J. King Saud Univ., Vol. 6, Computer & Information Sci., pp. 47-69 (A.H 1414/1994)

CAD of Weak Inherent Coupled Systems

I. Ziedan*, M. Moness, and M. Mohamed*****

* *Department of Electronic and Communication Engineering, Faculty of Engineering, Zagzig University, Egypt.*

** *On leave at the Department of Electronic Technology, Technical College of Riyadh, Riyadh, Saudi Arabia.*

*** *Department of Electrical Engineering, Faculty of Engineering, El-Minia University, Egypt*

(Received September 1992; accepted for publication 2 March 1993)

Abstract. The paper presents a general and simple CAD of weak inherent coupled systems. The developed algorithm uses the state-space description to avoid polynomial operations, and in turn the numerical stability is improved. On the other hand, it is more simple than the other state-space algorithms. Moreover, the designed algorithm makes use of the "column-essential" concept to guarantee the design of a minimum order precompensator. The constructed software uses "True BASIC" language.

1. Introduction

It is well known that any linear, time-invariant, mutivariable system can be decoupled if it is a right invertible one [1]. Such systems can be decoupled either by using linear state feedback (l.s.f.), or a combination of input dynamics and l.s.f. The later systems are called weak inherent coupled systems. The topic of decoupling weak inherent coupled systems has been studied by several authors and by using different approaches. Silverman [2], used the state-space representation and introduced the "invertible-algorithm" for designing precompensators. Panda [3], used also the time-domain approach to design such precompensators. Cremeer [4], illustrated by a numerical example that Panda's algorithm may lead to a precompensator of order higher than the minimum one. Cremer [5], developed an efficient techniqe for designing a minimum order precompensator. Sinha [6], tried to simplify the lengthy procedures technique of Cremer. He proposed that the minimum order of the precompensator must be equal to the system inputs and outputs. However, Moness and Amin [7], proved that Sinha's theorem is neither sufficient nor necessary. They set the infinite zero structure to develop simpler algorithms than Cremer's one. Moness employed the Wiener-Hopf factorization [8], and the interactor concept [9], to obtain alternative conditions and the corresponding algorithms.

The use of frequency domain approach for the analysis of weak inherent coupled systems is of special importance for practical problems. Wang [10] was the first who used the transfer function matrix description T(s) for designing a precompensator $T_c(s)$ such that the compensated system $T(s)T_c(s)$ satisfies the condition of decoupling by l.s.f. alone. Wolovich [1] constructed a more general algorithm than that one given by Wang. He did not only specify the precompensator, but also designed the associated feedback matrices. The main disadvantages of Wolovich's decoupling algorithm is the need of matrix fraction description for the algorithm. However, Amin-Moness algorithm [11] avoided this problem by using the transfer function matrix directly.

The previous contributions can be classified into two main techniques. The first one consists of two steps: First a precompensator $T_c(s)$ is designed such that the cascaded system $T(s)T_c(s)$ satisfies the condition of decoupling through l.s.f. alone. Among the contributions using this technique are the work of Panda [3], and Cremer [5]. The second technique attacks the problem directly. A precompensator and a suitable l.s.f are designed at the same time to achieve the diagonal form. The contributions of Moness and Amin [7], and Moness [8, 9] belong to this technique.

In this paper a developed CAD for decoupling weak inherent coupled systems will be introduced in section 3. It belongs to the first technique. The developed algorithm collects the individual advantages of the previous methods. Moreover, it is more general than those ones, which were designed by Gilbert and Pivincy [12], and Melsa and Johnes [13]. Section 2 covers the preliminary material, while section 4 illustrates the application of the introduced CAD to a practical example.

2. Preliminary

Consider an p-input, m-output and n th order system S(A, B, C, E). The time-domain approaches make use of the following state-space description

$$X(t) = AX(t) + BU(t) \tag{1.a}$$

$$Y(t) = CX(t) + EU(t) \tag{1.b}$$

where X(t) is an n-state vector, U(t) is an p-input vector, Y(t) is an m-output vector and A, B, C, and E are nxn, nxp, mxn and mxp constant matrices respectively. Alternatively, the frequency-domain approaches make use of the input-output form of equation (1), i.e., the mxp transfer function matrix T(s) which is given by:

$$T(s) = C(sI - A)^{-1}B * E \tag{2}$$

Definition 1 (Falb and Wolovich [14])

The mxp matrix B* is defined as follows:

$$B^* = \left[b_i^* \right],\ i = 1, 2, \ldots\ldots, m; \tag{3.a}$$

$$b_i^* = c_i \qquad \text{if } c_i \neq 0 \tag{3.b}$$

$$= c_i\ A^{d_i} B \qquad \text{if } c_i = 0 \tag{3.c}$$

where b_i^*, c_i and e_i are the i^{th} row of B^*, C and E respectively. The scalar constants d_i are called the decoupling indices and are defined as follows:

$$d_i = \min \left\{ j \,|\, c_i\ A^{j-1} B \neq 0 \right\},\ j = 0, 1, \ldots, n\text{-}1 \ \text{ if } e_i = 0 \tag{4.a}$$

$$= 0 \ \text{ if } \ c_i \neq 0 \tag{4.b}$$

Theorem 1 (Falb and Wolovich [14])

A linear, time-invariant, invertible (p = m) system S can be decoupled by 1.s.f. alone if and only if B* is nonsingular.

Definition 2 (Gilbert [15])

The matrix B* and the decoupling indices d_i are related to the i^{th} row $t_i(s)$ of the transfer function matrix T(s) by the following relations:

$$b_i^* = \lim_{s \to \infty} s^{d_i + 1}\, t_i(s), \tag{5.a}$$

$$d_i = \min_{i \in m} \left\{ j \,|\, \lim_{s \to \infty} s^{j+1}\, t_i(s) \neq 0 \right\} \tag{5.b}$$

Theorem 2 (Gilbert [15])

Singular systems cannot be decoupled at all. However, nonsingular (invertible) systems can be decoupled by using precompensators in addition to 1.s.f.

Definition 3 (Gilbert [15])

A given system S (A,B,C,E) is said to be weakly inherent coupled if:

$$\text{a)} \quad |\ T(s)\,| \neq 0 \tag{6.a}$$

$$\text{b)} \quad |\ B^*\,| \neq 0 \tag{6.a}$$

where | (.) | denotes the determinant of the given matrix.

Definition 4 (Vardoulakis [16])

Let t(s) = n(s)/d(s) such that d(s) ≠ 0, where t(s) is a rational function and n(s), d(s) are polynomials, then

$$\delta_\infty \ (t(s)) = \deg d(s) - \deg n(s) \ ; \ t(s) \neq 0$$

$$= + \infty \ ; \ t(s) = 0 \tag{7}$$

is called the valuation at infinity of t(s). The valuation at infinity of an $1 \times m$ rational vector $t_j(s)$;

$$t_j(s) = [\ t_{j,1}(s) \quad t_{j,2}(s) \quad \ldots\ldots \ t_{j,m}(s)\ ] \tag{8}$$

is defined as:

$$\delta_\infty\left(\ t_j(s)\ \right) = \min_{i \in m} \left\{\ \delta_\infty\left(\ t_{j,i}(s)\ \right)\ \right\} \tag{9}$$

The valuation at infinity of an pxm rational function matrix T(s), whose rank equals to r is:

$$\delta_\infty \ (T(s)) = \min \text{ (valuation of all the rth minors of } T(s)), \text{ if } r > 0$$

$$= \ +\infty \qquad \text{if} \quad r = 0 \tag{10}$$

For simplicity we will write the valuation at infinity as ≠ instead of $\neq_\infty$.

3. Development

Consider a linear, time-invariant and invertible system S(A,B,C,E). In this section we introduce a new time-domain algorithm for designing precompensators S_c to achieve decoupling by l.s.f. alone i.e., $B^*(SS_c)$ is nonsingular. The steps of the presented algorithm are based upon the following facts:

Theorem 3

Any weak inherent coupled system can be transformed into decouplable one by introducing cascaded precompensators $T_{c1}, T_{c2}, \ldots, T_{ck}$ in the proper terminals until:

$$\delta\left(\ S S_{c1}\ S_{c2} \ldots S_{ck}\right) = \left(\sum_{1}^{m} d_i^{(k)} + 1\right) \tag{11}$$

where δ (.) and d_i is the valuation and decoupling indices of the composite system.

Proof.

See [16], [10] and [11].

Theorem 4

Let W be a pxm scalar matrix. If $\bar{W}$ is the echelon form of W, then columns of W which have only one nonzero element are column essential.

Proof

By elementary column (or equivalently right) operations on W, the echelon form $\bar{W}$ can be obtained. The pxm matrix $\bar{W}$ has the following properties:

a) The first nonzero element in each column is 1.
b) If any row contains the first nonzero element of any column, then the element is the only nonzero element of that row.
c) The zero columns, if any, come last.
d) The leading 1's in nonzero columns are along a broken line, sloping downwards. Since any column $\bar{W}_i$ is called "column-essential", (Cremer [5]), if $\bar{W}_i$ cannot be represented by linear combinations of other columns. Then it is evident from the previous properties that any column which has only one nonzero element is column-essential.

The following algorithm describes the design procedures of the required precompensator.

Design objectives

The objectives of the algorithm are:

(i) To design a precompensator $T_c(s)$ such that the composite system $T(s) = T(s)T_c(s)$ is decouplable through 1.s.f. alone,
(ii) to assign δ poles arbitrarily, and
(iii) to compute the required 1.s.f. pair F,G such that the compensated system T(s,F,G) is diagonal and nonsingular.

Algorithm inputs

The plant system matrices A, B, C, and E, the number of inputs p, the number of outputs m and the system order n.

Algorithm outputs

The feedback matrix F, the input manipulaltion matrix G, the precompensator order n_c and its representation A_c, B_c, C_c, and E_c.

Algorithm steps

Step 1: Consider the original systems S(A,B,C,E), we denote the parameters of such systems by the subscript "0". So, B_o^* is given by equation (3). In the following steps we construct cascaded precompensators T_{c1}, T_{c2} .... until T_{ck} (termination as indicated by Theorem 3). Hence the composite precompensator is $T_c(S) = T_{c1}\, T_{c2} \ldots T_{ck}(S)$. Compute δ_o (Definition 4) and $d_j^{(o)}$, $j = 1, 2, \ldots, m$ (Definition 1).

Step 2: For i = 0, 1, 2, ..., k do: If B_i^* is nonsingular (rank $[B_i^*] = l_i < m$ and $\infty_i >$ + 1), then go to step 3, otherwise the compensated system can be decoupled by l.s.f. alone and go to Step 8.

Step 3: Perform elementry column operations on B_i^*, i.e., multiplying it from the right by a constant matrix $U_{K,i}$ to transform B_i^* to $\bar{B}_i^*$ which has the right echelon form:

$$\bar{B}_i^* = B_i^* U_{R\,i} = \begin{bmatrix} 1 & 0 & 0 & . & . & . & 0 & . & 0 & 0 & 0 & 0 \\ \chi & 0 & 0 & . & . & . & 0 & . & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & . & . & . & 0 & . & 0 & 0 & 0 & 0 \\ \chi & \chi & 0 & . & . & . & 0 & . & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & 0 & . & 0 & 0 & 0 & 0 \\ \chi & \chi & \chi & \chi & \chi & \chi & 0 & . & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & . & . & . & 1 & . & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underbrace{\qquad\qquad\qquad}_{l_i \text{ columns}} \qquad \underbrace{\qquad\qquad}_{m-l_i \text{ columns}}$$

Step 4: Some of nonzero columns of $\bar{B}_i^*$ have only one nonzero element, and the others have more than one nonzero elements. Denote the later set by $\bar{l}_i$. Mathematically the set $\{\bar{l}_i\}$ is the collection of the non "column-essential" (Theorem 4). Physically, this means that each corresponding inputs to $\bar{l}_i$ affect more than one output, i.e.; the input terminal corresponding to $\bar{l}_i$ have interaction. To decouple this interaction we must introduce input dynamics in such terminals. For purpose of illustration only we add integrators (poles at the origin). However dynamics having poles assigned arbitrarily (not necessary at the origin) can be also introduced (see Step 5). Mathematically this means right multiplication by $D_i^{-1}(S)$, where,

$$D_i^{-1}(s) = \mathrm{diag}\,[s^{i}]\,, \tag{12}$$

$$j = 0 \text{ if } j \in \{ l_i - \bar{l}_i \}$$

$$= 1 \text{ if } j \in \{ \bar{l}_i \} \tag{13}$$

so,

$$T_{ci}(s) = U_{Ri}\, D_i^{-1}(s) \tag{14}$$

This step increases decoupling indices of every row of the set $\{\bar{l}_i\}$, by one at least. Hence the repetition of this step will ensure the condition of decoupling (Theorem. 3).

Step 5: Realize the i^{th} precompensator T_{ci} to its parameter S_{ci} (A_{ci}, B_{ci}, C_{ci}, E_{ci}) as follows:

$A_{ci} = \bar{l}_i \times \bar{l}_i$ diagonal matrix with arbitrarily assigned eigenvalues i.e.;

$$A_{ci} = \begin{bmatrix} \lambda_{c,i}^{1} & 0 & . & . & . & 0 \\ \chi & \lambda_{c,i}^{2} & . & . & . & 0 \\ . & . & . & . & . & 0 \\ 0 & 0 & . & . & . & \lambda_{c,i}^{(\bar{l}_i)} \end{bmatrix} \quad (15.a)$$

$C_{ci} = m \times \bar{l}_{ci}$ with its columns equal to columns of $U_{R,o}$
$B_{ci} = \bar{l}_i \times m$, has the controllable companion form:

$$B_{c,i} = \underbrace{\begin{bmatrix} 1 & 0 & . & 0 \\ \chi & 1 & . & . \\ . & . & . & 0 \\ 0 & 0 & . & 1 \end{bmatrix}}_{\bar{l}_i} \quad \underbrace{\begin{bmatrix} 0 & . & 0 \\ . & 0 & 0 \\ . & . & . \\ 0 & 0 & 0 \end{bmatrix}}_{m - \bar{l}_i} \quad (15.b)$$

and $E_{c,i} = m \times m$, with last columns equal to $\{ l_i - \bar{l}_i \}$ columns of $U_{R,i}$, i.e.

$$E_{ci} = \Big[\underbrace{O}_{\bar{l}_i} \ \Big| \ \underbrace{U_{R,i}^{(j)}}_{m - \bar{l}_i} \Big] , \quad j \in \{ l_i - \bar{l}_i \} \quad (15.c)$$

Step 6: The composite system S_{i+1} has the following description:

$$A_{i+1} = \begin{bmatrix} A_i & B_i C_{ci} \\ O & A_{ci} \end{bmatrix}, \quad B_{i+1} = \begin{bmatrix} B_i E_{ci} \\ B_{ci} \end{bmatrix}, \quad (16)$$

$$C_{i+1} = \begin{bmatrix} C_i & E_i E_{ci} \end{bmatrix}, \quad E_{i+1} = \begin{bmatrix} E_i E_{ci} \end{bmatrix},$$

where $S_o = \{ A, B, C, E \}$ and $S_i = \{ A_i, B_i, C_i, E_i \}$.

Step 7: Compute the matrix B^*_{i+1} and the new value of the valuation δ_{i+1} and the new decoupling indices d_j^{i+1}. Go to Step 2.

Step 8: Realize the precompensator and compute the feedback pair F,G from the relations:

$$G = \Lambda \left(B_k^* \right)^{-1}, \; F = \left(B_K^* \right)^{-1} \left[\sum_{j=0}^{\delta} M_k \quad C A_k^j - A^* \right] \tag{17.a}$$

where Λ is a diagonal matrix, $\delta = \max \{ d_i \}$ and M_k are chosen to assign the available poles arbitrarily, and has the form:

$$M_k = \text{diag} \left[m_k^1 \;\; m_k^2 \;\; \ldots . \; m_k^m \right] \tag{17.b}$$

where m_k^i are the coefficients of the characteristic equations;

$$Y_i^{d_i^{(k)}+1} = m_0^i \;\; Y_i + m_1^i \;\; Y_i^{(1)} + \ldots . + m_{d_i}^i \;\; Y_i^{d_i^{(k)}}, \;\; i = 1, 2, \ldots, m \tag{17.c}$$

$Y_i^{(1)}$ denotes the j^th^ derivative of the i^th^ output, and

$$A^* = \begin{bmatrix} c_{1,k} & A_k^{\left(d_1^{(k)}+1\right)} \\ c_{2,k} & A_k^{\left(d_2^{(k)}+1\right)} \\ & \cdot \\ & \cdot \\ c_{m,k} & A_k^{\left(d_m^{(k)}+1\right)} \end{bmatrix} \tag{17.d}$$

With B* nonsingular then the composite system can be decoupled by 1.s.f. alone. Figure 1.a shows the composite system block diagram, while Fig. 1.b illustrates the flowchart for the given algorithm.

The structure of the precompensator

The structure of the precompensator can be detected from equation (16). For k = 1, the realization of the precompensator is:

$$A_c = A_{c1} \;\;, \;\; B_c = B_{c1}, \;\; C_c = C_{c1} \;\; \& \;\; E_c = E_{c1} \tag{18}$$

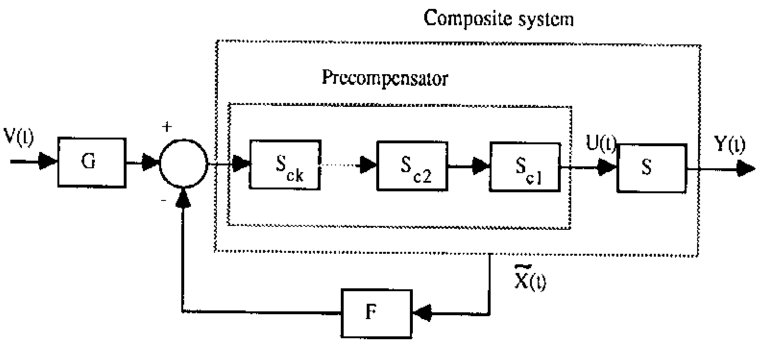

Fig. 1.a. The developed method block diagram

The matrices on the right hand side of equation (18) are described by equation (15). For k = 2, the precompensator consists of two stages and. In light of equation (16), the structure of the precompensator is:

$$A_c = \begin{bmatrix} A_{c1} & B_{c1}C_{c2} \\ 0 & A_{c2} \end{bmatrix},\ B_c = \begin{bmatrix} B_{c1}E_{c2} \\ B_{c2} \end{bmatrix} \tag{19.a}$$

$$C_c = \begin{bmatrix} C_{c1} & E_{c1}C_{c2} \end{bmatrix},\ E_c = \begin{bmatrix} E_{c1}E_{c2} \end{bmatrix} \tag{19.b}$$

For k = 3, we have the following structure:

$$A_c = \begin{bmatrix} A_{c1} & B_{c1}C_{c2} & B_{c1}E_{c2}C_{c3} \\ 0 & A_{c2} & B_{c2}C_{c3} \\ 0 & 0 & A_{c3} \end{bmatrix},\ C = \begin{bmatrix} B_{c1}E_{c2}E_{c3} \\ B_{c2}E_{c2} \\ B_{c3} \end{bmatrix} \tag{20.a}$$

$$C_c = \begin{bmatrix} C_{c1} & E_{c1}C_{c2} & E_{c1}E_{c2}E_{c3} \end{bmatrix},\ E_c = \begin{bmatrix} E_{c1}E_{c2}E_{c3} \end{bmatrix} \tag{20.b}$$

The previous procedures can be used to have the structure of a precompensator consisting of any number of stages.

It is obvious that the developed method belongs to the first technique for noninteraction (decoupling) of weak inherent coupled systems. It has the following advan-

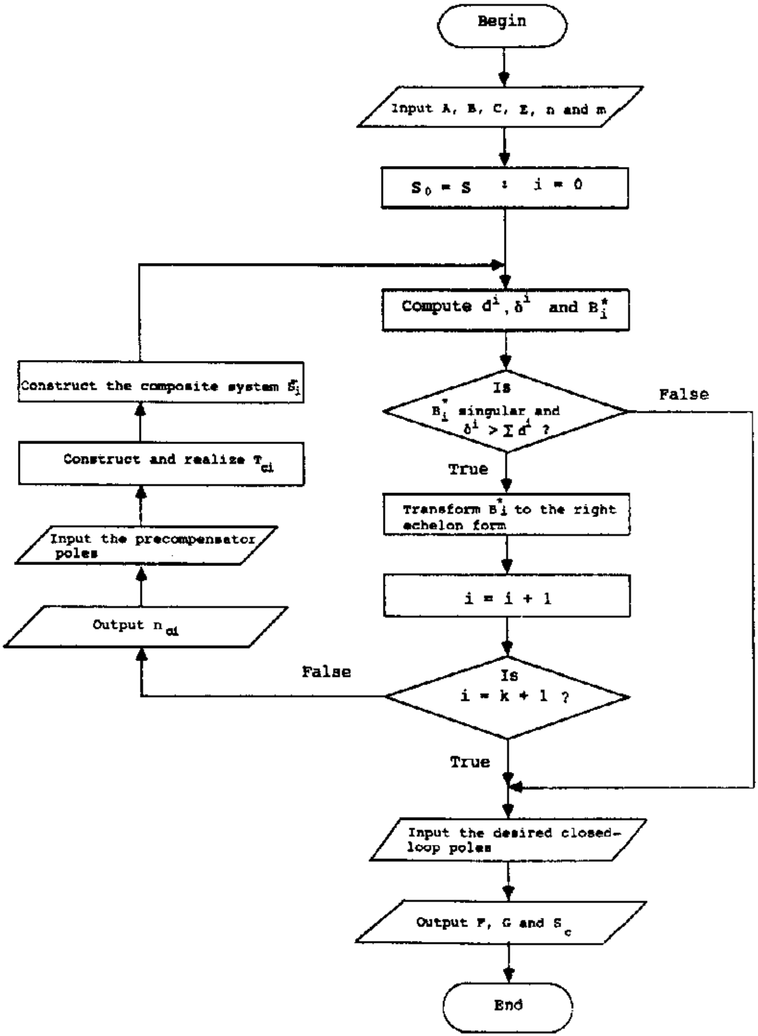

Fig. 1.b. The flowchart for the algorithm

tages over the well-known methods belonging to that technique:

i) It uses the state-space parameters and hence it is more numerically stable than Wang's algorithm which uses elementary polynomial operations.

ii) The use of row essential concept gurantees the minimality of the designed precompensator. It is well known that this minimality cannot be guaranteed by using Panda's algorithm (Cremer [6]).

iii) The steps of computations are simpler than Cremer's ones. Moreover, the developed algorithm needs less memory size than Cremer's one (Mohamed [17]).

Briefly speaking, the developed method is numerically stable, efficient and yields a minimal order precompensator. Numerical stability is evident from the facts that scalar operations only are used and avoiding the use of polynomial operations which may lead to numerical instability. As we illustrated before there is no other method which has all of these advantages at the same time.

A complete software is constructed by using "True BASIC", an entirely new and structured version of BASIC, developed by the originators of BASIC programming languages (Kemeny and Kurtz [18, 19]). True BASIC retains all the simplicity of the earliest version of BASIC, and has superb machine-independent graphics, making it arguably the best high level language for beginners. On the other hand, it has features which will be quite unrecognizable to old-time BASIC addicts: modern control structures (DO-WHILE, DO-UNTILE, Block IF, SELECT CASE), named subroutine with arguments and local variables, matrices operations, modules, pictures (graphics subroutines) and optional line numbers. These features enable it to take its place as a serious scientific problem-solving language of the first degree (Hahn, [20]).

A numerical example

Consider the following example (Moness-Amin, [7])

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 \end{bmatrix} \quad \& \quad E = 0$$

using the developed algorithm we obtain the following results:

Step 1:

$$B_0^* = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

The original decoupling indices are $d^{(0)} = \{ 0,0,0 \}$, and $\delta_0 = 5$

<u>i = 0 ;</u>

Step 2:

$$\rho\left(B_0\right) = 2 < 3\ (m)\ , \quad \left(\delta_0 > \Sigma\left(d_i^{(0)} + 1\right) = 3\right).$$

Step 3:

$$U_{R0} = \begin{bmatrix} 0.5 & 0 & 1 \\ -0.5 & 1 & -1 \\ 0.5 & 0 & -1 \end{bmatrix}$$

$$\overline{B}_0^* = B_0^* \, U_{R0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

It is clear that $l_o = \{ 1,2 \}$.

Step 4:

From step 3: $\bar{l}_o = \{ 1,2 \}$ and $\{ l_o - \bar{l}_o \} = \emptyset$ (the empty set),

$$D_0\ (s) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{c0}\ (s) = U_{R0}\ D_0^{-1}\ (s) = \begin{bmatrix} 1/2s & 0 & 1 \\ -1/2s & 1/s & -1 \\ 1/2s & 0 & -1 \end{bmatrix}$$

Step 5:

$$A_{c0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{c0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$C_{c0} = \begin{bmatrix} .5 & 0 \\ -.5 & 1 \\ .5 & 0 \end{bmatrix}, \quad E_{c0} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

Step 6:

$$A_0 = \left[\begin{array}{ccccc|cc} 0 & 0 & 0 & 0 & 0 & .5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.5 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .5 & 0 \\ - & - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \quad B_0 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ - & - & - \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$C_0 = \left[\begin{array}{ccccc|cc} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 & 0 \end{array}\right]$$

$$C_{01}\, B_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \quad C_{01}\, A_0\, B_0 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$
$$C_{02}\, B_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \quad C_{02}\, A_0\, B_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix},$$
$$C_{03}\, B_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \quad C_{03}\, A_0\, B_0 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

Step 7:

$$B_1^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \quad \delta_1 = 7, \quad d^{(1)} = 1, 1, 1 \ .$$

<u>i = 1 :</u>

Step 2:

$$\rho\left(B_1^*\right) = 2 < m \text{ and } \delta_1 = \sum_1^m \left(d_i^{(1)} + 1\right) = 6 \, .$$

Step 3:

$$U_{R1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\overline{B}_1^* = B_1^* U_{R1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad \& \quad I_1 = \{1, 2\}$$

Step 4:

From step 3: $\bar{I}_o = \{ 1,2 \}$ and $\{ I_o - \bar{I}_o \} = \emptyset$

$$D_1(s) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$T_{c1}(s) = U_{R1} D_1(s)^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1/s & 0 & 0 \\ 0 & 1/s & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 5:

$$A_{c1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; B_{c1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \; C_{c1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \; E_{c1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 6:

$$A_1 = \left[\begin{array}{ccccc|cccc} 0 & 0 & 0 & 0 & 0 & .5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .5 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right],$$

$$B_1^T = \left[\begin{array}{cccccc|cccc} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right],$$

$$C_1 = \left[\begin{array}{ccccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$C_{11}\,B_1 = C_{11}\,A_1\,B_1 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix},\ C_{11}\,A_1^2\,B_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$
$$C_{12}\,B_1 = C_{12}\,A_1\,B_1 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix},\ C_{12}\,A_1^2\,B_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix},$$
$$C_{13}\,B_1 = C_{13}\,A_1\,B_1 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix},\ C_{13}\,A_1^2\,B_1 = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$$

Step 7:

$$B_2^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix},\ \delta_2 = 9,\ \ d^{(2)} = 2, 2, 2\ .$$

<u>i = 2 :</u>

Step 2:

$$p\left(B_2^*\right) = 3 = m \text{ and } \delta_2 = \sum_{1}^{m} d_1^{(2)} + 1 = 9\ .$$

Step 8:

The precompensator matrices are:

$$A_c = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ \ B_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$C_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ \ E_c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

The 1.s.f. control law for assigning the closed loop poles at the origin is:

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ \ G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

5. A Practical Example

The proposed scheme will be applied to a fifth order model of a drum boiler (Bengtsson-Lindahl [21]). The design objective is to have non interaction (decoupling between drum pressure and drum liquid level. The linearized equation for the boiler around a certain operating point can be written as:

$$X(t) = AX(t) + U(t),\ Y(t) = CX(t)$$

where the state variables are:
x_1 = drum pressure (bar)
x_2 = drumliquid level (m)
x_3 = drum liquid temperature
x_4 = riser wall temperature
x_5 = steam quality (per cent).

The input variables are:
u_1 = heat flow to the risers (KJ/sec).
u_2 = feedwater flow (Kg/sec).

The output variables are:
$y_1 = x_1$ and $y_2 = x_2$.

Numerical values for A, B, and C for a power station boiler with a maximum steam flow of about 350 t/h are given by Bengtsson-Lindahl [21]. The drum pressure is 140 bar and the operating point is 90 per cent full load. From Bengtsson-Lindahl [21] we have:

$$A = \begin{bmatrix} 0.129 & 0.0 & 0.396 & 0.025 & 0.0191 \\ 0.329\text{x}10^{-2} & 0.0 & -0.779\text{x}10^{-4} & 0.122\text{x}10^{-3} & -0.621 \\ 0.0718 & 0.0 & -0.1 & 0.887\text{x}10^{-3} & -3.851 \\ 0.0411 & 0.0 & 0.0 & -0.0822 & 0.0 \\ 0.361\text{x}10^{-3} & 0.0 & 0.0350\text{x}10^{-4} & 0.426\text{x}10^{-4} & -0.0743 \end{bmatrix}$$

$$B = \begin{bmatrix} .0 & .139\text{x}10^{-2} \\ .0 & .359\text{x}10^{-4} \\ .0 & -.989\text{x}10^{-2} \\ .249\text{x}10^{-4} & .0 \\ .0 & -.543\text{x}10^{-5} \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The constructed software for the developed method yields the following results: The matrix B* is singular. Hence the system cannot be decoupled via l.s.f. alone. A complete decoupling can be achieved by l.s.f. in combination with a precompensator. The minimum order of this precompensator equals to "1", and its representation is:

$$A_c = 0\,, \quad B_c = 1 \quad 0\,, \quad C_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E_c = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

So, the order of the compensated system is "6". Among the six poles of the compensated system, there are four poles which can be assigned arbitrarily. If the closed loop poles are located at: $(-0.1 \pm j0.1)$ & $(-0.12 \pm j0.11)$, then the feedback matrices must be equal to:

$$F = \begin{bmatrix} -2.35613 & -910.127 & -3.54585 & -2.61598 & 3496.00 & -0.644359 \\ -18383.4 & 2.0325\text{x}10^{6} & 9761.4 & 6233.37 & -7.5612\text{x}10^{6} & 1909.59 \end{bmatrix}$$

$$G = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} \begin{bmatrix} -1.67601 & 34344.4 \\ 1.98067\text{x}10^{6} & -7.66888\text{x}10^{7} \end{bmatrix}$$

where γ_1 and γ_2 are any constants. For the previously selected closed loop poles, the compensated system has the following performances:

damping ratio, $\xi = 0.7$

undamped natural frequency, $\Omega_n = 0.143$,

maximum per cent overshoot = 4.3% and

the settiling time (if the 5% criterion is used), $t_3 = 30$ sec.

Figures 2-7 depict the system response for unit step inputs. It is obvious from Fig. 2 that the variation of the first input υ_1 (t) does not affect the second output (drum liquid level). Figure 3 shows that the variation of the second input υ_2 (t) does not affect the first output (drum pressure). Figures 2 and 3 indicate the accuracy of the developed software, since complete and exact decoupling is obtained. Figures 4 and 5 indicate the other state variable response (x_3 (t), x_4 (t), x_5 (t)) and the precompensator state (x_6(t)) for unit step inputs υ_1 (t) and υ_2 (t) respectively. Figures 6 and 7 indicate the control signal response U(t) for unit step inputs υ_1 (t) and υ_2 (t) respectively. In light of these figures it is clear that satisfactory performances are obtained.

6. Conclusion

A CAD of weak inherent coupled system has been developed. The designed algorithm is based upon the "valuation at infinity" and "column-essential" concepts.

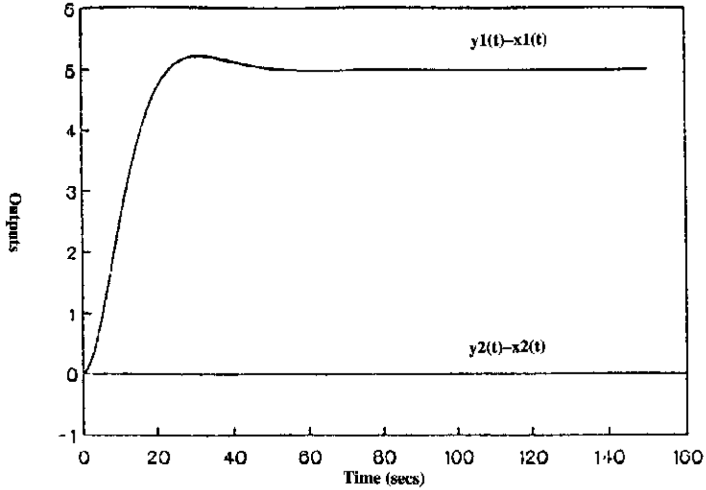

Fig. 2. Output responce υ(t) due to input υ(t)

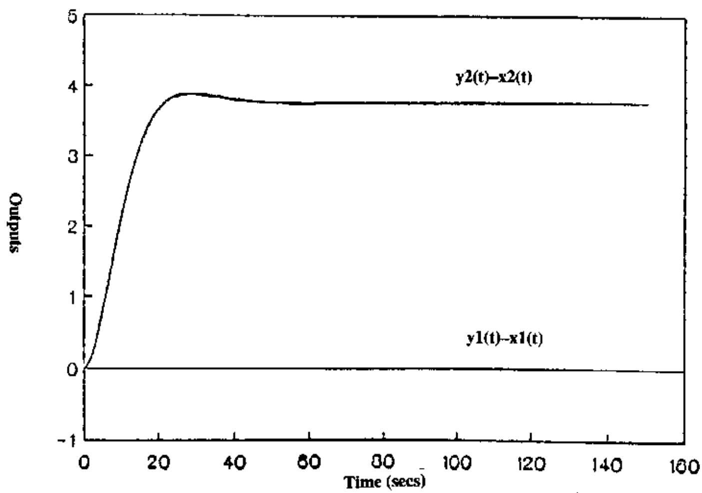

Fig. 3. Output responce υ(t) due to input υ_2(t)

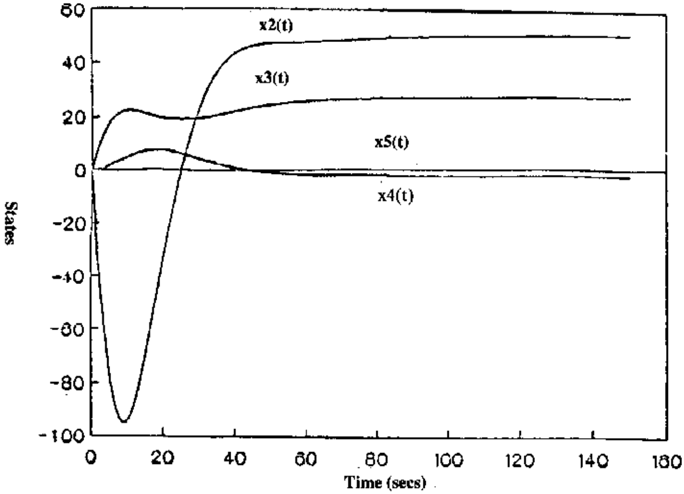

Fig. 4. States responce X(t) due to input $\upsilon_1(t)$

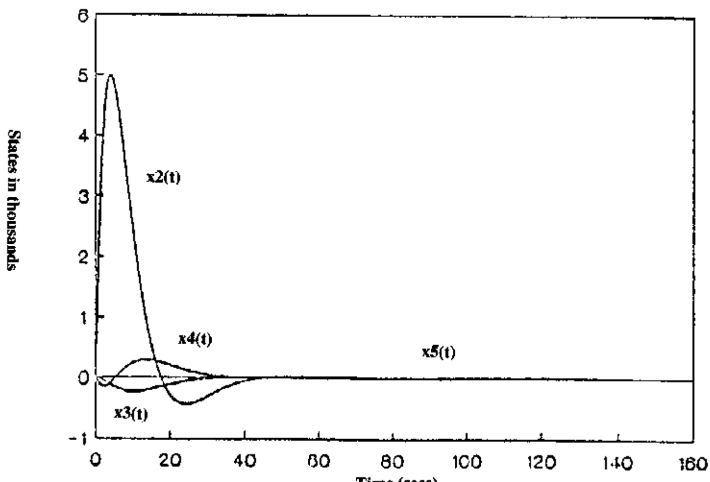

Fig. 5. States response X(t) due to input $\upsilon_2(t)$

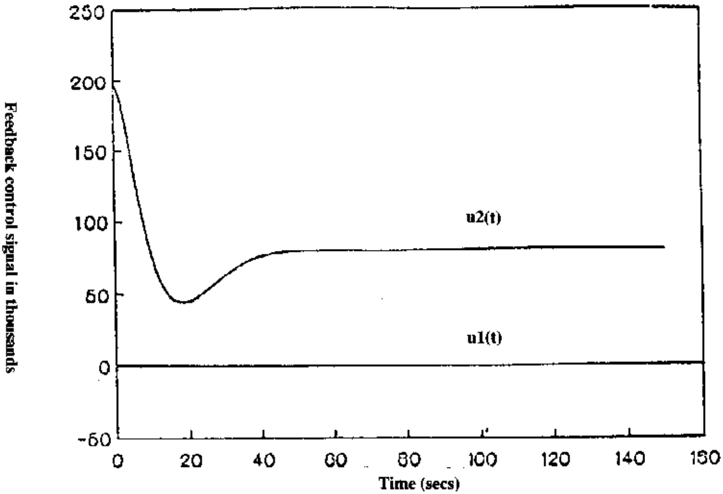

Fig. 6. Feedback control signal U(t) due to inpute $\upsilon_1(t)$

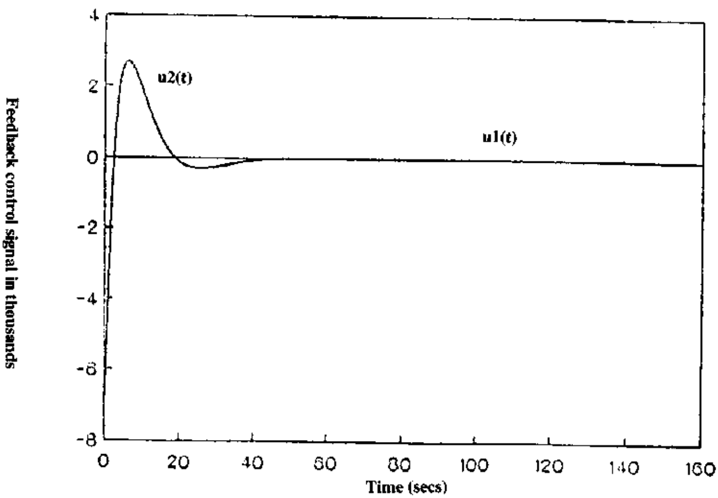

Fig. 7. Feedback control signal U(t) due to input $\upsilon_2(t)$

These concepts ensure the termination of the algorithm and the design of a minimum order precompensator. The algorithm has the following advantages:

i) It's procedures are simpler than Cremer's algorithm.

ii) It uses state-space description and hence it is more numerically stable than Wang's algorithm which uses elementary polynomial operations (polynomials addition, subtraction, multiplication and division).

iii) It yields a minimum order precompensator.

True BASIC has been used for the software design. Since BASIC is one of the most popular languages, then the designed software is suitable for any level of users.

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تصميم النظم ذات التداخل واهن التأصل بمساعدة الحاسوب

إبراهيم زيدان *، محمد مؤنس ** ومنتصر محمد ***

** قسم الإلكترونات والاتصالات الهندسية، كلية الهندسة، جامعة الزقازيق، جمهورية مصر العربية*

*** في إجازة التفرغ بقسم التقنية الإلكترونية، الكلية التقنية بالرياض، المملكة العربية السعودية*

**** قسم الهندسة الكهربائية، كلية الهندسة، جامعة المنيا، جمهورية مصر العربية*

ملخص البحث. يقدم البحث تصميمًا عامًا وبسيطًا لتصميم النظم ذات التداخل واهن التأصل بمساعدة الحاسوب. ويتم حساب الشبكة التعويضية ومصفوفات تغذية الحالة الخطية اللازمين لفك تداخل هذه النظم باستخدام معادلات فقط، ويتفادى الخوارزم المصمم بذلك استخدام العمليات المعتمدة على كثيرات الحدود والتي ينشأ عنها أحيانًا عدم الاتزان العددي في الطرق السابقة. ويتضمن الخوارزم أيضًا تصميم الشبكة التعويضية بأدنى رتبة ممكنة لاعتماده على مفهوم «العمود الأساسي». وقد استخدمت لغة «ترو- بيسك» في تصميم البرمجيات لإمكاناتها الجيدة من ناحية ولسهولة استخدامها من ناحية أخرى.