

*Reply to Dr. Al-Yousef Comment on:*

## **A Stochastic Approach to the Estimation of the Number of Survivals at Each Age[1]**

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We begin with the point that some studies of the number of survivals at each age, has approached the problem assuming it to be deterministic phenomenon as represented by Makeham's law. Dr. Al-Yousef has treated the problem in his paper entitled "*The analytical adjustment of life tables*"[2] by using deterministic model to represent the stochastic phenomenon of birth and death. How can we use a mathematical expression for a random process!. This is questionable and wrong. Dr. Al-Yousef has failed to take into account the random element explicit in his model. He considered the number of survivals at each age as a mathematical and not a random variable.

Next, we move to the point that the estimated values were not compared with the realized ones. For this, it is usual in the empirical work with regression analysis not to compare the estimated values with the actual ones, but to depend on both the statistical significance of the estimated parameters as a measure of confidence in the estimates and the value of  $R^2$  as a measure of goodness of fit. Dr. Al-Yousef has mentioned the estimated values and made the comparison with the actual ones, in his paper, because he has no tool to measure either the confidence in his estimated or the goodness of fit of the estimate values with the numerical method of estimation, he had adopted.

Third, for the point of the scientific proof of the simplicity, generality, and superiority of the non-linear least squares method of estimation. We repeat again what we have said in our paper. It is simple in the sense that it uses a very well known

algorithm of computation, namely the least squares method. It is general, in the sense that it can be applied to any model, that is intrinsically non-linear in one or two parameters. Finally, it is superior to the deterministic approach used by Dr. Al-Yousef in that it meets the criterion of statistical sufficiency by using all the available data in estimating the parameters of the model. This is in opposition to the numerical method of Dr. Al-Yousef's, where it is necessary in most cases to delete part of the raw data in order to obtain values for the parameters which satisfy the conditions of the model. Thus as has been indicated in the first page of Dr. Al-Yousef's paper.

Fourth, we reply on the criticism to our suggested method of estimation. As regards the correlations that might exist between the survivals numbers at different ages, it is very well known in the econometric literature that least squares and not the numerical method given by Dr. Al-Yousef that can be exploited for the detection of any autocorrelations among the residuals of the estimated models and can be corrected for if there is any. Thus for Dr. Al-Yousef to say that he knew the method but he did not use because of the possible correlations between the survivals at different ages is a wrong excuse. For the inaccuracy of the estimated parameter  $c$  of the stochastic model with an error at a range of  $10^{-2}$  as Dr. Al-Yousef says, we emphasized the objective of the non-linear least squares estimation procedure is not to get the exact values of the parameters but rather to obtain statistical significant reliable estimated parameters. We are not estimating the model for the whole population but a sample of it.

Fifth, as regards the comparison between the computed values of the parameters resulting from the numerical method with the estimated values obtained from the least squares using the same data, it seems unacceptable to compare computed values obtained from a deterministic expression with the estimated values  $j$  from a stochastic equation for the purpose of establishing the accuracy of the within forecasts. Although, we have got  $R^2 = -0.9$ , which means that the fit of the estimated values to the actual observations is nearly perfect, we would argue that generating accurate predictions of realized values, is in general, not a real problem. It is possible to ignore Makeham's law completely and fit a polynomial of a sufficient degree to the number of survivals at each age. This would generate highly accurate within period forecasts. The objective again and again is to develop an alternative and stochastic procedure for estimating the number of survivals at each age according to Makeham's law.

#### References

- [1] **Abou-El-Fetouh, M.F.** "A stochastic approach to the estimation of the number of survivals at each age." *J. of the College of Admin. Sci., King Saud University*, **11**, 2 (1986), 43-56.
- [2] **Al-Yousef, M.H.** "An analytical adjustment of life tables." *J. of the College of Admin. Sci., King Saud University*, **9** (1984), 73-84. (In Arabic language).