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Utilizing Structure Symmetry to Reduce the CPU Time for Computing [Z] Matrix Elements in the Method of Moments

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Abstract. The Method of Moments is often used to accurately compute the radiated or scattered fields from simple or complex structures. However, huge computer memory is required when utilizing it to solve electrically large structures. And, even if the available computers satisfy the memory requirements, the needed CPU time is often very long. In this paper a technique that can be used to reduce the required CPU time for computing the [Z] matrix elements is described. This technique utilizes the possible symmetry in the problem. An example involving the computation of the scattered fields from a large triangular dihedral is worked out in detail with CPU time reduction up to about 75%.

Introduction

The Method of Moments (MM) is often used to accurately compute the radiated or scattered fields from simple or complex structures. However, huge computer memory is required when utilizing it to solve electrically large structures. Even if the available computers satisfy the memory requirements, the needed CPU time is often very long. It is true that faster computers with large memory capacity, along with advanced processing algorithms can help in solving such problems in a direct manner. However, user demands to solve electrically larger problems often reach the limit of any available computing resources.

In some problems, such as scattering from plate structures, it was noted that the main portion of the CPU time is consumed in the process of computing the *[Z]* matrix elements. The rest of the CPU time is for inverting the matrix and computing the fields [1]. The matrix inversion share of the total CPU time increases with the increase of the number of matrix elements, and can become the main share for relatively large problems. That depends on the nature of the moment method code and the way it computes the matrix elements as well as on the computer vectorization capabilities.

The CPU time required for computing the *[Z]* matrix can be dramatically reduced by utilizing the possible symmetry in the problem. Such symmetry may not be very obvious, but it is not always difficult to observe. In this paper, a technique to reduce the required CPU time for computing the *[Z]* matrix elements is explained. This technique utilizes the possible symmetry in the problem. An example involving MM computation of the scattered fields from a large triangular dihedral using the ESP4 code [1] is worked out in detail with CPU time reduction of about 75%. Some notes are also given on the use of this code for solving plate-modeled structures. Note that this technique is only concerned with computing the *[Z]* matrix elements and hence it is valid for any direction of incidence.

Triangular dihedral

Figure 1 shows a triangular dihedral consisting of two triangular plates. The plates are such that $L_1=2m$, $L_2=1m$ and $\psi=90^{\circ}$. The back or bistatic scattering from such a structure is an interesting problem that involves different scattering mechanisms and has been investigated in several studies [2,3,4,5,6]. The need for accurately analyzing the scattered fields from dihedral structures arises in several situations; for example, when using the dihedral as a calibration target in scattering measurements or when a reference solution is needed for checking another method's result.

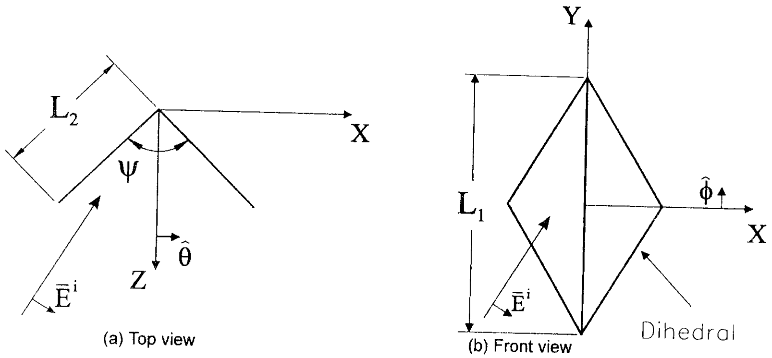
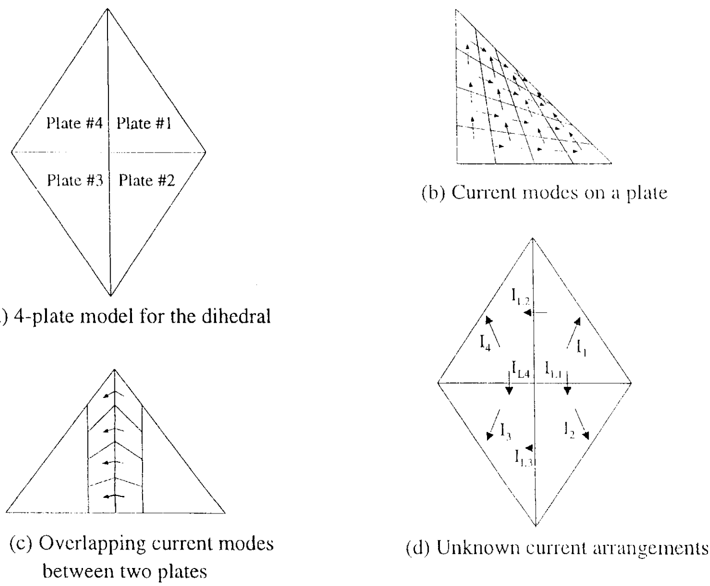


Fig. 1. Triangular dihedral.

For reasons to be clarified later, let us use a 4-plate model for the dihedral, such as shown in Figure 2(a). Also, let us use the ESP4 code, which is a MM code, to solve for the scattered fields from the dihedral. The code solves for the currents induced on the dihedral surface by an incident electromagnetic field, say a plane wave. The scattered fields are then computed using radiation integrals. This code uses the Electric Field Integral Equation to relate the unknown induced currents to the incident fields. The unknown currents are expanded in terms of N surface patch modes that cover each plate such as shown in Fig. 2(b). In Fig. 2(b), the plate is divided (segmented) into a set of quadrilateral surface patches. The current modes have a piecewise sinusoidal function that covers two patches. That is indicated in the figure by placing arrows (which represent modes) such that every arrow covers two patches, i.e., the patches covered by that mode. Overlapping modes that cover the junction between any two plates are also included in the expansion of the unknown currents such as shown in Fig. 2(c). That leaves us with the following unknown current vectors $[I_1]$,$[I_2]$,$[I_3]$,$[I_4]$,$[I_{L1}]$,$[I_{L2}]$,$[I_{L3}]$, and $[I_{L4}]$ that correspond to the currents on the plates in the way shown in Figure 2(d). Then, by enforcing the Electric Field Integral Equation for N linearly independent test modes, one obtains an NXN simultaneous linear equations, which can be written as

$$[Z]\, I = V$$

where $[Z]$ is the NXN impedance matrix, V is the voltage vector with length N, and I is the unknown current vector with length N. These linear equations can then be solved for the unknown currents.



(a) 4-plate model for the dihedral

(b) Current modes on a plate

(c) Overlapping current modes between two plates

(d) Unknown current arrangements

Fig. 2. 4-plate model and current arrangements for the triangular dihedral.

The impedance matrix is the main concern of this paper and can be written as shown in Fig. 3. The element Z_{mn} of *[Z]* is called the mutual impedance between the current modes *m* and *n*. Computing the elements of such a matrix involves many computations and can consume a long CPU time. This, however, can be reduced by utilizing the symmetry of the structure under consideration. For example, one can say that $Z_{mn} = \pm Z_{kl}$ if mode *m* is identical to mode *k* and mode n is identical to mode *l* and the relative position of mode *k* to mode *l* is the same as that of mode *m* to mode *n*. The ± sign is to accommodate for differences in the directions of the assumed reference currents, as will be seen towards the end of the next paragraph.

1/1 A	1/2 B	1/3 C	1/4 D	1/L_1 E	1/L_2 F	1/L_3 G	1/L_4 H
2/1 B	2/2 A	2/3 D	2/4 C	2/L_1 -E	2/L_2 G	2/L_3 F	2/L_4 -H
3/1 C	3/2 D	3/3 A	¾ B	3/L_1 H	3/L_2 -F	3/L_3 -G	3/L_4 E
4/1 D	4/2 C	4/3 B	4/4 A	4/L_1 -H	4/L_2 -G	4/L_3 -F	4/L_4 -E
L_1/1 I	L_1/2 -I	L_1/3 L	L_1/4 -L	L_1/L_1 M	L_1/L_2 N	L_1/L_3 -N	L_1/L_4 O
L_2/1 J	L_2/2 K	L_2/3 -J	L_2/4 -K	L_2/L_1 P	L_2/L_2 Q	L_2/L_3 R	L_2/L_4 -P
L_3/1 K	L_3/2 J	L_3/3 -K	L_3/4 -J	L_3/L_1 -P	L_3/L_2 R	L_3/L_3 Q	L_3/L_4 P
L_4/1 L	L_4/2 -L	L_4/3 I	L_4/4 -I	L_4/L_1 O	L_4/L_2 -N	L_4/L_3 N	L_4/L_4 M

Fig. 3. The impedence matrix |*Z*|.

The *[Z]* matrix of the triangular dihedral problem can be defined by blocks such as those shown in Fig. 3. Block (1/1) (the upper left) contains the mutual impedances between the current modes of the first plate. Block (1/2) contains the mutual impedances between the current modes of plate #1 and the current modes of plate #2. Likewise, block (L_1/L_1) contains the mutual impedances between the first overlapping current modes. Block (L_1/1) contains the mutual impedances between the first overlapping current modes and the first plate current modes. Every block is labeled with a capital letter (A,B,C,....). Only blocks labeled with an underlined bold letter need to be computed. The other blocks can then be filled according to the labels. For example, block (2/1) is labeled with "B"; that means that the elements of block (2/1) are identical to block (1/2) elements. Also, the elements of block (L_3/4) are equal to the negative of the elements of block (L_2/1). The relations of Figure 3 are evident from the symmetry of the problem. For example, blocks (1/1), (2/2), (3/3), and (4/4) are the same because plates 1,2,3, and 4 are identical and have identical segmentation. Also, block (2/3) is the same as block (1/4) because plate 2 current modes are identical to plate 1 current modes, plate 3 current modes are identical to plate 4 current modes, and the locations of plate 2 modes relative to plate 3 modes are the same as the locations of plate 1 modes relative to plate 4 modes. Likewise, block (L_4/4) elements equal the negative of (L_1/1) elements. That is because L_4 modes are identical to L_1 modes, plate 4 modes are identical to plate 1 modes and the locations of L_4 modes relative to plate 4 modes are the same as the locations of L_1 modes relative to plate 1 modes. The only difference between the two cases is the reference current direction. L_1 modes reference current is flowing out of plate 1 while L_4 modes reference current is flowing into plate 4, leading to the (-) sign in the above relation. Note that, in general, the *[Z]* matrix is not symmetric. It is symmetric if the test modes used in enforcing the integral equation are the same as the expansion modes [1]. So, the relation $Z_{mn}=Z_{nm}$ only holds when modes *m* and *n* are identical.

One can see that this technique can save up to about 75% of the CPU time required for computing the *[Z]* matrix elements. Figure 4 shows the CPU time required to compute the *[Z]* matrix elements versus frequency, with and without using symmetry. The figure should be viewed as if it shows only a relative time, since it greatly depends on the machine and its compiler and vectorization capabilities. But, for the sake of an example, on a CRAY YMP computer, the CPU time needed to compute a 264X264 matrix was 68 seconds. The use of the symmetry mentioned above reduced that time to 17.1 seconds. A 1680X1680 matrix computation consumed about 650 seconds on the same computer, using symmetry. Without using symmetry that time would have been about 2600 seconds. For reference, the CPU time to invert this matrix and to compute the radiated fields was about 100 seconds. Note that this latter time is relatively short because of the available vectorization on the CRAY computer. On a non-vector computer, the matrix inversion time would be more than the computation time for such a large matrix. For the interested reader, Figure 5 shows the X-Z plane, -Polarized back scattered field from the dihedral at 1GHz.

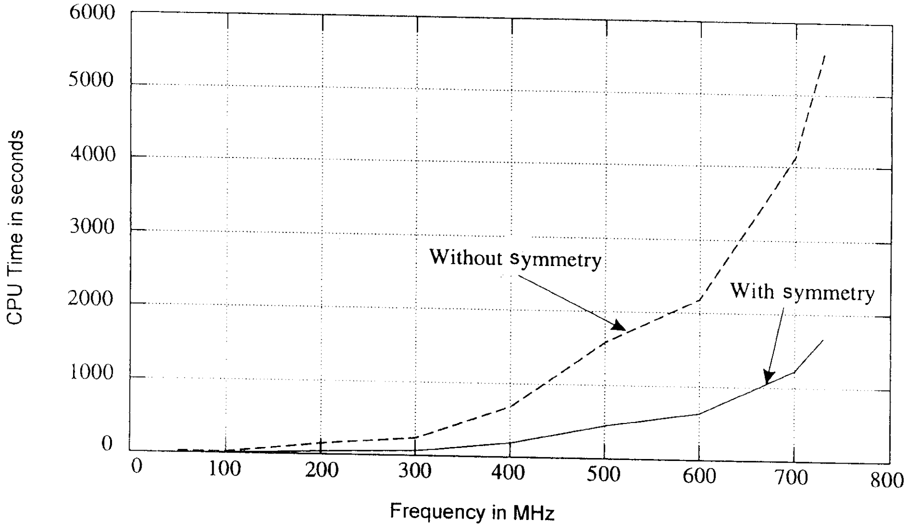


Fig. 4. The CPU time required to compute the *[Z]* matrix elements versus frequency with and without using symmetry. The dihedral dimensions are L_1=2*m*, L_2= 1*m* and $\psi = 90^{\circ}$.

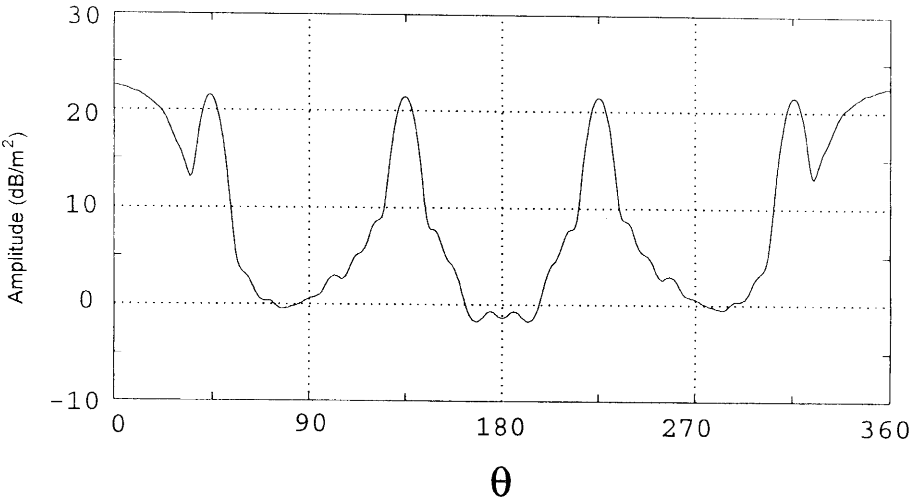


Fig. 5. The θ-polarized backscattered field from the dihedral at 1GHz, X-Z plane.

Modeling and Segmentation Issues

The ESP4 code utilizes one method for plate segmentation, such as shown in Figure 2(b). Such a method is general and does not take advantage of the nature of the plate that needs to be segmented. That results in an inefficient segmentation of the plates, i.e., more unknown current modes, and hence requires more computer memory, when compared to other possible segmentation methods. In fact, the above 4-plate model for the triangular dihedral results in fewer modes than a 2-plate model, using the default ESP4 segmentation method (routine) in both cases. This is because of the general nature of the segmentation routine. Using a 10-plate model for the dihedral, such as the one shown in Fig. 6(a), results in an even fewer unknown current modes. A better approach is to "custom make" a segmentation routine for the problem under consideration. For example, one can segment the dihedral plates as shown in Fig. 6(b). Such a segmentation approach results in about a minimum number of unknown current modes. But, one needs to make sure that such segmentation methods produce convergent results before the resulting solution can be trusted.

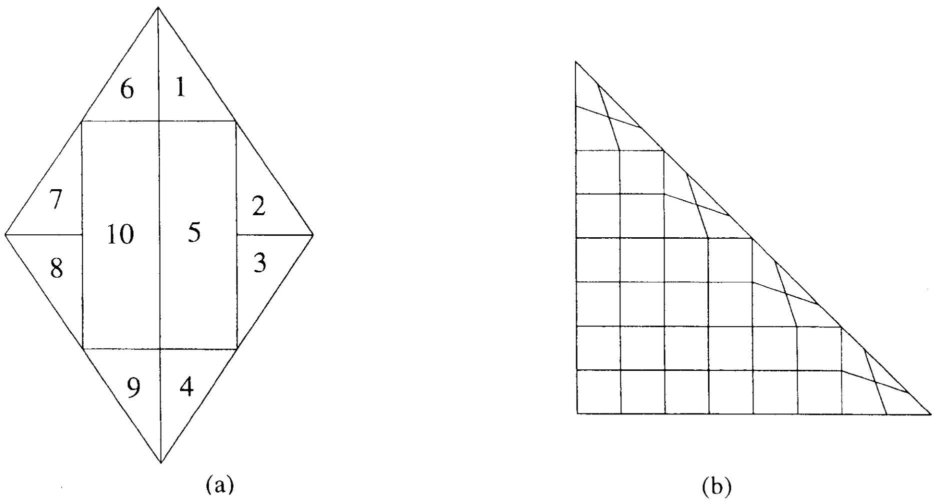


Fig. 6. a) 10-plate model for the triangular dihedral. b) Custom segmentation for the triangular plate.

Figure 7 shows the number of unknown dihedral current modes versus frequency for the above mentioned four segmentation methods. In all cases the maximum segment width is 0.2 wavelength. One can see that the 4-plate model results in a smaller number of modes when compared with the 2-plate model. Note that unlike the 2-plate model, the 4-plate model permits an efficient use of the structure symmetry, which is the reason behind choosing the 4-plate model at the beginning of this section. The 10-plate model requires less computer memory and CPU time when compared to the 4-plate model. However, utilizing the structure symmetry in the 10-plate model is more complicated. The custom made segmentation with the 4-plate model results in the smallest number of unknown current modes, compared to the other 3 models, and hence the minimum required computer memory and CPU time.

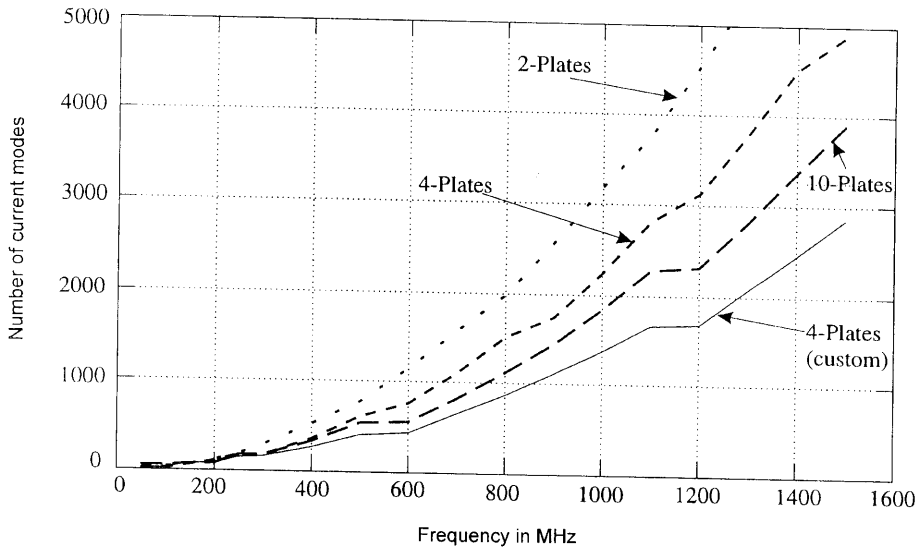


Fig. 7. Number of current modes versus frequency for 2-plate, 4-plate, and 10-plate models for the triangular dihedral with the ESP4 code segmentation, as shown in Fig. 2(b). Also shown in the figure is the number of current modes for the 4-plate model with custom segmentation, as shown in Fig. 6(b). In all cases the maximum segment width is 0.2 wavelength.

This technique can be utilized to solve other electrically large problems. One should start by dividing the structure into smaller substructures that show some similarity in the size and shape, and relative locations to each other. The *[Z]* matrix is then formed of blocks for self and mutual impedance of each substructure elements as well as blocks for the mutual impedance between different substructure elements. One should carefully review the problem and try to find similarity between the *[Z]* matrix blocks and, only compute the minimum number of blocks. Changes and refinements to the substructure choices may be needed.

Conclusions

A technique that can save about 75% of the CPU time required for computing the MM *[Z]* matrix elements is described. This technique utilizes the structure symmetry that exists in the problem. It requires extra work and specific treatment for every new problem. However, accurate solutions to some problems are always desired, especially in the absence of measurement results. Lack of measurement's facility or the unacceptable cost or time required for building measurement's model(s) could make this technique's extra work justifiable for several interesting problems. Also, the process of utilizing the symmetry in the problem can be simplified or built in the moment method codes, leaving the user with minimum work to do.

It was noted that the *[Z]* matrix size depends on the method used for segmenting the plates making the structure under consideration. Designing a specific segmentation routine may result in a reduced number of unknowns and hence less memory and CPU time requirements. Further CPU time and memory reduction may be achieved by considering current symmetry, which exist .for specific planes of incidence, i.e., the principle planes of the dihedral (Y-Z and X-Z planes).

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ملخص البحث. كثيرا ما تستخدم طريقة العزوم في حساب المجالات المشعة أو المرتدة من هياكل بسيطة أو مركبة. و يتطلب استخدامها -في حالة الهياكل الكبيرة- حيزا كبيرا من ذاكرة الحاسب الآلي، بالإضافة إلى وقت طويل نسبيا لحساب عناصر المصفوفة [Z]. ويقدم هذا البحث أسلوبا للتقليل من زمن حساب عناصر تلك المصفوفة، وذلك باستخدام التناظر المتاح والذي قد لا يكون بالضرورة واضحا. وتم تقديم الأسلوب من خلال حل مفصّل لمسألة تتضمن حساب المجالات المرتدة من عاكس زاوية مثلث كبير نسبيا. وتم توفير حتى ما يقارب ٧٥٪ من زمن حساب عناصر المصفوفة[Z].