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Calculation for Six-gluon Scattering to Leading Order in the Number of Color

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Abstract. At present there is no simple analytic expression for this scattering. Existing lengthy programs were used to study this kind of scattering. In this paper we developed some mathematical methods in which we hope to get a simple analytic form.

Also we discovered that the non-leading terms are not important in the relative size of the cross-section for this scattering and these terms are ignored.

We compared the analytic approximation form for the cross-section which we obtained with the exact form and we found that there is 96% agreement. We can use this analytic form to study the four jets.

Introduction

Present and future hadron colliders will generate many multi-jet events. It is important to model these theoretically so that the results may be used to test perturbative QCD, and also that one can estimate the conventional QCD background to new physics processes.

Exact QCD matrix elements for $2 -> n$ parton – parton scattering are known for $n \leq 4$ [1-8]. For four jet production the existing expressions are rather lengthy and require large amount of cpu time when used in computer Monte Carlo programs.

By decomposing the n/gluon amplitude into sub-amplitudes weighted by Chan-paton like traces of color matrices, Mangano, Parke and Xu [6], and independently Berends and Giele [7], showed how the calculations could be simplified and how compact expressions could be obtained in terms of spinors for the unsquared sub-amplitudes. The purpose of this paper is to give a compact expression for the sub-amplitudes squared for the six-gluon scattering and also to reduce the number of the poles which are required in the gg − > gggg process. This expression has been evaluated in terms of kinematical invariants and it is useful when coded in a program; it leads to a moderate gain in computer time by a factor of 2.

We shall check the accuracy of the result and confirm that the non-leading terms typically constitute less than 5% of the full result, making this compact expression, leading in the number of color, useful in simulating four jet events.

We shall write the six-gluon scattering matrix element squared, $|M6|^2$, as

$$|M_6|^2 = |M_6^{PT}|^2 + |M_6^{rest}|^2 + \quad \text{Non-leading terms} \tag{1}$$

$|M_6^{PT}|^2$ is the contribution of $(- - + + + +)$, and permuted, helicity orderings which is given by the formula of Parke and Taylor [9].

$$|M_6^{PT}|^2 = g^8 N_c^4 (N_c^2 - 1) \sum_{i<j} (ij)^4 \sum_{P} \frac{1}{(12)(23)(34)(45)(56)(61)} \tag{2}$$

Here (ij) means the dot product of the four-momenta $(P_i\, P_j)$. The matrix elements squared of the expression in (2) are summed over color and helicity averaging factor should be supplied to these expressions. P denotes a sum over the 60 distinct non-cyclic permutations of 1,2,3,4,5,6 up to cyclic and reverse reorderings.

For the remaining helicity configurations we shall write following ref. [6].

$$|M_6^{rest}|^2 = 2g^8 N_c^4 (N_c^2 - 1) \sum_{P_6} \left[\frac{1}{6} H_1(1 \ldots 6) + H_2\ (1 \ldots 6) + \frac{1}{2}\ H_3(1 \ldots 6) \right] \tag{3}$$

H_1, H_2 and H_3 in equation (3) are understood as contributions of the $(+ - + - + -)$, $(+ + - + - -)$ and $(+ + + - - -)$ helicity combinations, respectively. P_6 means a sum over all 720 permutations of the external gluon momenta, (1,2,3,4,5,6). The expressions for H_1 and H_2 are summarized in Table 1.

Table 1. The expressions for H_1 and H_2 in terms of kinematical invariants

Poles	$H_1(123456)$	$H_2(123456)$
T_1^2	$\frac{3}{4} a_1^2 S_{13}^2 S_{46}^2$	$\frac{1}{4} b_1^2 S_{12}^2 S_{56}^2$
T_2^2	0	$\frac{1}{2} b_2^2 S_{24}^2 S_{56}^2$
$T_2 T_3$	$3[(a_4+a_5)^2 + 2a_5 t_{135}^2 S_{34} S_{16} + a_5^2]$	$[(b_4+b_5)^2 + 2t_{124}^2 b_5 S_{16} S_{34} + b_5^2]$
$T_1 T_3$	0	$S_{12}^2[(b_6+b_7)^2 + b_7(3b_7+2b_6)$ $+ t_{124}^2(U_{3546}^2 - 4S_{35}S_{45}S_{46}S_{36})]$
$T_1 S$	$3t_{234} a_1 S_{13} S_{46}(a_6+2a_7)$	$b_1 \beta_1 t_{234} S_{12}^2 S_{56}$
$T_2 S$	0	$b_2 S_{24} S_{56}[\beta_2 t_{345} S_{56} + \beta_3 t_{123}]$
S^2	$[3S_{34}S_{16}((a_6+a_7)(a_8+a_9) +$ $a_7 a_9 + t_{135}^2 S_{13} S_{46} a_{10}) +$ $\frac{3}{2} t_{234} t_{345} S_{13} S_{46} a_1 (a_4+2a_5)$ $-\frac{3}{4} t_{123}^2 a_2 a_3 a_5]$	$\{S_{12}S_{23}S_{56}[\beta_1\beta_3 - t_{124} b_4 u_{3564}$ $-2t_{124}^2 U_{1246} S_{35} S_{34} S_{56}] +$ $S_{12}S_{34}S_{56}S_{16}[\frac{1}{2}\beta_1\beta_2 + t_{124}^2 b_{12}] -$ $\frac{1}{4} t_{123}^2 b_2 b_3 b_5 +$ $\frac{1}{2} t_{234} t_{345} S_{12} S_{56} b_1 \beta_3 +$ $\frac{1}{2} t_{234} b_3 b_7 (S_{12} t_{234} b_1 + 2t_{123}\beta_2)\}$

The poles in Table 1 are defined by the following:

$$T_1 = t_{123} S_{12} S_{23} S_{45} S_{56} \tag{4}$$

$$T_2 = \pi_+ T_1, \; T_3 = \pi_- T_1 \tag{5}$$

$$S = S_{12} S_{23} S_{34} S_{45} S_{56} S_{61} \tag{6}$$

where we assume a convention in which all the particles are on mass - shell.

$$S_{ij} = (P_i + P_j)^2 = 2 (P_i P_j) \tag{7}$$

π_+ and π_- in equation (5) denote permutations of the external momenta according to the following rules

$$\pi_+ :(123456) \rightarrow (234561)$$

$$\pi_+ :(123456) \rightarrow (345612)$$

Our experience suggested the use of the following kinematical quantities:

$$t_{ijk} = S_{ij} + S_{iK} + S_{jK} \tag{8}$$

$$U_{ijKL} = S_{ij} S_{KL} - S_{iK} S_{jL} + S_{jK} S_{iL} \tag{9}$$

$$t_{ijKL} = S_{ij} S_{KL} - S_{jK} S_{iL} \tag{10}$$

For H_1 there are ten kinematical quantities grouped into three triplets which transform cyclically under the permutation π_+. (a_1, a_2, a_3) ; (a_4, a_6, a_8) ; (a_5, a_7, a_9) and a quantity a_{10} which is invariant under π_+.

$$a_1 = 2 (t_{135} t_{123} - S_{13} S_{46}) ; a_2 = \pi_4 a_1 ; a_3 = \pi_+ a_2$$

$$a_4 = - t_{135} (t_{135} t_{234} t_{345} - t_{135} S_{34} S_{16} - t_{345} S_{15} S_{24} - t_{234} S_{35} S_{26})$$

$$a_5 = - S_{15} S_{35} S_{26} S_{24} ; a_6 = \pi_+ a_4 ; a_7 = \pi_+ a_5$$

$$a_8 = \pi_+ a_6 = \pi_+ a_7 ,$$

$$a_{10} = - \frac{1}{2} (t_{1526} t_{2345} + t_{1524} t_{2563} + t_{2315} t_{2465} + t_{2315} S_{26} S_{45}$$

$$+ t_{2465} S_{12} S_{35} + t_{2614} S_{52} S_{53} + t_{4563} S_{12} S_{25} + S_{12} S_{35} S_{24} S_{56})$$

For H_2 there are ten b_i. They can be grouped into pairs which are related to each other by permutation π_r.

$$b_1 = \pi_j a_1 \quad ; \quad b_2 = \pi_i b_1 \quad ; \quad b_3 = \pi_r b_2$$

$$b_4 = \pi_L a_4 \quad ; \quad b_5 = \pi_L a_5.$$

$$b_6 = - t_{124} (t_{123} S_{35} - t_{124} S_{45} + t_{345} S_{56})$$

$$b_7 = S_{12} S_{56} S_{35}$$

$$b_8 = \pi_r b_6 \quad ; \quad b_9 = \pi_r b_7 \quad ;$$

$$b_{12} = \frac{1}{2} (t_{2346} S_{14} S_{35} + t_{1345} S_{24} S_{36} + t_{1634} S_{23} S_{45} + t_{1463} S_{34} S_{52} + t_{1643} S_{35} S_{24} + t_{2615} S_{34}^2 - S_{23} S_{15} S_{46} S_{34} - S_{26} S_{13} S_{34} S_{45} - 2S_{13} S_{46} S_{34} S_{25}).$$

π_j, π_t, π_r and π_L are permutations acting on the external momenta according to the following rules:

$$\pi_j : (123456) \rightarrow (132546)$$

$$\pi_i : (123456) \rightarrow (241356)$$

$$\pi_r : (123456) \rightarrow (654321)$$

$$\pi_L : (123456) \rightarrow (154326)$$

$$\beta_1 = (b_6 + 2b_7) \quad ; \quad \beta_2 = (b_8 + 2b_9)$$

$$\beta_3 = (b_4 + 2b_5).$$

The expression for H_3 is also in a compact form,

$$H_3 (1..6) = t_{123}^3 \quad \frac{(t_{123} S_{34} S_{16} + 2t_{234} S_{45} S_{12})}{t_{234} t_{345} S_{12} S_{23} S_{34} S_{45} S_{56} S_{61}}$$

$$+\frac{(t_{123}t_{234}t_{345}-2t_{234}S_{45}S_{12})^2}{t_{234}^2t_{345}^2S_{34}^2S_{16}}-\frac{4t_{123}^2}{t_{234}t_{345}S_{34}S_{16}}, \tag{11}$$

We have compared our compact expressions to the FORTRAN Code supplied by the authors of ref. [6] and found complete agreement. One detail should be mentioned. Our results for H_1 and H_2 in equation (3) are not identical to the squared matrix element for these helicity combinations. We have exploited the summation over all permutations in equation (3), and permutation invariance of various parts of the expressions, to reduce the number of terms.

It is of interest to compare the compact leading N_c expression with the exact squared matrix element for six – gluon scattering. For this purpose we generated 2 – → 4 events using the phase space generator RAMBO [10], and applied cuts in the parton – parton C.M. to select hard, well – separated jets. The cuts applied were as follows $\sqrt{\hat{S}} = 100\text{GeV}$, $P_T^i > 15\text{GeV}$, $E_T > 70\text{GeV}$, for each pair of jet $\cos\theta_{ij} < .643$ and $|\eta| < .80$ with E_T denoting the transverse energy of the four final jets. These are the cuts used in ref. [11]. We find that the compact leading N_c expression (using equation (3) and Table 1 for H_1 and H_2, and equation (11) for H_3) is within 20% of the exact result for all of the generated events, 93% of generated events are within 10% of the exact result and 47% of generated events with 5%. The ratio of leading N_c to the exact cross-section is .960 *i.e.*

$$\frac{\sigma_{approx}}{\sigma_{exact}} = 0.960\text{ , where}$$

σ_{approx} is our result (short expression),

σ_{exact} is the result of ref. [4] (lengthy expression).

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حســاب التشـــتت لســتة فولونـــات

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(سُلِّمَ في ١ محـرم ١٤١٣هـ؛ وقُبل للنشر في ١٠ ربيع الآخر ١٤١٤هـ)

ملخص البحث. في الوقت الحاضر لا توجد صيغة تحليلية بسيطة لهذا التشتت. لكن هناك بعض البرامج المطوّلة تستخدم لدراسة مثل هذا النوع من التشتت. في هذا البحث طوّرنا بعض الأساليب الرياضية التي نأمل من خلالها الحصول على صيغة تحليلية بسيطة.

أيضًا اكتشفنا أن الحدود الوسطية لا تلعب دورًا مهمًا في المقطع العرضي لهذا التشتت وبالتالي يمكن إهمالها.

قارنـا الصيغـة التحليلية التقريبية للمقطع العرضي التي حصلنا عليها مع المقطع العرضي الكلي فوجدنا أن هناك توافقًا في حدود ٩٦٪ وبالتالي يمكن استخدام هذه الصيغة التحليلية لدراسة ٤ـ جت.