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Hunting Characteristics of Reluctance Motors

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Abrtract. This paper describes the characteristics of damping and synchronising torques which are produced during hunting of a synchronous reluctance motor. The damping torque is divided into its individual components as produced by various windings. Theoretical results are presented which show the influence of various parameters on hunting performance of reluctance motors.

Expressions are developed for the direct – and quadrature – axis components of damping torque. These are obtained by analysing the d– and q– axis equivalent circuits. The expressions lead to a much improved understanding of the influence of the different parameters on stability, and indicate direction in which improved behaviour may be obtained. The results obtained using these expressions are found to be in reasonably good agreement with those obtained using more detailed modelling.

List of Symbols

G	=	torque matrix
H	=	inertia constant, Sec.
i, I	=	current, p.u.
J	=	inertia, p.u.
L,M	=	p.u. self and mutual inductances, respectively.
v,V	=	voltage, p.u.
R	=	resistance, p.u.
X,x	=	p.u. total and leakage reactances, respectively.
T	=	torque, p.u.
T_d,T_s	=	p.u. damping and synchronising torques, respectively.
Z	=	impedance, P.u.
ψ	=	flux linkage, p.u.
δ	=	load angle
θ	=	angle turned through, rad.

Ω = frequency of oscillation, p.u.
ω_0 = synchronous speed of equivalent 2–pole machine at rated frequency, rad/sec.
ω = angular frequency of supply, p.u.
$\Delta V, \delta V$ = apparent and absolute changes in voltage, respectively.
$\Delta i, \delta i$ = apparent and absolute changes in current, respectively.
ΔT = change in torque.
p = differential operator.

Subscripts

a = armature
d,q = direct and quandrature axes, respectively.
kd, kq = rotor (d–,q – axes)
ad, aq = mutual (d–,q – axes)
L = load
M = mechanical
T = transpose

1. Introduction

Synchronous reluctance motors are used in some industrial applications. This is due to the fact that such motors offer the possibility of precise speed and position control as well as synchronisation between multi-motor drive systems.

Steady state stability of reluctance motors is of considerable importance. Lipo and Krause [1] have used Nyquist's stability criterion to analyze the stability of reluctance motors. Liapnuov's method has been applied by Hoft [2] to analyze unstable modes in polyphase reluctance motors. Lawrenson and Bowes [3] have used the D–decomposition approach for the multiparameter stability analysis of reluctance machines. Honsinger [4] has developed a simplified treatment based on polynomial analysis for stability studies.

It is well known that the damping and synchronising torques influence the steady state stability of synchronous machines [5,6]. This paper describes a method to compute these torques for a three-phase reluctance motor. Furthermore, the contributions of the individual motor windings to the total damping torque are separated. The results point out clearly the role of different windings as well as its parameters on the overall damping characteristics of the motor. The results also illustrate how mutual interactions between various winding influence the stability of the machine. Using d– and q–axis equivalent circuits, expressions are derived for the different contributions to the damping torque and the conditions under which maximum damping can occur. It is further shown that like other methods the present method of analysis permits the computation of damping and synchronizing torques and the frequency of oscillations. However, this method has the advantage that it also permits evaluation of the contribution of the individual machine members to the total damping torque. Such information is helpful for optimum design of the reluctance motor from the stability point of view. Furthermore, the analysis of the equivalent circuit provides a clear understanding of the influence of various parameters on the hunting characteristics of reluctance motors.

2. Mathematical Analysis

2.1. Linearized Equations in Park's d–q Axes

The equations of Park's d–q reference frame describing a synchronous reluctance motor operating from a balanced three–phase supply can be expressed as follows [7].

$$\begin{aligned} v_d &= R_a i_d + p\psi_d - \omega\psi_q \\ v_q &= R_a i_q + p\psi_q + \omega\psi_d \\ 0 &= R_{kd}\, i_{kd} + p\,\psi_{kd} \\ 0 &= R_{kq}\, i_{kq} + p\psi_{kq} \end{aligned} \qquad (1)$$

$$\begin{bmatrix} \psi_d \\ \psi_{kd} \\ \psi_q \\ \psi_{kq} \end{bmatrix} = \begin{bmatrix} L_d & M_{ad} & \cdots & \cdots \\ M_{ad} & L_{kd} & \cdots & \cdots \\ \cdots & \cdots & L_q & M_{aq} \\ \cdots & \cdots & M_{aq} & L_{kq} \end{bmatrix} \begin{bmatrix} i_d \\ i_{kd} \\ i_q \\ i_{kq} \end{bmatrix} \qquad (2)$$

$$v_d = V \operatorname{Sin} \delta \qquad (3)$$

$$v_q = -V \operatorname{Cos} \delta$$

$$T_M = \psi_d\, i_q - \psi_q\, i_d \qquad (4)$$

$$Jp^2\theta_M = T_M - T_L \qquad (5)$$

$$\omega_M = p\theta \qquad (6)$$

For steady state stability analysis of such machines, these non-linear equations are linearized about a steady state operating point. Consequently, the following motional-impedance matrix, which describes the machine behaviour during hunting oscillations, is obtained.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \Delta T \end{bmatrix} = \left[\begin{array}{cccc|c} R_a + L_d p & -L_q\omega & M_{ad}p & -_{aq}\omega & -L_q i_q p - V\cos\delta \\ L_d\omega & R_a + L_q p & M_{ad}\omega & M_{aq}p & L_d i_d p - V\sin\delta \\ M_{ad}p & & R_{kd} + L_{kq}p & & \\ & M_{aq}p & & R_{kq} + L_{kq}p & \\ \hline -(L_d - L_q)i_q & -(L_d - L_q)L_d & -M_{ad}i_q & M_{aq}i_d & 2\omega_0 H p^2 \end{array}\right] \begin{bmatrix} \Delta i_d \\ \Delta i_q \\ \Delta i_{kd} \\ \Delta i_{kq} \\ \Delta\theta \end{bmatrix} \tag{7}$$

or

$$\begin{bmatrix} 0 \\ \\ \Delta T \end{bmatrix} \begin{matrix} \text{elect.} \\ = \\ \text{mech.} \end{matrix} \left[\begin{array}{c|c} Z_1 & Z_2 \\ \hline Z_3 & Z_4 \end{array}\right] \begin{bmatrix} \Delta i \\ \\ \Delta\theta \end{bmatrix} \tag{8}$$

where Z_1 is the electrical impedance matrix for a reluctance machine, Z_2 is the voltage deviation vector $[\Delta V]$ and $Z_3 = -i_T[G+G_T]$ where G is the torque matrix, comprised of terms containing ω in the Z_1 matrix.

Using matrix reduction techniques, equation (8) can be reduced to:

$$\Delta T = [Z_4 - Z_3 Z_1^{-1} Z_2]\,[\Delta\theta] \tag{9}$$

which defines the equation of motion. The general torque equation of the rotor of an electrical machine during free oscillations is given by;

$$Jp^2\,\Delta\theta + T_d p\,\Delta\theta + T_s\,\Delta\theta = 0 \tag{10}$$

where T_d and T_s are the damping and synchronising torque coefficients respectively. From equations (9) and (10), we can write

$$\begin{aligned} T_d &= -\text{ Imaginary } [Z_3 Z_1^{-1} Z_2]/\Omega \\ T_s &= -\text{ Real } [Z_3 Z_1^{-1} Z_2] \\ Jp^2 &= Z_4 = 2\omega_0 H p^2 \end{aligned} \tag{11}$$

Since equation (7) is expressed in per unit time [8] and contains complex numbers, the operator p can be replaced by $j\Omega$ where Ω is the per unit hunting frequency.

2.2. Calculation of the Hunting Frequency

The hunting frequency can be obtained by using an iterative procedure. Assuming an initial value for Ω_i, the damping and synchronizing torque coefficients T_{di}, and T_{si} are evaluated from equation (11). Then a new value Ω_n is calculated by using the following expression [6];

$$\Omega_n = \sqrt{\frac{T_{si}}{J} - \left(\frac{T_{di}}{2J}\right)^2} \tag{12}$$

Ω_n is then compared with Ω_i. When the difference between two successive value of Ω is within a specified limit the solution is reached.

2.3. Linearized Equations in Kron's Reference System

The relationship between absolute changes (δi , δV) of Kron's axes and the apparent changes (Δi, ΔV) of Park's axes can be obtained as follows. The absolute and apparent changes in the armature current are related by [6]

$$\begin{aligned} \delta i_d &= \Delta i_q - i_q \, \Delta\lambda \\ \delta i_q &= \Delta i_q + i_d \, \Delta\lambda \end{aligned} \tag{13}$$

These equations may be rewritten in the matrix form as

$$\delta i = \Delta i + \rho . i . \Delta\lambda \tag{14}$$

$$\rho = \begin{bmatrix} & -1 \\ 1 & \end{bmatrix}$$

The relationship between absolute and apparent changes in the voltages is as follows

$$\delta V = \Delta V - \rho_T . V . \Delta\lambda \tag{15}$$

The hunting equations expressed in Kron's axes are obtained by substituting δi and δV from equations (14) and (15) into Park's hunting equations [6]. This leads to the following system of equations:

$$\begin{bmatrix} \delta V_d \\ \delta V_q \\ 0 \\ 0 \\ \Delta T \end{bmatrix} = \begin{bmatrix} & (L_d-L_q)i_q p-(L_d-L_q)i_d p\theta \\ Z_1 \text{ from eqn. (7) \& (8)} & (L_d-L_q)i_d p+(L_d-L_q)i_q p\theta \\ & M_{ad}i_q p \\ & -M_{aq}i_d p \\ Z_3 \text{ from eqn. (7) \& (8)} & Jp^2+(L_d-L_q)(i_d i_d-i_q i_q) \end{bmatrix} \begin{bmatrix} \delta i_d \\ \delta i_q \\ \Delta i_{kd} \\ \Delta i_{kq} \\ \Delta\lambda \end{bmatrix} \quad (16)$$

An analysis similar to that applied for equation (7) can also be used for equation (16) to calculate the total damping and synchronizing torque coefficients. Consequently,

$$\begin{aligned} T_d &= -\text{ Imaginary } [Z_3 Z_1^{-1} Z_2]/\Omega \\ T_s &= -\text{ Real } [Z_3 Z_1^{-1} Z_2] + (L_d - L_q)(I_d I_d - I_q I_q) \end{aligned} \quad (17)$$

2.4. Components of the Damping Torque

The contribution of the different windings can be found by splitting the total damping torque into its direct and quadrature axis components. The total damping torque is given by:

$$T_d = \Sigma \frac{\delta i^2}{\Omega} r \quad (18)$$

where δi represents the real deviation in current occuring in various windings, and r is given by R/Ω or $R/(\Omega \pm 1)$ depending upon whether the winding is on the rotor or the stator, respectively.

The current deviation (Δi) in Park's axes does not represent the real changes (δi) in the armature current. It is, therefore, necessary to use Kron's system to compute the contribution of the armature winding to the damping torque.

Alternatively, the damping torque contributed by the armature winding may be calculated by an indirect method. In such an approach, the total damping torque is calculated using Park's equations as described earlier. Then the damping torque contributed by the rotor circuits are calculated from $\Delta i^2 R/\Omega$. For the rotor circuits $\Delta i = \delta i$ in both Park's and Kron's axes. The difference between damping of the rotor and the total damping torque gives the damping torque contributed by the stator winding. Hence, the armature damping torque is:

$$= T_d - \frac{1}{\Omega^2}\left[(\Delta i_{kd})^2 R_{kd} + (\Delta i_{kq})^2 R_{kq} \right] \quad (19)$$

Both of the above methods give exactly the same results.

The oscillating current $[\Delta i]$ or $[\delta i]$ may be obtained from equations (7) or (16), respectively. The respective equations may be written as:

$$[\Delta i] = [Z_1]^{-1} [\Delta V]$$
$$[\delta i] = [Z_1]^{-1} [\delta V] \qquad (20)$$

2.5. Simplified Analysis Using the Equivalent Circuits

Although detailed system modelling of the reluctance motor is available, it is always of great assistance to represent the machine in the simplest possible way. The equivalent circuit of complex electromagnetic systems, has always appealed to engineers as it allows an insight into some of the more complicated physical phenomena inside the machine, without resorting to the tedious mathematics or computing necessary to obtain results.

Consider the q-axis equivalent circuit of a synchronous reluctance motor, Figure 1a. To find the conditions for maximum damping torque contributed by the quadrature axis rotor winding (*i.e.* a condition for maximum I^2R loss in the q-axis rotor resistance), the equivalent circuit is reduced to the circuit shown in Figure 1b, where

$$E_{th} = \psi_d \frac{X_{aq}}{Z_q} = X_d I_d \frac{X_{aq}}{\sqrt{R_a^2 + X_q^2}} \qquad (21)$$

$$Z_{th} = R_{th} + jX_{th} \qquad (22)$$

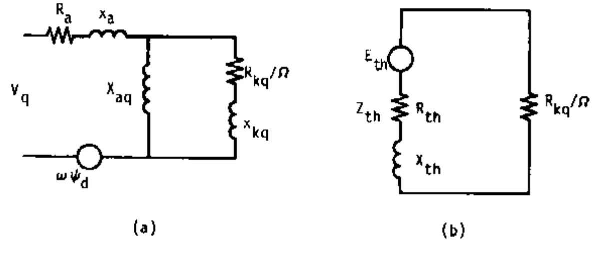

Fig. 1. (a) q-axis equivalent circuit of reluctance motors.
(b) Thevenin's Equivalent circuit of Fig.1 (a).

where

$$R_{th} = \frac{R_a \quad X_{aq}^2}{R_a^2 + X_q^2} \text{ and } X_{th} = X_{kq} - \frac{X_{aq}^2 \quad x_q}{R_a^2 + X_q^2}$$

The maximum power loss in the rotor resistance occurs when

$$\frac{R_{kq}}{\Omega} = |Z_{th}|$$

or

$$R_{kq} = \Omega \sqrt{\frac{(X_{kq} X_q - X_{aq}^2)^2 + R_a^2 X_{kq}^2}{Z_q^2}} \tag{23}$$

The damping torque contributed by the q-axis rotor circuit is given by;

$$T_d = \frac{E_{th}^2}{(\frac{R_{kq}}{\Omega} + R_{th})^2 + X_{th}} \quad \frac{R_{kq}}{\Omega^2} \tag{24}$$

The maximum contribution can be obtained from equation (24) by substituting R_{kq} from equation (23).

The above analysis may be further simplified by neglecting the armature resistance, in which case:

$$\left. \begin{aligned} E_{th} &= V \cos \delta \frac{X_{aq}}{X_q} \\ X_{th} &= X_{kq} - \frac{X_{aq}^2}{X_q} \end{aligned} \right| \tag{25}$$

Maximum damping by the rotor q-axis occurs when $\frac{R_{kq}}{\Omega} = X_{th}$, therefore,

$$\left.\begin{aligned} T_{d(max)} &= \frac{E_{th}^2}{2R_{kq}} = \frac{(V\cos\delta)^2}{2\,\Omega\,X_{th}}\,\frac{X_{aq}^2}{X_q} \\ \text{or} \quad &= \frac{I_d^2\,X_d^2\,X_{aq}^2}{2\Omega\,X_{th}\,X_q^2} \end{aligned}\right| \qquad (26)$$

If $X_{kq} = X_q$ then, $X_{th} = X_q''$ where X_q'' is the quadrature-axis sub-transient reactance as defined for synchronous alternators. Equation (26) demonstrates clearly the dependence of the damping torque on direct-axis/quadrature-axis reactance ratio, frequency of oscillations, and load angle. It also shows the dependence on the leakage reactance as included in the term X_{aq}.

To obtain corresponding expression for the d-axis rotor circuit contributions to the total damping, replace suffix q in equation (21-26) with suffix d.

3. Results and Discussion

The present approach was used to calculate the hunting performance of a three-phase reluctance motor. The parameters of this motor are given in the Table (1). Unless otherwise stated, system inertia is taken as 10 times motor inertia. Consideration has been only given to fixed normal frequency operation.

Table 1. Per unit parameter of a 4-pole reluctance motor [7].

X_d	X_q	$x_a = x_{kd} = x_{kq}$	X_{kd}	X_{kq}	R_a	$R_{kd} = R_{kq}$	J
1.82	0.365	0.0546	1.82	0.365	0.0685	0.0365	34.4

Figure 2 shows the variation of the synchronizing torque as well as different components of the damping torque. The contribution of the q-axis rotor decreases while that of the d-axis increases with an increase of the load angle δ. The contribution of the armature winding is always negative. The total damping torque increases marginally with load angle whereas the total synchronising torque decreases (reducing also the frequency of oscillations) until it reaches zero at the pull-out load angle of about 38°, in which case the machine becomes unstable.

Figure 3 and 4 shows the effect of the q-axis rotor resistance on the hunting characteristic of the reluctance motor. In Fig. 3 the total damping and synchronising torques are shown for two different load angles. In Fig. 4 the contributions of individual windings to the total damping torque are shown. It is clear that the q-axis produces maximum damping at a value of R_{kq} which agrees with that given by equation (23).

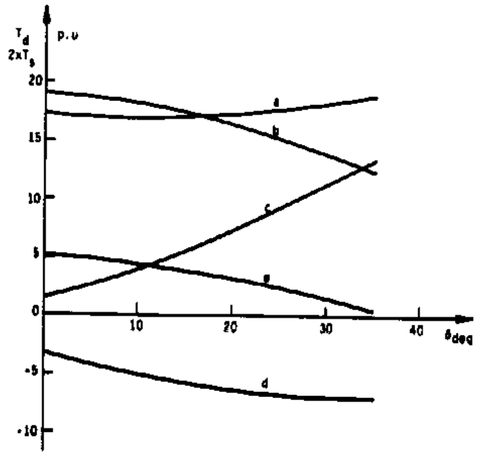

Fig. 2. Influence of load angle on damping and synchronising torques.

a) Total damping torque

b) Damping torque contributed by q-axis rotor resistance

c) Damping torque contributed by d-axis rotor resistance

d) Damping torque contributed by armature

e) Total synchronising torque

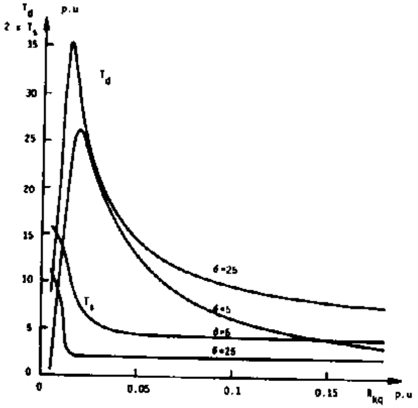

Fig. 3. Effect of quadrature-axis rotor resistance on damping and synchronising torques

At very small values of R_{kq}, the contributions of both the rotor d-axis and the armature windings show positive and negative peak values respectively. After that these torques decreases to low constant values. This is due to the fact that a low value of R_{kq} causes an increase in the q-axis current during hunting oscillations which increases the speed voltages induced in the d-axis circuits. The total damping torque is negative for values of R_{kq} below 0.005 p.u., indicating instability.

The influence of the direct axis rotor resistance on the damping and synchronising torques is shown in Fig. 5. It is clear from Fig. 4 and 5 that as expected the behaviour of d and q-axis rotor circuits are similar, although the q-axis contributes more damping than the d-axis does.

Figures 6 and 7 respectively show the influence of the stator resistance and leakage reactance on the damping characteristics. Both of these figures show reduction in the total damping torque when R_a or x_a are increased respectively. It is generally assumed that the armature leakage reactance equals the damper leakage reactance on both the d- and q-axes. The dotted line in Fig. 6 shows the total damping (d-q axis rotor) obtained from the equivalent circuits *i.e.* using equation (24) and a corresponding expression for the d-axis rotor contribution. It is clear from Fig. 6 that the results from the equivalent and detailed analysis are in complete agreement when the armature resistance is neglected ($R_a = 0$). When R_a is present, there is some difference between the two curves. However, for R_a values of practical interest the difference between the two curves is not significant and the equivalent circuits gives result of reasonable accuracy.

The influence of system inertia on the damping and synchronising torques is shown in Fig. 8. The damping torque increases while the synchronising torque decreases when the inertia is increased. The synchronising torque in all cases decreases gradually to zero with increasing load angle indicating instability at the pull-out load angle.

4. Conclusions

The total damping and synchronising torque are calculated for a 3-phase reluctance motor. The contribution of the rotor to the damping torque is split into its d-q axis components. The effects of motor parameters on the hunting performance have been assessed quantitatively.

d- and q-axis equivalent circuits have been used to analyze the damping characteristics of reluctance motors. The derived expressions demonstrate the influence of motor parameters on damping. The results obtained from equivalent circuits are in reasonable agreement with those obtained from more detailed studies.

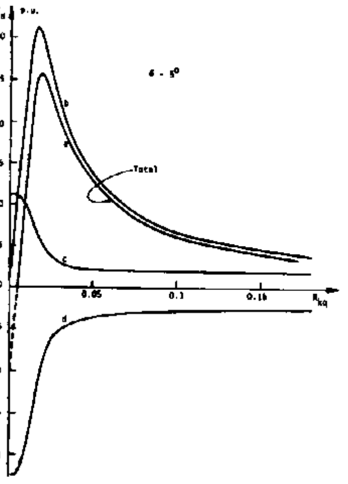

Fig. 4. Effect of quadrature-axis rotor resistance on damping torque components.

a) Total damping torque

b) Damping torque contributed by q-axis rotor resistance

c) Damping torque contributed by d-axis rotor resistance

d) Damping torque contributed by armature

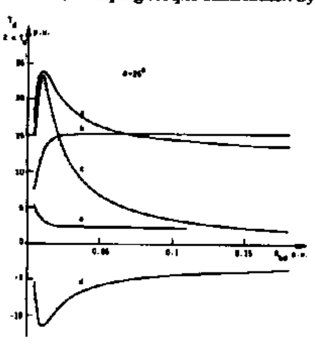

Fig. 5. Effect of direct-axis rotor resistance on damping and synchronising torques.

a) Total damping torque

b) Damping torque contributed by q-axis rotor resistance

c) Damping torque contributed by d-axis rotor resistance

d) Damping torque contributed by armature

e) Total synchronising torque.

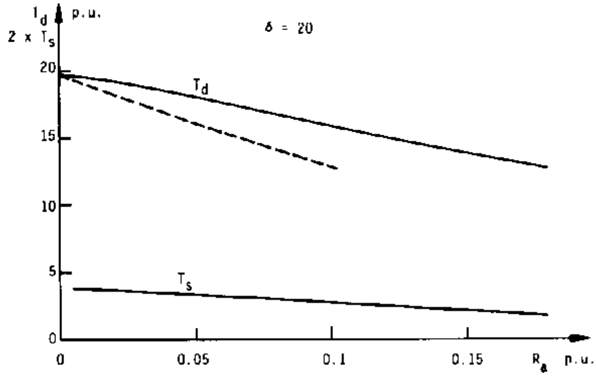

Fig. 6. Influence of stator resistance on damping and synchronising torques.

—— detailed system model

- - - analytical expression

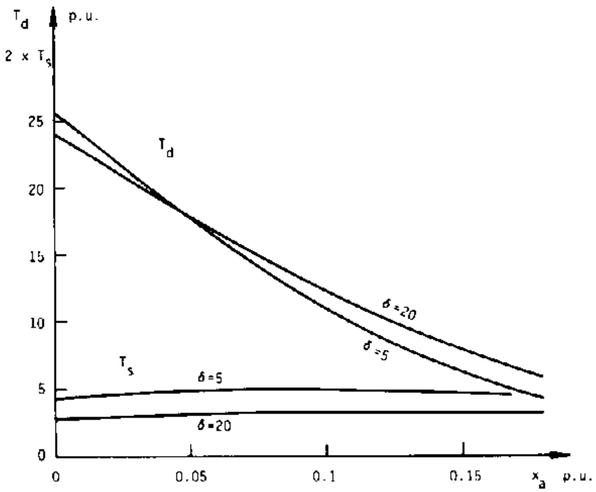

Fig. 7. Influence of leakage reactance on damping and synchronising torques

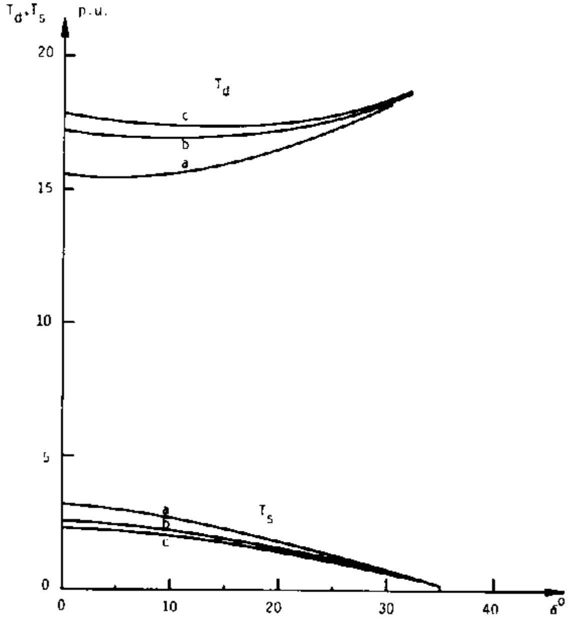

Fig. 8. Influence of inertia constant on damping and synchronising torques.

a) Inertia constant H equal to 5 x that of rotor

b) Interia constant H equal to 10 x that or rotor

c) Inertia constant H equal to 20 x that of rotor

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خصائص التأرجح لمحركات الممانعة المغناطيسية

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ملخص البحث: تتناول هذه الورقة دراسة خصائص عزميْ الاضمحلال والدوران التوافقي الناتجين أثناء تأرجح محركات الممانعة المغناطيسية. تم تقسيم عزم الاضمحلال إلى مركباته الأساسية الناتجة عن كل عضو في المحرك. كما تم أيضا استعراض تأثير جميع عناصر ودلائل المحرك على خصائص التأرجح. ثم استنتاج علاقات معادلات رياضية بسيطة لعزم الاضمحلال باستخدام الدوائر الكهربائية المكافئة للمحرك. هذه المعادلات أدت لفهم أدق لخصائص هذا العزم وتأثير جميع العناصر عليه بصورة أكثر وضوحا.

النتائج المستخلصة من هذه المعادلات متفقة تمامًا مع النتائج المستخلصة من حل النماذج الرياضية التي تمثل المحرك باستخدام الحاسب الآلي.