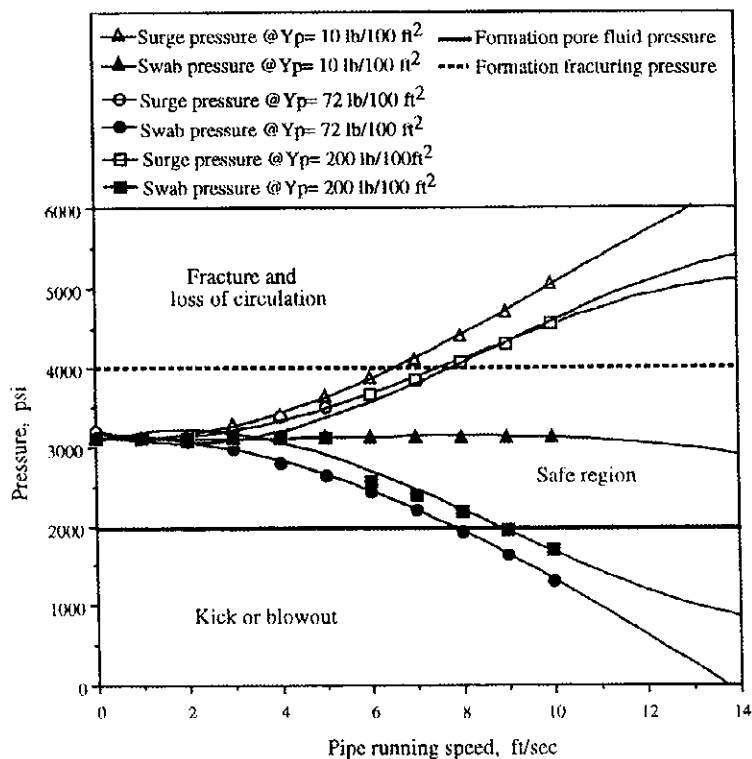


Fig. 4. The effect of mud yield point on surge and swab pressures magnitude.

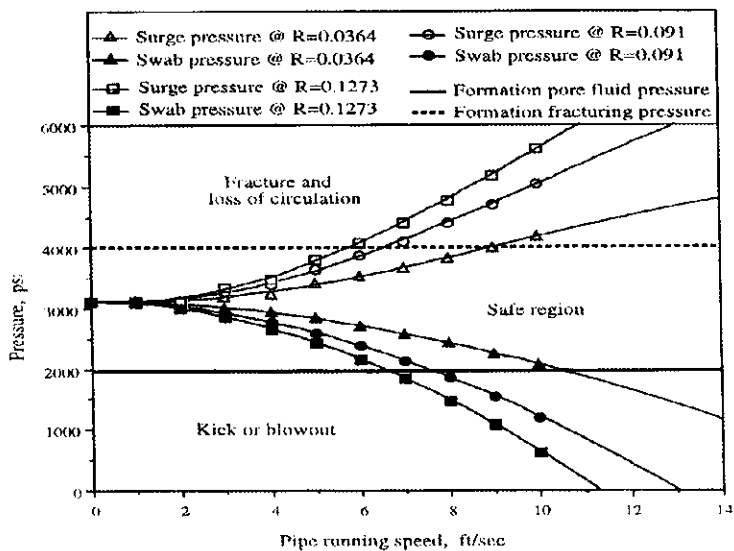


Fig. 5. The effect of drillcollars-to-drillpipe length ratio on surge and swab pressures magnitude.

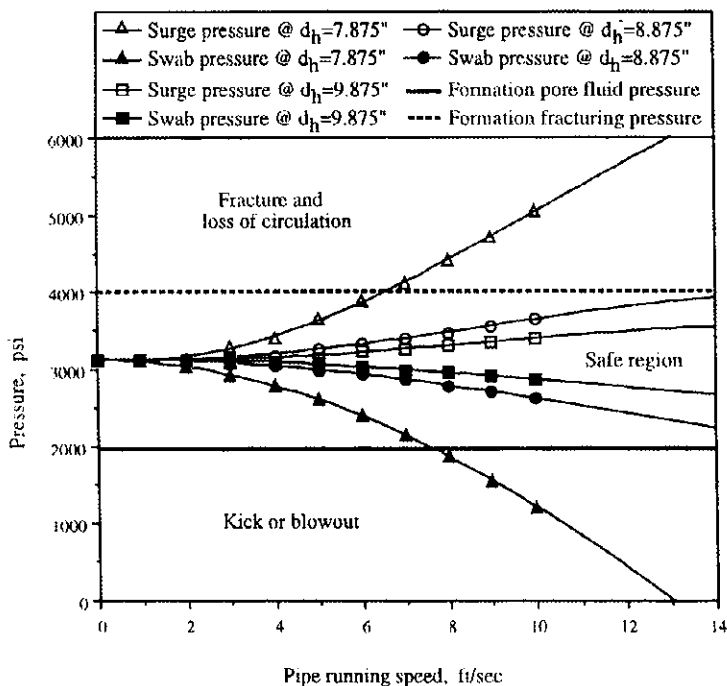


Fig. 6. The effect of hole size on surge and swab pressure magnitude.

#### Critical running-in speed:

$$V_p = a_0 + a_1 \rho_m + a_2 \mu_p + a_3 Y_p + a_4 R + a_5 d_h + a_6 P_f \quad (12.a)$$

Where,

$$P_f = G_f * h \quad (12.b)$$

#### Critical running-out speed:

$$V_p = b_0 + b_1 \rho_m + b_2 \mu_p + b_3 Y_p + b_4 R + b_5 d_h + b_6 P_p \quad (13.a)$$

Where,

$$P_p = G_p * h \quad (13.b)$$

Using the above mentioned basic fluid flow equations (Eqs. 1 to 8) and the result of the sensitivity analysis (Figs. 2 to 6), the correlation coefficients shown in Eqs. 12 and 13 were evaluated using regression analysis and the results are tabulated in Table 3. Therefore these correlations can be applied to rapidly predict the critical speeds during tripping-in or -out of the wellbore to avoid blowouts or loss of circulation problems caused by excessive surge or swab pressures.

**Table 3. Correlation coefficients and accuracy constants**

Critical running-in speed correlation constants Eq. 12.	$a_0$	= -90.650644	Coefficient of linear correlation
	$a_1$	= -2.03	
	$a_2$	= 0.00654655	$r^2 = 0.875$
	$a_3$	= 0.040705	Standard error of estimate
	$a_4$	= 42.302	
	$a_5$	= 11.9403	
$a_6$	= 4.663E-03		
Critical running-out speed correlation constants Eq. (13).	$b_0$	= -48.293	Coefficient of linear correlation
	$b_1$	= -1.1625	
	$b_2$	= 3.749E-03	$r^2 = 0.955$
	$b_3$	= 2.231E-02	Standard error of estimate
	$b_4$	= 24.225	
	$b_5$	= 6.9378	
$b_6$	= 2.6703E-03		

### Discussion

Drilling fluid properties including density, yield point, and viscosity are important factors which greatly affect the magnitude of surge and swab pressures generated during pipe running-in and -out of the hole respectively. Figure 2 shows the effect of mud density on critical pipe running-in and -out speeds range. The critical pipe running speed is the speed beyonds which loss of circulation or blowout could happen. When the mud density increases the rang of safe pipe running-in speed decreases. This effect is attributed to the large increase in mud pressure (in addition to the piston-cylinder action caused by the drillstring) opposing the formation being drilled and the results will be serious fractures leading to loss circulation problem. On the other hand, the increase in mud density can easily control formation pore fluid pressure, therefore, wide range of safe pipe running-out speeds can be applied as shown in Fig. 2. Safe pipe running-in and -out speeds are affected by mud rheological properties such as mud plastic viscosity and yield point. This effect could be due to the fact that viscous mud

and -out speeds are affected by mud rheological properties such as mud plastic viscosity and yield point. This effect could be due to the fact that viscous mud magnify the piston-cylinder action produced by dillstring up and down movements as shown in Figs. 3 and 4. Dillrcollars has a diameter bigger than the normal drillpipes. Therefore it adds an extra pressure on the formation during pipe running-in and drain the formation pore fluid into the wellbore during pipe running-out due to the piston-cylinder action as shown in Figures 5. Hole size have a large effect on safe pipe running speeds. This is because during pipe running-out, large hole diameter allows the drilling fluid to rapidly fill in the place which was occupied by the drillstring , therefore the formation pore fluid pressure is easily controlled and avoid blowouts.

During pipe running-in, large hole diameter provides bigger passage area for the mud, therefore, the piston-cylinder action acting opposite to the formation is minimized and the formation fracturing can be avoided as shown in Fig. 6. Thus, by plotting data obtained from the developed computer program, safe pipe running-in and -out speeds can be predicted. For simplicity and time saving as required in the field, two correlations have been developed. These correlations can be applied to rapidly predict safe pipe running-in and -out speeds required to avoid loss of circulation or blowout problems. Figures 7 and 8 shows comparisons between the critical pipe running-in and -out speeds predicted using the basic calculations and plotting technique and using the developed correlations.

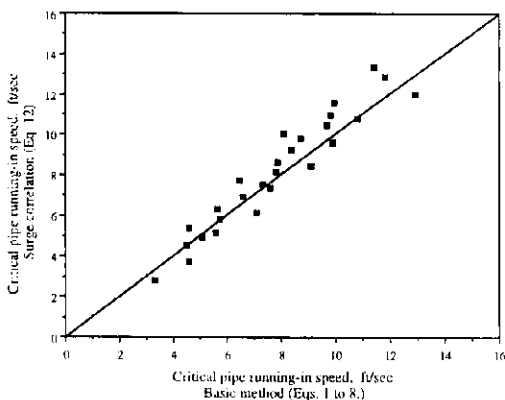


Fig. 7. Crossplot of the critical pipe running-in speed using the basic fluid flow equations (Eqs. 1 to 8.) and the developed surge correlation (Eq. 12).

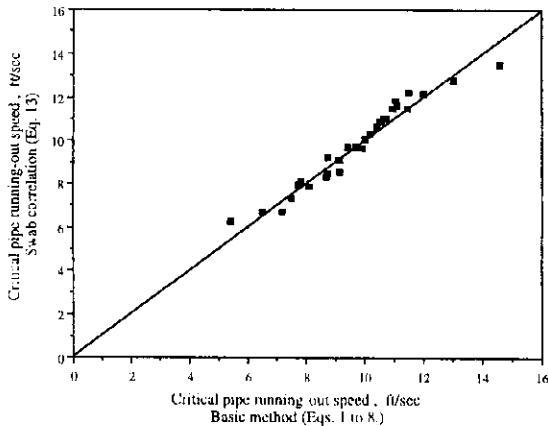


Fig. 8. Crossplot of the critical pipe running-out speed using the basic fluid flow equations (Eqs. 1 to 8.) and the developed swab correlation (Eq. 13).

### Conclusions

Based on the performed analysis, the following conclusions can be drawn:

1. Mud weight, and rheology are important factors which must be chosen carefully and maintained at the desired level during drilling and tripping operations in order to avoid blowout or loss of circulation problems.
2. Safe tripping-in and -out speeds are greatly influenced by hole size.
3. Drillcollars length and size can affect the safety during tripping operations due to the initiation of piston-cylinder action which will drain the formation during tripping-out causing a kick and will fracture the formation during tripping-in leading to a loss of circulation.
4. The developed computer program (based on the basic fluid flow equations) can be used to predict safe pipe running speeds.
5. As an alternative way to predict the critical pipe running speeds, two correlations have been developed. Firstly for the critical pipe running-in speed

and secondly for the critical pipe running-out speed. These correlations can be evaluated for infinite combinations of possibilities existed in oil well drilling operations.

6. The developed correlations require simple input data which can be measured in laboratory using the API testing equipment.

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## التنبؤ بالسرعة الحرجة لأنابيب الحفر خلال عمليات الإنزال والسحب أثناء حفر آبار النفط والغاز

مساعدة ناصر جاسم العواد

قسم هندسة النفط، كلية الهندسة، جامعة الملك سعود، ص.ب. ٨٠٠،  
الرياض ١١٤٢، المملكة العربية السعودية

(استلم في ١٩٩٦/٧/٧ م، قبل النشر في ١٩٩٨/١/٧ م)

**ملخص البحث .** إن العوامل المؤثرة على ضغوط الإنزال والسحب خلال عمليات حفر آبار الزيت والغاز يجب أن تتنازل بدقة وذلك للسيطرة على اندفاع السوائل إلى البئر أو فقدان دورة سائل الحفر وهما مشكلتين خطيرتين من الممكن حدوثهما خلال عمليات إنزال وسحب أنابيب الحفر خلال عمليات حفر آبار الزيت والغاز. حيثما لا تتم السيطرة على السرعة وينعدم التحكم في خواص سائل الحفر المستخدم. إن العوامل المؤثرة على ضغوط الإنزال والسحب خلال عمليات حفر آبار الزيت والغاز يجب أن تتنازل بدقة وذلك للسيطرة على أي اندفاع مفاجئ للسوائل من الطبقات المحفورة إلى البئر أو العكس. وعلى ذلك فإن التنبؤ بالسرعة الحرجة لأنابيب الحفر خلال عمليات إنزال وسحب أنابيب الحفر من وإلى البئر يعتبر أمراً في غاية الأهمية. إن عملية التنبؤ بالسرعة الحرجة لأنابيب الحفر تعتمد على تقويم قوانين سريان السوائل في الفراغ الموجود بين أنابيب الحفر وجدار البئر. وتم كتابة برنامج كمبيوتر لإجراء تلك الحسابات. يحتاج هذا البرنامج إلى إدخال بيانات من الممكن قياسها في المعمل باستخدام الأجهزة القياسية الخاصة بسوائل الحفر بالإضافة إلى أبعاد كل من البئر وأنابيب الحفر. ومن ثم تحويل نتائج ذلك البرنامج إلى رسومات بيانية تمكننا من التنبؤ بالسرعة الحرجة لأنابيب الحفر. وكطريقة بديلة للتنبؤ بالسرعة الحرجة لأنابيب الحفر تم تطوير علاقات رياضية مسطحة تؤدي نفس الغرض الذي يؤديه البرنامج المذكور أعلاه وسرعة وسهولة أكبر. إن العلاقات الرياضية المطورة تأخذ بعين الاعتبار جميع العوامل المؤثرة على السرعة الحرجة لأنابيب الحفر خلال عمليات الإنزال والسحب. ولقد تم عمل مقارنة للتنبؤ بالسرعة الحرجة لأنابيب الحفر باستخدام كلتا الطريقتين ووجد أن هناك توافق كبير في النتائج. ووجد أن السرعة الحرجة لأنابيب الحفر تتأثر بشكل كبير بالوزن النوعي لسائل الحفر، قطر البئر ونسبة طول أنبوب الحفر الثقيل إلى طول أنبوب الحفر الاعتيادي