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Second Order Eigensensitivities of Singularly Perturbed Systems

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Abstract. In this paper, second order eigenvalue sensitivities are derived with respect to the singular perturbation parameter whose variation changes the reduced system order. These sensitivities are used to identity boundaries between fast and slow subsystems. This is applicable to systems that possess two-time scale property. Accurate estimates for the system modes without solving the eigenproblem of the whole system are obtained.

List of Symbols

x, z, x_d, z_d, χ	$\triangleq$	System states.
U	$\triangleq$	System input.
ε	$\triangleq$	Singular perturbation parameter.
n_s, n_f	$\triangleq$	Subsystems order.
$R(\varepsilon), D(\varepsilon)$	$\triangleq$	Diagonal form of slow and fast subsystems.
λ_i, μ_j	$\triangleq$	Eigenvalues of slow and fast subsystems.
u_i, v_i	$\triangleq$	Eigenvectors of $R(\varepsilon)$ and $R^T(\varepsilon)$.
q_j, p_j	$\triangleq$	Eigenvectors of $D(\varepsilon)$ and $D^T(\varepsilon)$.
$A, A_1, B, C, D, D_1, A, B$	$\triangleq$	Matrices of appropriate dimensions.
$\hat{\lambda}_i$	$\triangleq$	Estimated eigenvalues due to a change in ε.
$e'_q, e'_d, \delta, \omega$	$\triangleq$	Machine states.
R_j, E_{fd}, V_R	$\triangleq$	Automatic voltage regulator states.
V, S	$\triangleq$	Voltage and complex power.
H, D	$\triangleq$	Machine inertia and damping constants.

X_d, X_q, X'	$\triangleq$	Machine and transformer reactances.
X_e	$\triangleq$	Transmission line reactance.
T'_{d0}, T'_{q0}	$\triangleq$	Transient time constants in d and q axes.
d, q	$\triangleq$	Direct and quadrature axes.
A_k, C_k, D_k	$\triangleq$	Determined matrices for states-space description.
A_{kj}, C_{kj}, D_{kj}	$\triangleq$	Determined matrices for states-space description.
L_k, H_{kj}	$\triangleq$	Determined coupling matrices.
ξ	$\triangleq$	System parameter.
A^{-1}	$\triangleq$	Inverse of matrix A.

Introduction

Eigenvalue and Eigenvector sensitivities have been for the last two decades, among the most common tools in circuit and system analysis [1]. They are used to determine the influence of system components and parameters on individual system modes. The study in Reference [3] demonstrated successfully the advantages of employing first order eigensensitivity with respect to a parameter ε whose variation changes the system order. Using the expressions derived in Reference [3], it was possible to analyze modes sensitivities with respect to neglected parasitics and to the fast modes eliminated from a reduced order model. While this was a good achievement, yet misleading results in the case of the turning points may result. Moreover, most of the time, the value of ε is not usually known explicitly and this results in wrong estimates for the boundaries between the slow and fast subsystems.

This paper extends the notion of sensitivities to include second order sensitivity terms in addition to the first ones. Consequently, these sensitivities are used to determine the boundary level between the slow and the fast modes without solving the eigenproblem of the whole system. Moreover, Taylor's expansion around a base value of ε including the second terms are used to obtain better estimates for the slow and the fast subsystems modes. The method is demonstrated by considering small power system as an illustrative example [3].

Second Order Eigensensitivities

It is shown in [4] that linear system which possesses the two time scale property can be written in the form

$$\dot{x}_d = R(\varepsilon)x_d \tag{1}$$

$$\varepsilon \dot{z}_d = D(\varepsilon) z_d \tag{2}$$

where $x_d \in R^{ns}$ and $z_d \in R^{nf}$ represent the slow and fast subsystem states respectively, ε is a small positive scalar. Expressions for *R* and *D* are derived in Appendix A to facilitate the derivation of the second order eigenvalue sensitivities of the singular perturbed system given by equations (1) and (2). Expressions for 1st order eigensensitivities had been derived in Reference [3]. These expressions are used to derive the second order sensitivities as shown in Appendix B.

Eigensensitivities for the slow subsystem can be stated as

$$\frac{\partial \lambda_i}{\partial \varepsilon} = -\lambda_i \upsilon_i^T B D_1^{-1} D^{-1} C u_i \tag{3}$$

$$\frac{\partial u_i}{\partial \varepsilon} = \sum_{k=1}^{n_s} \phi_{ik} u_k, \quad k \neq i \tag{4}$$

$$\frac{\partial^2 \lambda_i}{\partial \varepsilon^2} = -2\lambda_i \upsilon_i^T A_1 B D_1{}^2 D^{-1} C u_i - 2\upsilon_i^T B D_1^{-1} D^{-1} C A_1 \frac{\partial u_i}{\partial \varepsilon} \tag{5}$$

Also eigensensitivities for the fast subsystem may be written as

$$\frac{\partial \mu_j}{\partial \varepsilon} = \frac{1}{\mu_j} p_j^T C B q_j \tag{6}$$

$$\frac{\partial q_j}{\partial \varepsilon} = \sum_{k=1}^{n_f} \psi_{jk} q_k, \quad k \neq j \tag{7}$$

$$\frac{\partial^2 \mu_j}{\partial \varepsilon^2} = \frac{2}{\mu_j} p_j^T D^{-1} C A_1 B q_j + \frac{2}{\mu_j} p_j^T C B \frac{\partial q_j}{\partial \varepsilon} \tag{8}$$

where

$$\phi_{ik} = \frac{\lambda_i}{\lambda_k - \lambda_i} \upsilon_k^T B D_1^{-1} D^{-1} C u_j, \quad \phi_{ii} = 0$$

$$\psi_{jk} = \frac{-1}{\mu_k(\mu_k - \mu_j)} p_k^T C B q_j \quad \psi_{jj} = 0$$

An estimate of a specific eigenvalue with respect to a change in the parameter ε can be obtained using Taylor's series expansion around a nominal value of ε (ε = 0). This value represents the quasi-steady state representation of the system.

For example, if the base value for the mode is λ_{i0}, the estimated value $\hat{\lambda}_i$ due to a change in $\varepsilon = 0$ can be stated as

$$\hat{\lambda}_i = \lambda_{i0} + \varepsilon \left.\frac{\partial \lambda_i}{\partial \varepsilon}\right|_{\varepsilon=0} + \frac{\varepsilon^2}{2} \cdot \left.\frac{\partial^2 \lambda_i}{\partial \varepsilon^2}\right|_{\varepsilon=0} + 0\left(\varepsilon^{n \geq 3}\right) \tag{9}$$

where $0(\varepsilon^{n \geq 3})$ are *neglected trerms* of order $\varepsilon^{n \geq 3}$.

The improvements in the estimates can be measured by calculating the error involved by comparison with the exact modes values. This requires calculating the whole eigenpattern of the large system which is computationaly cumbersome. However, it can be avoided by calculating the error with respect to quasi-steady state estimates. This is mainly useful in the process of determining the appropriate partitioning boundaries as it will be demonstrated in an illustrative example.

Single Machine-Infinite Bus System

The single machine-infinite bus system, described in Reference [5], is shown in Fig. 1 and used as an illustrative example. The data of the system is shown in Table 1. The synchronous machine is modeled by the power angle δ, the machine speed ω,

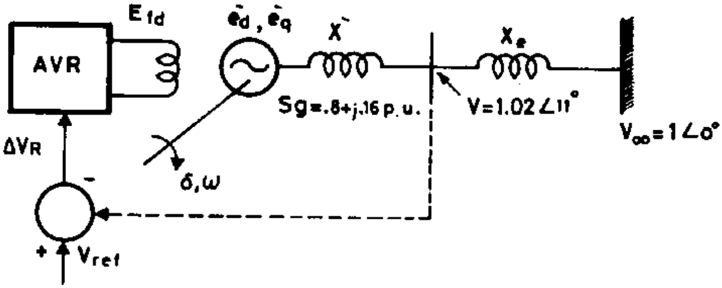

Fig. 1. Single machine-infinite bus system

Table 1. Synchronous machine data

H	=	5.0 sec	X'	=	0.25 pu
D	=	2.0 pu	X_e	=	0.25 pu
X_d	=	1.2 pu	T'_{d0}	=	5.0 sec
X_q	=	1.0 pu	T''_{q0}	=	0.5 sec

and the voltages induced by the flux linkages in the direct and the quadrature axes e'_d, e'_q, respectively. The automatic voltage regulator is modeled by the feedback signal R_f, the exciter output E_{fd}, and the amplifier output V_R.

The system states are e'_q, R_f, e'_d, δ, ω, E_{fd}, and V_R.

The system linearized model is written as

$$\dot{\chi} = A\chi + BU \tag{10}$$

where χ and U are the system states and input variables respectively. The system matrix A is calculated in Reference [5] as

$$\begin{bmatrix} A & B \\ \frac{C}{\varepsilon} & \frac{D}{\varepsilon} \end{bmatrix} = \begin{bmatrix} -0.58 & 0.0 & 0.0 & -0.269 & 0.0 & 0.2 & 0.0 \\ 0.0 & -1.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 5.0 & 2.12 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 377 & 0.0 & 0.0 \\ -0.141 & 0.0 & 0.141 & -0.2 & -0.28 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.084 & 2.0 \\ -173 & 66.7 & -116 & 40.9 & 0.0 & -66.7 & -16.7 \end{bmatrix}$$

Table 2 shows the exact and quasi-steady state (q.s.s.) system eigenvalues solutions by considering e'_q and R_f as slow states. These are given in columns 1 and 2 while column 3 shows the involved *error*. The error is of the order of 7%. The percentage error is considered as

Table 2. Qusi-steady state eigenvalues ($n_s = 2$)

Modes	Eigenvalues		Percentage errors
	Exact	q.s.s.	
Slow	$-0.362 \pm j0.557$	$-0.399 \pm j0.580$	6.577
Fast	-3.939	-3.723	5.491
	$-0.859 \pm j8.379$	$-0.779 \pm j8.390$	0.957
	$-8.548 \pm j8.204$	$-8.308 \pm j7.939$	3.038

$$\text{Error} = \left| \frac{\lambda_i(\text{Exact}) - \hat{\lambda}(\text{Estimated})}{\lambda_i(\text{Exact})} \right| \times 100 \tag{11}$$

where $\hat{\lambda}_i$ (Estimated) is obtained using equation (9).

The first and second order eigenvalue sensitivities of both the slow and fast modes have been calculated using equations (3) to (8). These sensitivities are given in Table 3 and Table 4.

Table 3. Modes Estimates Using 1st Eigensensitivities ($n_s = 2$)

Modes	Eigenvalues		Sens. and percentage errors	
	Exact	**Estimated #1**	**Sensitivity # 1**	**Error # 1**
Slow	$-0.362 \pm j0.557$	$-0.359 \pm j0.559$	$0.040 \mp j0.021$	0.705
Fast	-3.939	-3.951	-0.229	0.314
	$-0.859 \pm j8.379$	$-0.851 \pm j8.384$	$-0.072 \mp j0.006$	0.109
	$-8.548 \pm j8.204$	$-8.513 \pm j8.217$	$-0.205 \pm j0.282$	0.315

Table 4. Modes Estimates Using 2nd Eigensensitivities ($n_s = 2$)

Modes	Eigenvalues		Sens. and percentage errors	
	Exact	**Estimated #2**	**Sensitivity # 2**	**Error # 2**
Slow	$-0.362 \pm j0.557$	$-0.362 \pm j0.556$	$0.003 \mp j0.004$	0.129
Fast	-3.939	3.949	-0.002	0.271
	$-0.859 \pm j8.379$	$-0.859 \pm j8.380$	$-0.008 \mp j0.004$	0.011
	$-8.548 \pm j8.204$	$-8.549 \pm j8.206$	$-0.037 \mp j0.011$	0.022

The second order sensitivities are much smaller than the first order sensitivity which implies that ε is very small and that a good choice for the separation boundaries has been considered. These sensitivities are used in the Taylor's formula to obtain estimates for the system modes. The errors involved in these estimates are given in the last column of Table 3 and Table 4. The error has been reduced substantially from that of the q.s.s. solution. Moreover, the tables show improvement in the estimates using the 2nd order terms. The reduction in the 2nd order sensitivity compared to the 1st order and the improvement in their estimates compared to the q.s.s. are mainly due to the appropriate grouping in the slow and fast states. This is clear since ε is approximately equal to 0.17 (for $n_s = 2$). ε is estimated as the max $|\lambda_s|$ / min $|\lambda_f|$. Moreover, $\varepsilon = 0.17$ is the minimum of $|\lambda_i|/|\lambda_{i+1}|\ \forall_i$, which indicates the largest gap between the system dynamics. The errors involved in the 1st and the 2nd order estimates calculated with respect to the q.s.s. solution are listed in Table 5. Examining Table 5 shows the 2nd order errors are very close to the 1st order errors for such a healthy partitioned case [6].

Table 5. Sensitivities Estimates errors referred to q.s.s. solution

Modes	q.s.s. eigenvalues	Percentage errors	
		1st order	2nd order
Slow	$-0.399 \pm j0.580$	6.434	6.322
	3.723	6.142	6.097
Fast	$-0.779 + j8.390$	0.858	0.960
	$-8.308 \pm j7.936$	3.030	3.157

To indicate the usefulness of the 2nd order term in determining better modes estimates, the partitioning boundary has been moved such that the slow subsystem is of the third order. In this case, e_q', R_f, and e_d' describe the slow system dynamics. Table 6 shows the errors obtained by considering the q.s.s. solution and those obtained using Taylor's expansion with the 1st and 2nd order terms included.

Table 6. Errors in system modes estimates ($n_s = 3$)

Modes	Exact eigenvalues	Percentage errors		
		q.s.s.	1st order	2nd order
Slow	$-0.362 \pm j0.557$	5.122	0.132	0.017
	-3.939	5.952	5.330	0.087
Fast	$-0.859 \pm j8.379$	9.258	3.582	0.343
	$-8.548 \pm j8.204$	3.039	0.238	0.042

The results shown indicate the importance of evaluating the 2nd order terms which reduces the overall errors compared to both the q.s.s. and 1st order errors estimates.

We have indicated the importance of the 1st and the 2nd order eigensensitivities in obtaining better modes estimates. It is remained to show the role of these sensitivities in the partitioning boundary decisions. The states are ordered intentionally wrong in such a way to give the sequence e'_q, R_f, δ, ω, e'_d, E_{fd}, and V_R. This may be used to construct plant controller for stability purpose. In this case, e'_q, R_f, δ, ω are considered as slow states, which leads to a wrong partitioning. Table 7 shows the eigensensitivities solution with respect to the parameter ε ($\varepsilon = 2.138$). The value of the 2nd order terms for some modes in the slow and fast subsystems increases largely compared to the 1st order terms. This indicates that the inherent value of ε is large and the choice of the partitioning boundary is not a good one. This is a good achievement since a decision can be made using these sensitivities regarding the states sequence and the partitioning boundaries before proceeding in performing more computations.

Moreover, this decision can be guessed without calculating the exact eigenvalues, for the large system. Instead of calculating the errors with respect to the exact eigenvalues, the q.s.s. estimates can be used as a reference.

Table 7. 1st and 2nd order Eigensensitivities ($n_s = 4$)

Modes	Exact eigenvalues	Eigensensitivities 1st order	Eigensensitivities 2nd order
Slow	$-0.362 \pm j0.557$	$0.042 \mp j0.020$	$-0.005 \mp j0.006$
	$0.859 \pm j8.379$	$-2.268 \pm j0.118$	$0.125 \pm j2.944$
Fast	-3.939	0.000	4.508
	$-8.548 \pm j8.204$	$-0.243 \pm j0.255$	$-0.030 \mp j0.006$

Table 8 shows the errors in the mode estimates of the proceeding case ($n_s = 4$) using sensitivities as compared to the q.s.s. solution. The errors involved in the 2nd order estimates are very large compared as to the 1st order estimates errors. This concludes the same previous remarks regarding the state sequence and partitioning boundaries.

Table 8. Sensitivities Estimates errors referred to q.s.s. solution.

Modes	q.s.s eigenvalues	Percentage errors 1st order	2nd order
Slow	$-0.399 \pm j0.581$	6.610	6.474
	$0.288 \pm j7.256$	31.274	51.479
Fast	-5.000	0.000	90.154
	$-8.308 \pm j7.936$	3.064	3.209

Conclusion

Second order eigenvalue sensitivities with respect to the singular perturbation parameter ε are derived. The usefulness of these sensitivities in determining better modes estimates is highlighted. It has been shown that the relative value of the *second* to *first order sensitivities* can be used as a test to obtain *appropriate* boundaries between *slow* and *fast subsystems*.

Equations (40) and (44) represent the actual *second-order* eigensensitivities of the system eigenvalues which remains finite as $\varepsilon \to 0$.

Appendix A

Consider the linear, time-invariant system with $u = 0$ (*free response*) for simplicity

$$\dot{x} = Ax + Bz, \quad x(0) = x_0 \tag{12}$$

$$\varepsilon \dot{z} = Cx + Dz, \quad z(0) = z_0 \tag{13}$$

where x and z are vectors of order n_s and n_f respectively. ε is the ratio of *slow* to *fast* system modes dynamics. It is known as the singular perturbation parameter and can be written as

$$\varepsilon = \left|\lambda_{n_s}\right| / \left|\lambda_{n_{s+1}}\right| \tag{14}$$

Applying the iterative scheme for separation of time scale [4], the system equations (12) and (13) after k^{th} L-iterations and j^{th} H-iterations has the form of equations (1) and (2), where the matrices are now defined as

$$R(\varepsilon)=A_{kj} \tag{15}$$

$$D(\varepsilon)=D_k \tag{16}$$

where

$$A_k=A-BL_k \tag{17}$$

$$C_k=C-DL_k+\varepsilon L_k A_k \tag{18}$$

$$D_k=D+\varepsilon L_k B \tag{19}$$

$$L_{k+1}=D^{-1}C+\varepsilon D^{-1}L_k\left(A-BL_k\right),\quad L_i=D^{-1}C \tag{20}$$

$$A_{kj}=A_k-H_{kj}C_k \tag{21}$$

$$B_{kj}=B-H_{kj}D_k+\varepsilon A_{kj}H_{kj} \tag{22}$$

$$D_{kj}=D_k+\varepsilon C_k H_{kj} \tag{23}$$

$$H_{kj+1}=BD_k^{-1}+\varepsilon\left(A_k-H_{kj}C_k\right)H_{kj}D_k^{-1},\quad H_{kl}=BD_k^{-1} \tag{24}$$

For Simplicity, the block diagonal form of the fast subsystem $D(\varepsilon)$ may have terms or order $\varepsilon^{n\leqslant 2}$ [7,8], which can be obtained by evaluating equation (19) for k = 2 to give

$$L_2=D^{-1}C+\varepsilon D^{-2}CA_1 \tag{25}$$

$$D(\varepsilon)=D_2+0\left(\varepsilon^{n\geq 3}\right) \tag{26}$$

$$D(\varepsilon)=D+\varepsilon D^{-1}CB+\varepsilon^2 D^{-2}CA_1B+0\left(\varepsilon^{n\geq 3}\right) \tag{27}$$

where $0(\varepsilon^{n\leqslant 3})$ are neglected terms of order $\varepsilon^{n\leqslant 3}$.

Consequently, for adequate *accuracy* of separating the *fast variables;* while some *inaccuracy* is involved in the *slow variables* (which can be tolerated by requiring more H-*iterations* that L-*iterations*), the block diagonal form of the *slow subsystem* $R(\varepsilon)$ should be obtained for k = 1 and j = 2 in equation (21) to have terms of order $\varepsilon^{n\leqslant 2}$ [7,8].

Evaluating equation (21) for $k = 1$ and $j = 2$ yields

$$H_{12}=BD_1^{-1}+\varepsilon A_1BD_1^{-2}-\varepsilon BD_1^{-1}C_1BD_1^{-2} \tag{28}$$

$$R(\varepsilon)=A_{12}+0\left(\varepsilon^{n\geq 3}\right) \tag{29}$$

$$R(\varepsilon)=A_1-\varepsilon BD_1^{-1}D^{-1}CA_1-\varepsilon^2 A_1BD_1^{-2}D^{-1}CA_1+0\left(\varepsilon^{n\geq 3}\right) \tag{30}$$

where $0(\varepsilon^{n\leq 3})$ are neglected terms of order $\varepsilon^{n\leq 3}$.

Appendix B

In general, the *eigenproblem* solution of the linear multi-variable system described by equation (10) for $\mathbf{u} = 0$ is

$$Au_i = \lambda_i \upsilon_i, \tag{31}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the *eigenvalues* of A and $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n$ are the corresponding *eigenvectors*. Let $v_1, v_2, \ldots, v_n$ are the corresponding eigenvectors of the transpose of the A matrix. The folowing equations hold:

$$A^T \upsilon_i = \lambda_i \upsilon_i, \tag{32}$$

$$\upsilon_i^T A = \lambda_i \upsilon_i^T \tag{33}$$

Derivation of the *second-order eigenvalue sensitivity* proceeds [2] by differentiating equation (31) two times with respect to a small parameter ξ which leads to

$$\frac{\partial^2 A}{\partial \xi^2} u_i + 2\frac{\partial A}{\partial \xi}\cdot\frac{\partial u_i}{\partial \xi} + A\frac{\partial^2 u_i}{\partial \xi^2} = \frac{\partial^2 \lambda_i}{\partial \xi^2} u_i + 2\frac{\partial \lambda_i}{\partial \xi}\cdot\frac{\partial u_i}{\partial \xi} + \lambda_i \frac{\partial^2 u_i}{\partial \xi^2} \tag{34}$$

Premultiplication of equation (34) by υ_i^T and substituting from equation (33) gives

$$\frac{\partial^2 \lambda_i}{\partial \xi^2}\upsilon_i^T u_i = \upsilon_i^T \frac{\partial^2 A}{\partial \xi^2} u_i + 2\upsilon_i^T \frac{\partial A}{\partial \xi}\cdot\frac{\partial \upsilon_i}{\partial \xi} - 2\frac{\partial \lambda_i}{\partial \xi}\upsilon_i^T\cdot\frac{\partial u_i}{\partial \xi} \tag{35}$$

By considering the fact that $\upsilon_i^T u_i = 1$, so $\upsilon_i^T \frac{\partial u}{\partial \xi}$ equals *zero*. The above equation (35) reduces to

$$\frac{\partial^2 \lambda_i}{\partial \xi^2} = \upsilon_i^T \frac{\partial^2 A}{\partial \xi^2} u_i + 2\upsilon_i^T \frac{\partial A}{\partial \xi}\cdot\frac{\partial u_i}{\partial \xi} \tag{36}$$

which is in general; the *second-order eigenvalue sensitivity*.

Derivation of the *second-order eigenvalue sensitivity* of the slow subsystem proceeds by evaluating $\frac{\partial^2 R}{\partial \varepsilon^2}$ and $\frac{\partial R}{\partial \varepsilon}$ for $\varepsilon = 0$ from equation (30) and then substituting in equation (36) to give

$$\frac{\partial R(\varepsilon)}{\partial \varepsilon}\Big|_{\varepsilon=0} = -BD_1^{-1}D^{-1}CA_1 \tag{37}$$

$$\frac{\partial^2 R(\varepsilon)}{\partial \varepsilon^2}\Big|_{\varepsilon=0} = -2A_1BD_1^{-2}D^{-1}CA_1 \tag{38}$$

$$\frac{\partial^2\lambda_i}{\partial\varepsilon^2} = -2\upsilon_i^T A_1 B D_1^{-2} D^{-1} C A_1 u_i - 2\upsilon_i^T B D_1^{-1} D^{-1} C A_1 \frac{\partial u_i}{\partial\varepsilon} \tag{39}$$

Replacing $A_1\, u_i$ by $\lambda_i u_i$ in equation (39) and divide by two yields to

$$\frac{1}{2}\frac{\partial^2\lambda_i}{\partial\varepsilon^2} = -\lambda_i \upsilon_i^T A_1 B D_1^{-2} D^{-1} C u_1 - \upsilon_i^T B D_1^{-1} D^{-1} C A_1 \frac{\partial u_1}{\partial\varepsilon} \tag{40}$$

Analogously, the *second-order eigenvalue sensitivity* of the fast subsystem can be obtained by evaluating $\frac{\partial^2 D}{\partial\varepsilon^2}$ and $\frac{\partial D}{\partial\varepsilon}$ for $\varepsilon = 0$ from equation (30) and then substituting in equation (36) to get

$$\frac{\partial D(\varepsilon)}{\partial\varepsilon}\Big|_{\varepsilon=0} = D^{-1}CB \tag{14}$$

$$\frac{\partial^2 D(\varepsilon)}{\partial\varepsilon^2}\Big|_{\varepsilon=0} = 2D^{-2}CA_1B \tag{42}$$

$$\frac{\partial^2\mu_j}{\partial\varepsilon^2} = 2p_j^T D^{-2} C A_1 B_{qj} + 2p_j^T D^{-1} C B \frac{\partial q_j}{\partial\varepsilon} \tag{43}$$

Replacing $p_j^T D^{-1}$ by $\mu_j^{-1} p_j^T$ in equation (43) and divide by two yields to

$$\frac{1}{2}\frac{\partial^2\mu_j}{\partial\varepsilon^2} = \frac{1}{\mu_j^2} p_i^T C A_1 B q_j + \frac{1}{\mu_j} p_j^T C B \frac{\partial q_j}{\partial\varepsilon} \tag{44}$$

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الحساسية الثنائية لأطوار أنظمة الاضطراب المفرد

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ملخص البحث. تم في هذا البحث استنباط معادلة للحساسية الثنائية لأطوار أنظمة الاضطراب المفرد بالنسبة لمعامل الاضطراب الذي يؤثر في حجم النماذج المصغَّرة للنظام.

ولقد استخدمت هذه الحساسية لتعيين الحدود بين الأجزاء السريعة والبطيئة المُكَوِّنة للنظام وخاصة للأنظمة ذات خاصية تنوع مقياس الزمن. وبهذه الطريقة أمكن إيجاد تقديرات دقيقة لأطوار النظام من غير اللجوء للحل الكامل للنظام الكبير.