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PHYSICS

Computational Aspects of Cu/Sc Ratio in a Composite Superconductor

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Abstract. In this paper, the optimum copper to superconductor (Cu/Sc) ratio is calculated for composite superconductor operating at 1.9K. The goal is to guarantee the composite superconductor stabilizability via selecting the optimum Cu/Sc ratios associated to the maximum values of the minimum quench energy (MQE). All the calculations as well as the simulation studies are carried out based on the one-dimensional heat conduction formula of Dresner. It has been shown that the optimum values of the Cu/Sc ratio depend on the operating conditions concerning both the current density and the magnetic field. Furthermore, these optimum ratios are found to be feasible and acceptable from the economic view point in the manufacturing of the composite superconductor.

Keywords: Optimum Cu/Sc ratio; Composite superconductor modelling; Guaranteed stabilization; MQE.

Introduction

The present confidence in the reliability of composite superconductors has led to the design and construction of large and complex magnet system [1-3].

However, the stability problem of a superconductor against small fluctuations is an open question. That is because the critical state in a superconductor represents a nonequilibruim configuration. Such a fluctuation may be caused internally (thermally activated flux creep) or externally (change of applied magnetic field or current mechanical movements, ...etc.). The power dissipation associated with the small perturbation can cause a small temperature rise locally. Since the pinning force density usually decreases with increasing temperature, the perturbation may become larger, resulting in an instability and thermal runaway. This can lead to catastrophic consequences, if the magnetic energy stored in a superconducting magnet energy is suddenly converted into thermal energy. Hence, the development of stability criteria becomes highly important for applications of superconductors [1-3].

The minimum quench energy (MQE) is one of the key elements that affects the stability of the composite superconductor. For magnet designers, the main problem is to maximize the MQE, i.e. to make it more difficult for random energy disturbances such as wire movements, flux motion, ...etc. to make quench the magnet. That is to make the magnet more stable against the above discussed perturbations.

Recently, the problem of finding an optimum Cu/Sc ratio in a composite superconductor has received little attention [4-6]. For example, Yoda et. al. [5] have studied the addressed problem. However, their analysis and calculations suffer from strong approximation which limit the range or the applicability of their formulas. In [6], Ishuyamn and Shimizu have presented an optimal design method to calculate the optimum Cu/Sc ratio that minimizes the winding volume of superconductor magnets. Cu/Sc ratio that minimizes the MQE. However, the resulting ratios are not feasible from the economic point of view.

The main objective of this paper is threefold:

- First, link the meaning of the superconductor stability and maximization of the MQE as shown in the above discussion.
- Second, formulate more accurate expressions with less approximations than those given in [5].
- Third, calculate, via simulation study, the optimum Cu/Sc ratio corresponding to the maximum values of the MQE, which is a good indicator of the composite superconductor stability. We have focused on the feasibility of the resulting optimum Cu/Sc ratios for different values of the current density ($J=100 \sim 600$ Amp/mm^2) and background magnetic filed ($B = 2 \sim 10$ Tesla). In order to fulfill these requirements, we move to the next section to formulate the MQE as a nonlinear function of the Cu/Sc ratio.

Problem formulation

Our starting point is the one-dimensional heat conduction equation that describes the winding as an anisostropic continuum [4]:

$$S\frac{\partial T}{\partial t} = K_Z \frac{\partial^2 T}{\partial Z^2} + g(T) \quad (1a)$$

where the definitions and the units of the variables can be found in the appendix at the end of the manuscript, and g(t) is the current-sharing function given by:

$$g(T) = \begin{cases} 0 & T_b < T < T_{CS} \\ (T - T_{CS})/(T_C - T_{CS}) & T_{CS} T < T_C \\ 1 & T > T_C \end{cases} \quad (1b)$$

Dresner [ ] solved this equation and gave the following MQE formula:

$$MQE = \left(\frac{\Pi SA\sqrt{K_Z}\left(T_C - T_{CS}\right)^{1.5}}{\sqrt{\zeta \quad J_C^2}} \right) \left(\frac{1 - I/I_C}{\sqrt{I/I_C}} \right) \left(\frac{T_{CS}}{T_b} \right)^{3.5} \quad (2)$$

where the heat conduction is ignored in the epoxy region inside the superconducting magnet.

It is worth mentioning that the formula (2) is somewhat different from equation (26) given by Dresner [4]. The reason is that we add the term $(Tcs/Tb)^{3.5}$ which serves as one of the correction factors for the Dresner's solution (26) in [4] in order to enhance the Scott's experimental data [10]. This was discussed by Dresner in Table 3 in his paper [4].

Now, if X denotes the Cu/Sc ratio, then we have:

$$A_{CU} = A\frac{X}{1+X} \quad (3)$$

$$I_C = AJ_C\frac{1}{1+X} \quad (4)$$

Using the correlation of Wilson [3] and considering the average heat capacity and thermal conductivity of the composites, S and Kz at 4.2K are expressed as:

$$S = (5400 + 890X)/(1 + X) \quad (5)$$

$$K_Z = K_{CU} X/(1+X) \tag{6}$$

where Kcu is the thermal conductivity of the copper matrix. Further, Tcs is given by [4]:

$$T_{CS} = T_b + (T_C - T_b)/(1 - I/I_C) \tag{7}$$

and Tc and Jc under the background magnetic field B are practically given by [3]:

$$T_C = 9.2(1 - B/14.5)^{0.59} \tag{8}$$

$$J_C = (J_{CO} - 7.99*10^8 B)(1 - T/T_C) \tag{9}$$

where $J_{CO} = 10^{10}\ \mathrm{A/M^2}$ and finally, ρ and Kcu are typically expressed as [3]:

$$\rho = \frac{1.7*10^{-8}}{rrr} + 0.5*10^{-10} B \tag{10}$$

$$K_{CU} = 2.45*10^{-8} \frac{T}{\rho} \tag{11}$$

At this point, it is worth mentioning the following remarks:

(a) Equations (2)-(11) provide the complete solution of the one-dimensional MQE as a nonlinear function of the Cu/Sc ratio X under a given composite current I, composite area A, and background magnetic field B.

(b) The approximation adopted in equations (8)-(11) is valid in the range of (B = 2 ~ 10 T). Therefore, these equations are more accurate than those given in [5]. This is clear from the dependence of Jc in (g) in Jco, B, T and Tc instead of B only in [5]. Also, the dependence of Kcu in (11) on T and ρ_{CU} rather than B in [5].

(c) The satisfaction of condition I ≤ Ic besides equation (4) give the inequality constraint:

$$\mathrm{X} + 1 - J_C \, \mathrm{A}/\mathrm{I} \le 0 \tag{12}$$

This constraint will play an important rule to find the optimal ratio X^* as shown in the following section.

Simulation Results

In this section, we have simulated the system of equations (2)-(12) using the MATLAB package along with its nonlinear toolbox facility [8]. The MQE is plotted against X for the composite superconductor operating at 1.9K for different values of J and B, and the results are depicted in figures (1)-(5). By inspection of these plots besides the constraint given by (12), one can easily obtain the optimum value of Cu/Sc ratio, X^* corresponding to the maximum value of the minimum quench energy, MQE^*. The results are summarized in Table (1).

Table 1. Optimum Ratio X^* and Maximum MQE^* Corresponding to Different Values of J and B

J (Amp./mm^2)	B (Tesla)	X^*	$MQE^* \times 10^{-6}$ (Joules)	Left hand side of (12)
100	2	6.29	733.5	-58.08
200	2	3.95	403	-28.54
300	2	3.06	272.2	-17.69
400	2	2.1	203	-13.27
500	2	1.94	156.5	-10.016
600	2	1.85	122.4	-7.847
100	4	5.16	289.855	-44.879
200	4	3.06	157.971	-21.456
300	4	2.02	102.898	-13.997
400	4	1.953	75.185	-9.807
500	4	1.797	56.296	-7.411
600	4	0.937	42.592	-6.57
100	6	4.68	100.7	-32.326
200	6	3.1	52.76	-14.663
300	6	2.81	34.48	-9.442
400	6	2.74	23.73	-6.332
500	6	1.13	17.27	-5.465
600	6	0.968	13.5	-4.221
100	8	3.87	25.38	-20.117
200	8	2.26	12.82	-9.059
300	8	1.94	7.693	-5.039
400	8	0.97	5.167	-4.029
500	8	0.95	3.71	-2.824
600	8	0.93	2.7	-2.019
100	10	2.91	1.688	-8.821
200	10	0.98	0.724	-2.91
300	10	0.92	0.412	-1.94
400	10	0.91	0.221	-0.955
500	10	0.89	0.089	-0.364
600	10	0	0	-0.97

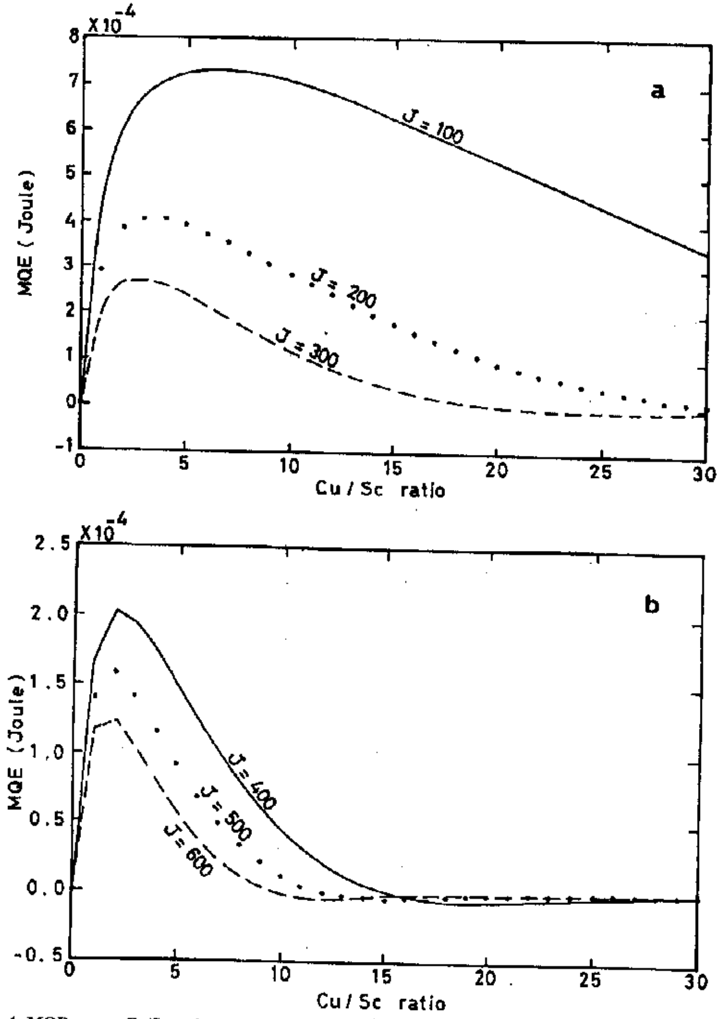

Fig. 1. MQE versus Cu/Sc ratio at B=2T and A=1mm²
(a) J = 100, 200, 300 Amp./mm², (b) J = 400, 500, 600 Amp./mm².

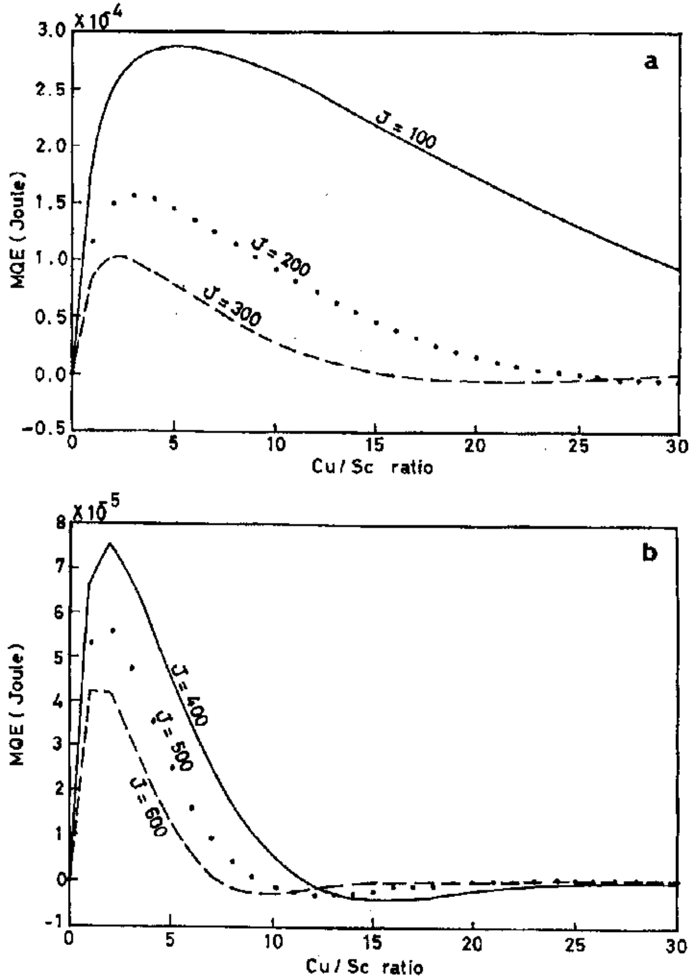

Fig. 2. MQE versus Cu/Sc ratio at b =4T and A = 1 mm².
(a) J = 100, 200, 300 Amp./mm²; (b) J = 400, 500, 600 Amp./mm².

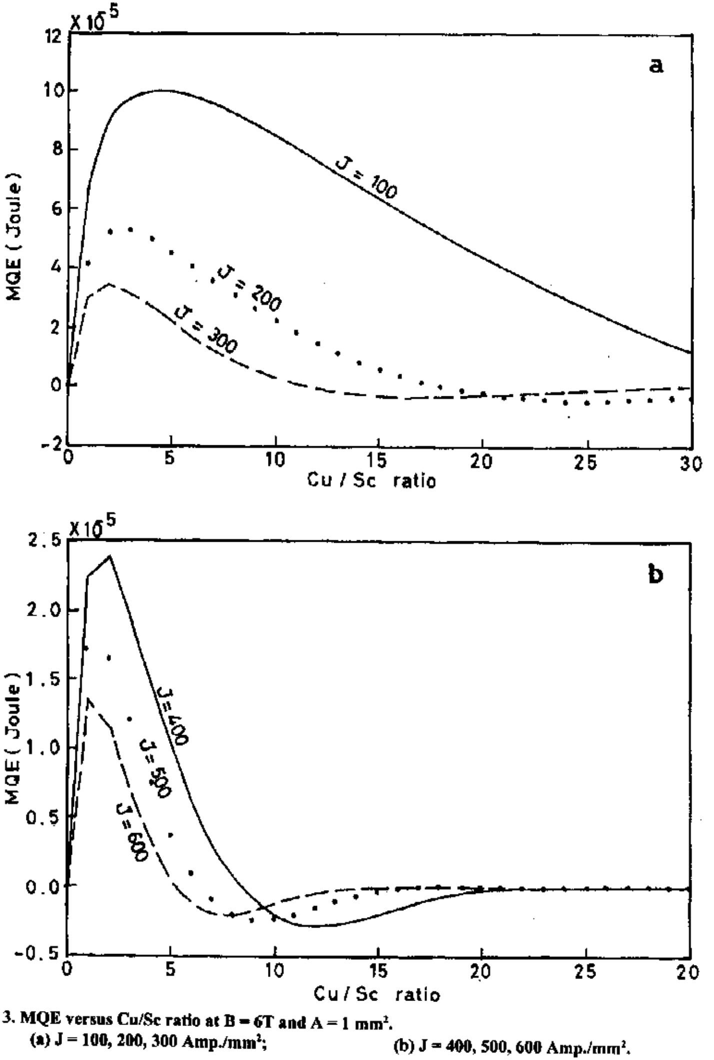

Fig. 3. MQE versus Cu/Sc ratio at B = 6T and A = 1 mm².
(a) J = 100, 200, 300 Amp./mm²; (b) J = 400, 500, 600 Amp./mm².

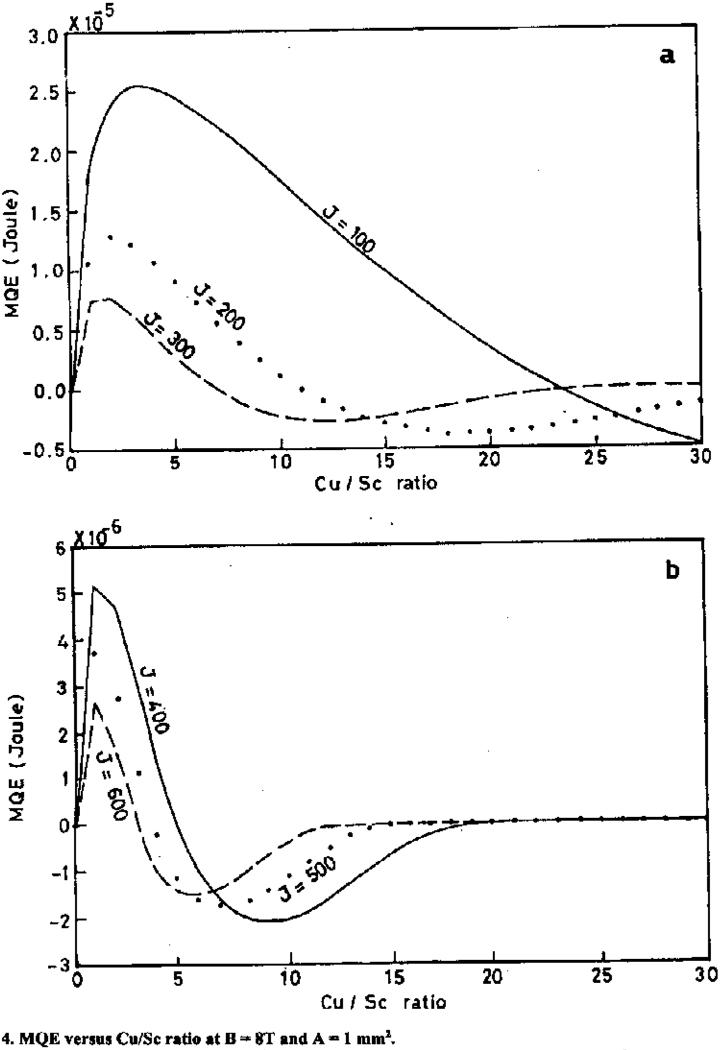

Fig. 4. MQE versus Cu/Sc ratio at B = 8T and A = 1 mm^2.
(a) J = 100, 200, 300 Amp./ mm^2; (b) J = 400, 500, 600 Amp./mm^2.

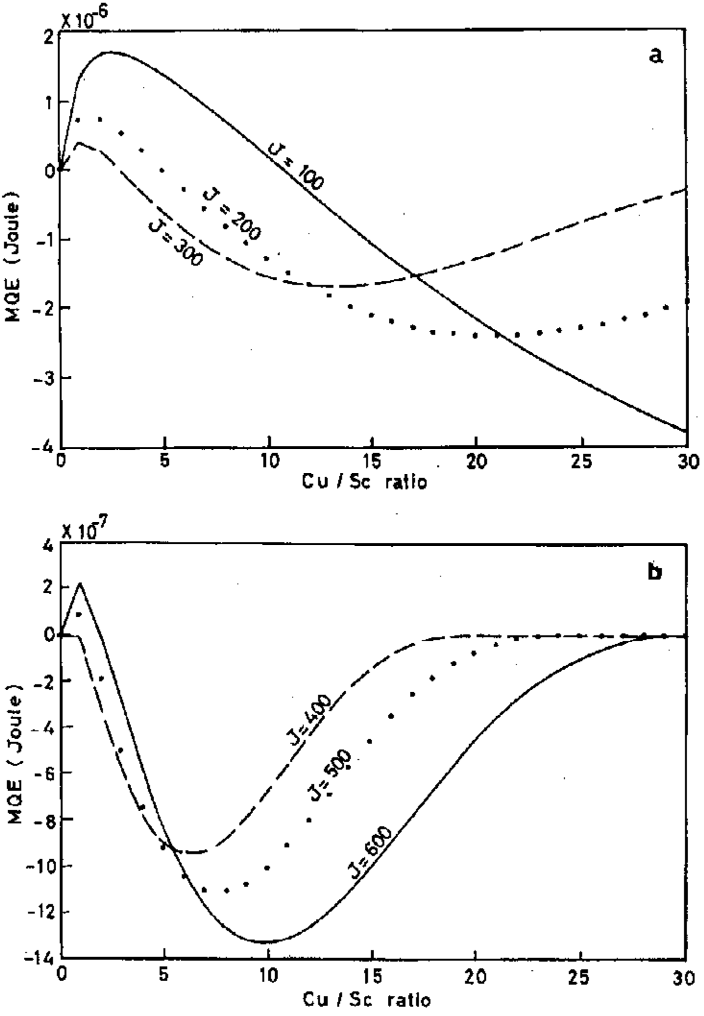

Fig. 5. MQE versus Cu/Sc ratio at b = 10 T and A = 1 mm².
(a) J = 100, 200, 300 Amp./mm²; (b) J = 400, 500, 600 Amp./mm².

From these results, one can easily show that the optimum Cu/Sc ratio (here, $X^*=0\sim6.29$) can be obtained via a proper selection of J and B parameters of the problem at hand. This means that the ratio X depends highly on the composite current density and the background magnetic field. Recalling the discussion adopted in the introduction, the magnet designer should take this result in his mind in order to guarantee the stabilization of the composite superconductor.

On the other hand, from the economic point of view, the resulting values of X^* seem to be well in the design of the composite superconductor. The reason is that the ratio X^* is greater than one in many cases of the simulations and the cost of copper is less expensive than that of preparing the superconductor in the fabrication of the composite superconductor. This point will be stressed in our future research.

Conclusions

(1) The results of numerical calculations and simulation study of the optimum Cu/sc ratio corresponding to the maximum values of the MQE for the composite superconductor are reported.

(2) Correction of many equations in the previous relating research are made. This leads to a wide range of the applicability of both current density and background magnetic field.

(3) A focus has been given for the feasibility of the obtained results in the economic fabrication of the composite superconductor.

(4) Further progress is being under our consideration concerning the poi t (3).

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APPENDIX

List of parameters and their definitions

$A[m^2]$	Area of superconductor composite.
$A_{Cu}[m^2]$	Area of the copper matrix.
$A_{Sc}[m^2]$	Area of the superconductor filaments.
B[T]	Background magnetic field.
g[T]	Current-sharing function defined by equation (1b).
I[A]	Operating current.
$I_c[A]$	Critical current of the superconducting composite.
$J[A/m^2]$	Current density in the superconducting composite
$J_c[A/m^2]$	Critical current density in the superconducting composite
$K_{cu}[W/mK]$	Longitudinal thermal conductivity of the copper matrix at the bath temperature.
$K_z[W/mK]$	Longitudinal thermal conductivity of the composite at the bath temperature.
MQE [J]	Minimum quench energy.
$S[J/m^3K]$	Volumetric heat capacity of the composite at the bath temperature.
T[K]	Operating temperature.
$T_b[K]$	Bath temperature (4.2K).
$T_c[K]$	Critical temperature of the superconducting composite.
$T_{cs}[K]$	Current-sharing threshold temperature.
X	Copper to superconductor ratio $[=A_{cu}/A_{sc}]$.
$\rho\,[\Omega m]$	Normal resistivity of the superconducting composite.
λ	Volume fraction of the composite superconductor in the winding.
rrr	Ratio of copper resistivity at 300K and 4.2K (rrr = 100 in the simulation).

حساب القيمة المثلى لنسبة النحاس داخل المواد فائقة التوصيل

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ملخص البحث. تم في هذا البحث حساب النسبة المثلى للنحاس في المواد فائقة التوصيل عند درجة حرارة ١,٩ كلفن مع ضمان استقرار النسبة Cu/Sc عند الحصول على أقل قيمة أطفأ للطاقة (MQE).

اعتمدت كل الحسابات والرسومات على معادلة أبعاد الحرارة المحاثية وتوصلنا إلى أن القيمة المثلى لنسبة النحاس في المواد فائقة التوصيل تعتمد على كثافة التيار والمجال المغناطيسي اللذان يمران بالعينة.