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Determination of the Determinant Operator Â by Using Fourier Transformation

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Abstract. The (det Â) is the main goal for evaluation of the path integral formula and the dynamical finite temperature, in quantum field theory. In this study, the Fourier representation is applied as a new treatment by using ς – function in order to find the determinant of any operator function Â (det Â) or any operator involving Dirac matrix γ^{μ}. This technique is explicitly demonstrated in two examples, and the results are in a good agreement with another technique used in this field.

Keywords: Determinant, Dirac, Fourier, Matrix, Operator, Zeta-function, 2000 mathematics subject classification.

1. Introduction

One of the most interesting problems in quantum field theory is finding the determinant of operator Â, which is very useful to find the path integral, and the finite temperature [1-3].

In order to find the path integral which takes a formula [4, pp. 188-195]:

$$\int_{x(t_a)=0}^{x(t_b)=0} \exp\left(\frac{i}{\eta} S[x(t)]\right) D[x(t)] = \Delta[Det[\hat{A}]]^{\frac{1}{2}} \tag{1}$$

and the partition function of dynamical system at finite temperature given by formula [5, pp. 201-204]:

$$Z = N(\det \hat{A})^{\frac{1}{2}} \tag{2}$$

It is clear that these formulae need to calculate determinant operator Â. This calculation can be done by using few techniques such as quadratic Lagrangian [4], complex semiclassical evaluation of the path integral [6], stander treatment [5] and the common one Riemann ζ – function [7, 8]. But, all these techniques have infinite product of eigen-values, and of course the determinant in general will be highly divergent which required special conditions and limiting for computing them .

In the Riemann's technique the $\varsigma_A(s)$ is defined by [3, 7, 8]:

$$\varsigma_A(s) = \sum_n \lambda_n^{-s} \tag{3}$$

where λ_n is a real positive eigen-value of a ginen operator Â(*x*).

Thus,

$$\varsigma_A(s) = \sum_n e^{-s\log\lambda_n}, \qquad \zeta'_A(s) = -\sum_n \log\lambda_n e^{-s\log\lambda_n} \tag{4}$$

so that,

$$\varsigma'_A(0) = -\sum_n \log\lambda_n = -\log\Pi_n\lambda_n = -\log(\det A) \tag{5}$$

i.e.

$$(\det A) = e^{-\varsigma'_A(0)} \tag{6}$$

This representation of det $\hat{A}$ would be useful if one could find a representation of $\varsigma_A(s)$ other than Eq. (3). This is found by introducing the function G(*x,y,τ*) defined by [1, 9-11]:

$$G(x,y,\tau) = \sum_n e^{-\lambda_n\tau} f_n^*(x) f(y) \tag{7}$$

Then [7],

$$\varsigma_A(s) = \frac{1}{\Gamma(s)} \int_o^\infty d\tau\ \tau^{s-1} H(\tau) \tag{8}$$

where,

$$H(\tau) = \int dx \quad G(x,x,\tau) \tag{9}$$

For:

$$\int_o^\infty d\tau\ \tau^{s-1} H(\tau) = \sum_n \int_o^\infty d\tau\ \tau^{s-1}\ e^{-\lambda\tau} \quad \bullet \int dx f_n(x) f_n^*(x)$$

$$= \sum_n \lambda_n^s \int_o^\infty du\ u^{s-1}\ e^{-u}$$

$$= \Gamma(s) \sum_n \lambda_n^{-s} \tag{10}$$

where the orthonormal property of the eigen functions [10]:

$$\int dx f_n^*(x) f_m(x) = \delta_{nm} \tag{11}$$

i.e., $<n|m> = \delta_{nm}$

Further, one notes that G(x,y,τ) satisfies:

$$A(x)G(x,y,\tau) = -\frac{\partial}{\partial\tau} G(x,y,\tau) \tag{12}$$

For:

$$A(x)G = \sum_n e^{-\lambda_n \tau} \lambda\, f_n(x) f_n^*(y) = -\frac{\partial}{\partial\tau} \sum_n e^{-\lambda_n \tau} f_n(x) f_n^*(y)$$

The completeness relation $\sum_n |\, n >< n \,| = 1$

i.e. $\sum_n < x \,|\, n >< n \,|\, y > = < x, y >$ or

$$\sum_n \ f_n(x) f_n^*(y) = \delta(x-y) \tag{13}$$

shows that Eq. (12) is to be solved subject to the boundary condition:

$$G(x,y,o) = \delta(x\text{-}y) \tag{14}$$

This completes the alternative representation of the function $\varsigma_A(s)$ associated with A: solving Eq. (12) subject to Eq. (14), then using Eq. (9) and Eq. (8) $\varsigma_A(s)$ obtained . Eq. (6) gives det A [3,12-15].

As an application of the above technique, take the Hermitian operator:

$$\hat{A} = -\partial^2 + m^2 \qquad (1-\dim) \tag{15}$$

then, one can find [8],

$$\varsigma_A(s) = (-ms + ...) \int dx \tag{16}$$

i.e.,

$$\log \det A = m \int dx \tag{17}$$

To see the significant effect of dimension, consider the same operator in 4-dim. Euclidean space:

(18)

where $\partial_E^2 = \partial_1^2 + \partial_2^2 + \partial_3^2 + \partial_4^2$. Then the solution of G(x,y,τ) by

using Eq. (12) and taking $G = e^{-m^2/\mu^2\tau}F$

where $F = \frac{C}{\tau^2} e^{-\mu^2(x-y)^2/4\tau}$ will be

$$G(x,y,\tau) = -\frac{\mu^4}{16\pi^2\tau^2} \exp\left[\frac{-\mu^2}{4\tau}(x-y)^2 - \frac{m^2}{\mu^2}\tau\right] \tag{19}$$

then, one can get value of $\varsigma_A(x)$ as :

$$\varsigma_A(s) = \frac{m^4}{16\pi^2}\left(\frac{m^2}{\mu^2}\right)^{-s} \frac{\Gamma(s-2)}{\Gamma(s)} \int d^4x$$

$$\varsigma_A(s) = \frac{m^4}{16\pi^2}\left(\frac{m^2}{\mu^2}\right)^{-s} \frac{\Gamma(s-2)}{\Gamma(s)}\int d^4x$$

i.e.,

$$\zeta'_A = \frac{m^4}{32\pi^2}\left(\frac{3}{2} - \log\, m^2/\mu^2\right)\int d^4x \tag{20}$$

and, therefore the determinate of the operator Â is equal to:

$$\log\,\det\,\left(-\partial_E^2 + m^2\right) = \frac{m^2}{32\pi^2}\left\{\frac{-3}{2} + \log\frac{m^2}{\mu^2}\right\}\int d^4x \tag{21}$$

in 4 dimensions.

In this paper, the new technique is applied by using the Fourier transformation instead of Riemann's ς – function [4] to calculate the det Â. This technique is more general, easy and ideal for description the dynamical systems at finite temperature [5, 16]. Also, it gives more information about the eigen function $f(x)$ and a complete orthogonal set over the same interval time with suitable boundary conditions for the system.

2. Mathematical Formulations

It is suitable to develop a new technique, that appears to be more general, for solving Eq. (12) subject to the boundary condition Eq. (14). One first observes that Eq. (14) restricts G(x,y,τ) to a function of x,y and τ.

Thus, G = G(x, τ) and , H(τ) = $\int$G(0, τ) dx
i.e.

$$H(\tau) = G(0, \tau) \int dx \tag{22}$$

always.

Solving Eq. (12) by using Fourier transformation, $G(x,\tau)$ is taking in the form:

$$G(x,\tau) = \frac{1}{(2\pi)^4}\int e^{i\lambda.x + h(\lambda)\tau}\, d^4\lambda \tag{23}$$

in 4 dimensions. Then Eq. (14), and G (x,0) = $\delta^{(4)}$x
immediately requires that g(λ) =1, so that:

$$G(x,\tau)=\frac{1}{(2\pi)^4}\int e^{i\lambda.x+h(\lambda)\tau}\ d^4\lambda \tag{24}$$

and one needs not to worry about Eq. (14) anymore. Next, apply the operator $A(x)+\partial/\partial\tau$ inside the integral sign,

$$\left(A(x)+\frac{\partial}{\partial\tau}\right)e^{i\lambda.x+h(\lambda)\tau}=0 \tag{25}$$

This equation determines *h(λ)* and completes the solution of Eq. (24) for *G(x,τ)* will give $\zeta_A(s)$.

3. Main Results and Applications

Example

Applying the new technique to Eq. (18):

$$A=-\partial_E{}^2+m^2 \quad \text{in 4-dim.}$$

gives

$$\lambda^2+m^2+\mu^2 h(\lambda)=0 \tag{26}$$

therefore,

$$G(o,\tau)=\frac{1}{(2\pi)^4}\int e^{-\frac{1}{\mu^2}(\lambda^2+m^2)\tau}\ d^4\lambda \tag{27}$$

Now, for integrals of the form $\int f(r,\theta)d^n x\, d^4x$ in n dim, the Euclidean space, for $f=f(r)$ [9]:

$$\int f(r)d^n x=\frac{\pi^{n/2}}{\Gamma(n/2)}\int_o^\infty f[x_1]x_1^{\frac{n-2}{2}}\,dx_1 \tag{28}$$

where $x_1=r^2$. Applying this to the integral in Eq. (27):

Therefore,

$$G(o,\tau) = \frac{\mu^4}{16\pi^2\tau^2}\, e^{-\frac{m^2}{\mu^2}\tau} \tag{29}$$

is in agreement with the result obtained from Eq. (19).

Now we can discuss the general case of any function operator Â in 4-dim.

$$\hat{A} = -\partial_E^2 + f(x) \tag{30}$$

Eq. (12) will be:

$$(-\partial_E^2 + f(x) + \frac{\partial}{\partial\tau})\; G(x,y,\tau) = 0 \tag{31}$$

write a Fourier representation:

$$G(x,y,\tau) = \frac{1}{(2\pi)^4}\int e^{i\lambda.(x-y)} g(\,\lambda,y,\tau) d^4\lambda \tag{32}$$

Subject to:

$$g(\lambda,y,o) = 1 \tag{33}$$

Also, the Fourier representation of *f(x)* is:

$$f(x) = \int e^{\sigma.x} 1(\sigma) d^4\sigma \tag{34}$$

Then,

$$f(x)G(x,y,\tau) = \frac{1}{(2\pi)^4}\int e^{i\lambda.(x-y)} \quad e^{i\sigma y} g(\lambda-\sigma,y,\tau) l\,(\sigma) d^4\sigma \tag{35}$$

and the Fourier transform of Eq. (31) will be:

$$\lambda^2 g(\lambda,y,\tau) + \frac{\partial g}{\partial\tau}(\lambda,y,\tau) + \int e^{i\sigma\lambda} \quad g(\lambda-\sigma,y,\tau) l\,(\sigma) d^4\sigma \tag{36}$$

For given $1(\sigma)$, this is a differential-integral equation to be solved for $g(\lambda, y, \tau)$, subject to the boundary condition Eq. (33).

Taking τ=0 in Eq. (36) and using Eq. (33), Eq. (34) then;

$$\frac{\partial g}{\partial \tau}(\lambda, y, \tau)\Big|_{\tau=0} = -\left(\lambda^2 + f(y)\right) \tag{37}$$

In view of Eq.(37) write

$$g(\lambda, y, \tau) = e^{-(\lambda^2 + f(y)\tau} \; h(\lambda, y, \tau) \tag{38}$$

Then $$h(\lambda, y, o) = 1 \quad \text{and} \quad \frac{\partial h}{\partial \tau}\Big|_{\tau=0} = 0 \tag{39}$$

Substituting Eq. (38) in to Eq. (36) one gets:

$$\frac{\partial h}{\partial \tau}(\lambda, y, \tau) - f(y)h(\lambda, y, \tau) = -\int e^{-(\sigma^2 - 2\lambda.\sigma)\tau} \; e^{i\sigma y} \; h(\lambda - \sigma, y, \tau) l(\sigma) \; d^4\sigma \; (40) \tag{40}$$

Repeated differentiation of this equation generates power series for *h(λ,τ)*, in *τ*, about *τ = 0,* write:

$$h(\lambda, y, \tau) = \sum_{n=0}^{\infty} c_n(\lambda, y)\tau^n \tag{41}$$

then,

$$C_o(\lambda, y) = 1,, \qquad C_1(\lambda, y) = 0$$

thus,

$$g(\lambda, y, \tau) = e^{(-\lambda^2 + f(y))\tau} \; \left[1 - C_2(\lambda, y)\tau^2 + C_3(\lambda, y)\tau^3 + ...\right] \tag{42}$$

where,

$$C_2 = \alpha^{(2)} y + \beta_\mu^{(2)}(y)\lambda_\mu, \tag{43}$$

$$C_3 = \alpha^{(3)} y + \beta_\mu^{(3)}\lambda_\mu, + \gamma_{\mu\upsilon}^{(3)}(y)\lambda_\mu\lambda_\upsilon$$

[See App. (A)].

Note that all the integrals are Euclidean (so that e.g. $g_{\mu\mu}=4$). The contribution of $\beta_\mu^{(2)}$ λ_μ and $\beta_\mu^{(3)}\lambda_\mu$, which are infact the imaginary terms in C_2 and C_3, vanish and then:

$$G(x,x,\tau)=\frac{e^{-f(x)\tau}}{16\pi^2}\left[\frac{1}{\tau^2}+\alpha^{(2)}(x)+\frac{1}{2}\gamma^{(3)}_{\mu\mu}(x)+\alpha^{(3)}(x)\tau+.....\right] \quad (44)$$

From App. (A):

$$\alpha^{(2)}+\frac{1}{2}\gamma^{(3)}_{\mu\mu}(x)=\frac{5}{6}\partial^2 f(x)\equiv a(x)$$

$$\alpha^{(3)}(x)=\frac{1}{3}\partial f(x)\partial f(x)-\frac{1}{6}\partial^2\partial^2 f(x)\equiv b(x) \quad (45)$$

Also, write $^{\tau}/_{\mu 2}$ for τ, so that the new τ is dimensionless and therefore,

$$G(x,x,\tau)=\frac{e^{-f(x)\tau/\mu^2}}{16\pi^2}\left[\frac{\mu^4}{\tau^4}+a(x)+\frac{b(x)\tau}{\mu^2}+.......\right] \quad (46)$$

Then,

$$\underset{A}{\varsigma}(s)=\frac{1}{\Gamma(s)}\int\tau^{s-1}G(x,x,\tau)\,d\tau d^4x \quad (47)$$

and do the τ integration and extract the term linear in s before the x-integration gives:

$$16\pi\zeta'_A(0)=\int[\frac{3}{4}f^2(x)+\frac{b(x)}{f(x)}-(\frac{1}{2}f^2(x)+a(x))\log\frac{f(x)}{\mu^2}d^4x \quad (48)$$

One recovers the result Eq. (20) as the special case $f(x)$= const.= m^2.

In order to apply this technique for solving the determinant operator involving Dirac matrix γ^{μ}, one requires to state and prove the following mathematical theorem.

Theorem

Determinant of operators involving the Dirac matrices γ^{μ} may be reduced to ones that do not involve γ^{μ} using the properties of the γ-matrices.

Proof:

Consider, first, the simple example:

$$\hat{A} = -ia.\gamma + mI \tag{49}$$

where a is a constant 4-vector. This may be calculated from

$$\det A = \exp(\mathrm{Tr}\log A) \tag{50}$$

$$\begin{aligned}\log A &= \log(mI - ia.\gamma) = \log\{(mI)(I - i\frac{a}{m}.\gamma)\} \\ &= \log(mI) + \log(I - \frac{i}{m}a.\gamma) \\ &= (\log m)I - [\frac{i}{m}a.\gamma + \frac{1}{2}(\frac{i}{m})^2(a.\gamma)^2 + \frac{1}{3}(\frac{i}{m})^3(a.\gamma)^3 + ...]\end{aligned} \tag{51}$$

since functions of matrices are defined by their series expansions and mI , $I - \frac{i}{m}a.\gamma$ commute. Therefore,

$$\log A = (\log m)I + \frac{1}{2m^2}(a.\gamma)^2 - \frac{1}{4m^4}(a.\gamma)^4 + ... - i[\frac{1}{m}a.\gamma - \frac{1}{3m^3}(a.\gamma)^3 + ...] \tag{52}$$

But,

$$(a.\gamma)^2 = a_\mu a_\nu \gamma^\mu \gamma^\nu = \frac{1}{2}a_\mu a_\nu(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu) = a_\mu a_\nu g^{\mu\nu} I = a^2 I$$

therefore, $(a.\gamma)^3 = a^2 a.\gamma$ and

$$\begin{aligned}\log A &= [\log m + \frac{1}{2}(\frac{a}{m})^2 - \frac{1}{4}(\frac{a}{m})^4 + ...] \\ &- \frac{i}{m}[1 - \frac{1}{3}(\frac{a}{m})^2 + \frac{1}{5}(\frac{a}{m})^4 - ....]a.\gamma\end{aligned} \tag{53}$$

also, $\mathrm{Tr}\gamma = 0 \quad \mathrm{Tr}a.\gamma = 0$

this gives the trace as,

$$\mathrm{Tr}\log A = [\log m + \frac{1}{2}(\frac{a}{m})^2 - \frac{1}{4}(\frac{a}{m})^4 + \ldots\ldots..]\mathrm{Tr}I \tag{54}$$

therefore in 4 dimensions,

$$\begin{aligned}\frac{1}{4}\mathrm{Tr}\log A &= \log m + (x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots\ldots) \\ &= \log m + \frac{1}{2}\log(1+x)\end{aligned} \tag{55}$$

i.e,

$$\mathrm{Tr}\log A = 4\log(a^2 + m^2)^{\frac{1}{2}} = \log(a^2 + m^2)^2 \tag{56}$$

$$\det(-ia.\gamma + m) = (a^2 \ \ + m^2)^2 \tag{57}$$

Now, consider the case:

$$A = -i\gamma.\partial + m \tag{58}$$

Following the same steps as above one gets:

$$\begin{aligned}\det(-i\gamma.\partial + m) &= \det(\partial^2 + m^2)^2 \\ &= (\det(\partial^2 + m^2))^2\end{aligned} \tag{59}$$

Since the right-hand side is a function of ∂^2 and m^2 one may also write:

$$\det(i\gamma.\partial + m) = \det(i\gamma.\partial - m) = \det(\partial^2 + m^2)^2 \tag{60}$$

It can be noted that the Euclidean version of this can be also obtained. It is important to appreciate the following observation concerning equations like (59) and (60) that:

On the L.H.S. $i\gamma.\partial + m$ acts on both coordinate and Dirac spaces, i.e, $u_\alpha(x)$, on R.H.S. $\partial^2 + m^2$ acts only on coordinate space, i.e., on $u(x)$. This result enables us to use the technique of sections (2) and (3) for the evaluation of determinant of the Dirac operator $-i\gamma.\partial + m$.

4. Conclusion

It is very clear that the determination of the operator Â by using Fourier transformation is in agreement with the solutions of the other technique like Riemann's technique (Eq. (17) and Eq. (21)) [Ref. 7, 8, pp. 130-133], but more general and easy when anyone needs to describe the dynamical system with any boundary conditions. Moreover, one can apply this technique to solve the determinant operators involving Dirac matrix γ^{μ} [Theorem] or any general operator function Â (Eq. (48)).

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Appendix (A)

To find the value of the coefficients, use the Eq. (41):

$$h(\lambda, y, \tau) = \sum_{n=0}^{\infty} c_n(\lambda, y)\tau^n$$

and substitution into Eq. (40) yields (on expanding the exponential) the recurrence relation.

$$(h+1)C_{n+1}(\lambda, y) = f(y)C_n(\lambda, y) + \sum_{k=0}^{n} \frac{(-1)^{n+1-k}}{(n-k)!} \int e^{i\sigma y}(\sigma^2 - 2\lambda.\sigma)^{n-k} 1(\sigma)C_k(\lambda - \sigma, y)d^4\sigma \qquad \text{(A-1)}$$

For $n = 1$ this gives:

$$2C_y(\lambda,, y) = \int e^{i\sigma y}(\sigma^2 - 2\lambda.\sigma)l(\sigma)d^4\sigma \qquad \text{(A-2)}$$

$$\partial_\mu(f(y) = i\int \sigma_\mu e^{ioy} 1(\sigma)d^4\sigma, \quad \partial_\mu\partial_v f(y) = -\int \sigma_\mu\sigma_v e^{i\sigma y} 1(\sigma)d^4\sigma$$

$$C_2(\lambda, y) = -\frac{1}{2}(\partial^2 - 2i\lambda.\partial)f(y) \qquad \text{(A-3)}$$

using

Similarly,

$$3C_3(\lambda, y) = -f(y)C_2(\lambda, y) - \frac{1}{2}\int e^{i\sigma.y}(\sigma^2 - 2\lambda.\sigma)^2 1(\sigma)d^4\sigma - \int e^{i\sigma.y} 1(\sigma)C_2(\lambda - \sigma, y)d^4\sigma, \qquad \text{(A-4)}$$

which gives:

$$C_3(\lambda, y) = \frac{1}{3}\partial f(y).\partial f(y) - \frac{1}{6}\partial^2\partial^2 f(y) - \frac{2}{3}i\partial^2\lambda.\partial f(y) - \frac{2}{3}\lambda.\partial\lambda.\partial f(y) \qquad \text{(A-5)}$$

Thus,

$$g(\lambda,,y,\tau)=e^{-(\lambda^2+f(y)\tau}\left[1+C_2(\lambda,y)\tau^2+C_3(\lambda,y\tau^3+........\right]. \tag{A-6}$$

where,

$$C_2=\alpha^{(2)}(y)+\beta_\mu^{(2)}(y)\lambda_\mu\ ,\quad C_3=\alpha^{(3)}(y)+\beta_\mu^{(3)}(y)\lambda_\mu+\gamma_{\mu\nu}^{(3)}(y)\lambda_\mu\lambda_\nu \tag{A-7}$$

The coefficients *α,β,γ* are to be read from (A-3) and (A-4). In calculating G(x,x,τ), given by:

$$G(x,x,\tau)=\frac{1}{(2\pi)^4}\int g(\lambda,x,\tau)d^4\lambda, \tag{A-8}$$

use,

$$\int e^{-\lambda^2\tau}d^4\lambda=\frac{\pi^2}{\tau^2}\ ,\ \int e^{-\lambda^2\tau}\lambda^2 d^4\lambda=\frac{2\pi^2}{\tau^{32}}$$

(which follows from coveriance)

$$\int \lambda_\mu e^{-\lambda^2\tau}d^4\lambda=0$$

$$\int \lambda_\mu\lambda_\nu e^{-\lambda^2\tau}d^4\lambda=ag_{\mu\nu}\qquad ,\qquad 4a=\int\lambda^2 e^{-\lambda^2\tau}d^4\lambda=\frac{\pi^2}{\tau^3} \tag{A-9}$$

إ اد مصفوفة أي مؤثر A باستخدام ثيل فورير

وى عناية

كلية العلوم، قسم الفيزياء، جامعة الملك عبدالعزيز

(قدم للنشر في ١٤٢٥/٣/٢٠هـ؛ وقبل للنشر في ١٤٢٦/٢/٢٠هـ)

ملخص البحث. تلعب مصفوفة المؤثر A دورا كبيرا في دراسة تكاملات المسار والديناميكية الحرارية المحددة لنظرية المجال الكمي، وفي هذه الدراسة تم تطبيق تمثيل فورير في دالة زيتا لإيجاد مصفوفة أي مؤثر A. أو أي مؤثر يحتوي على مصفوفة ديراك. كما أظهرنا ذلك في مثاليين ووجدنا أن النتائج كانت مشابهة للتقنيات الأخرى المستخدمة في هذا المجال.